Computer Algebra Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.3-Tangent/107-4.3.9-trig^m-a+b-tan^n+c-tan^-2-n-^p

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September 5, 2023 Compiled on September 5, 2023 at 8:51pm

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [51]. This is test number [107].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (51)	0.00 (0)
Mathematica	100.00 (51)	0.00 (0)
Fricas	82.35 (42)	17.65 (9)
Maple	64.71 (33)	35.29 (18)
Mupad	0.00 (0)	100.00 (51)
Giac	0.00 (0)	100.00 (51)
Maxima	0.00 (0)	100.00 (51)
Sympy	0.00 (0)	100.00 (51)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

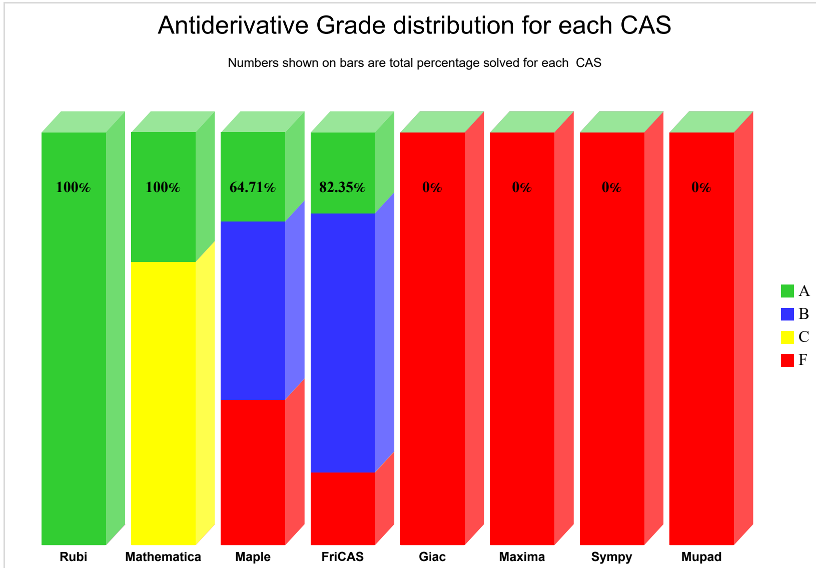
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

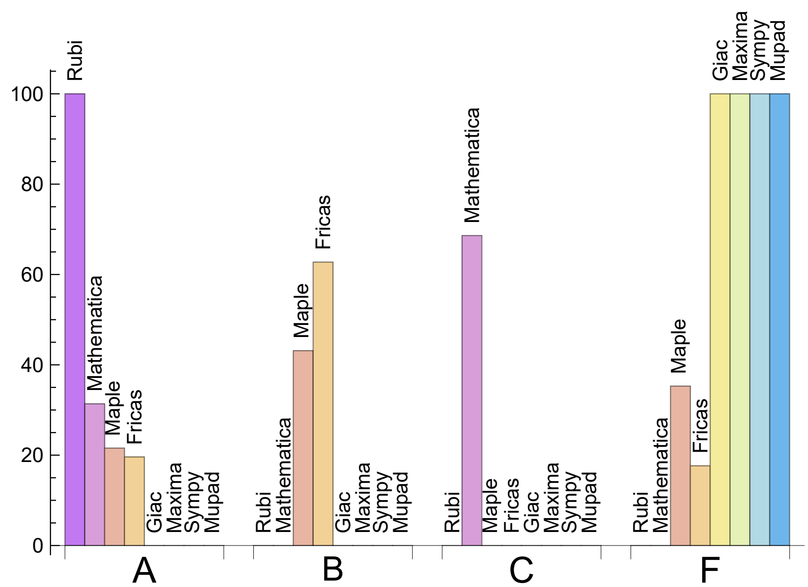
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	31.373	0.000	68.627	0.000
Maple	21.569	43.137	0.000	35.294
Fricas	19.608	62.745	0.000	17.647
Giac	0.000	0.000	0.000	100.000
Mupad	0.000	0.000	0.000	100.000
Maxima	0.000	0.000	0.000	100.000
Sympy	0.000	0.000	0.000	100.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	9	33.33	66.67	0.00
Maple	18	50.00	50.00	0.00
Mupad	51	0.00	100.00	0.00
Giac	51	33.33	66.67	0.00
Maxima	51	66.67	15.69	17.65
Sympy	51	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maple	1.08
Fricas	2.57
Rubi	5.57
Mathematica	5.68
Sympy	-nan(ind)
Maxima	-nan(ind)
Giac	-nan(ind)
Mupad	-nan(ind)

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	488.61	0.88	283.00	0.71
Rubi	533.49	1.00	500.00	1.00
Fricas	13607.79	22.23	4957.00	13.89
Maple	6665599.03	11091.65	7300729.00	10981.87
Sympy	-nan(ind)	-nan(ind)	nan	nan
Maxima	-nan(ind)	-nan(ind)	nan	nan
Giac	-nan(ind)	-nan(ind)	nan	nan
Mupad	-nan(ind)	-nan(ind)	nan	nan

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

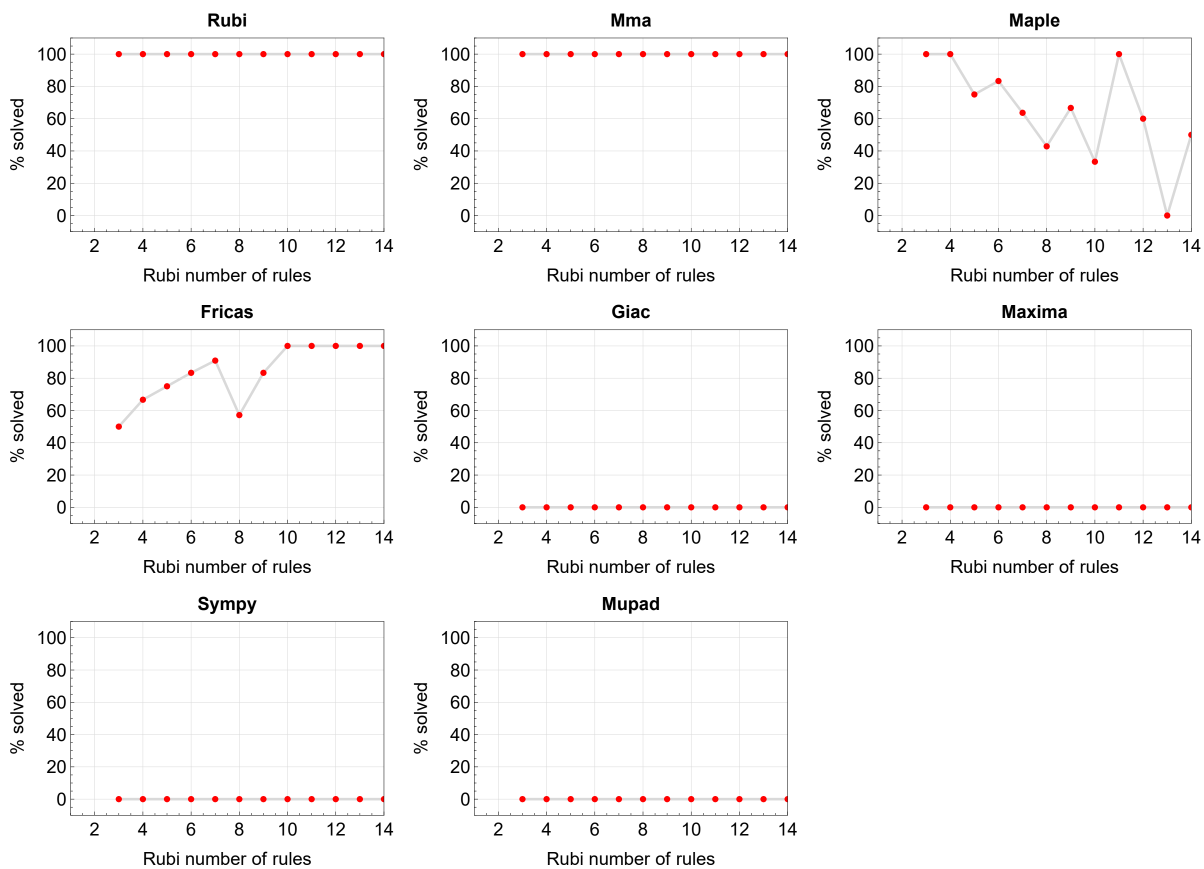


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

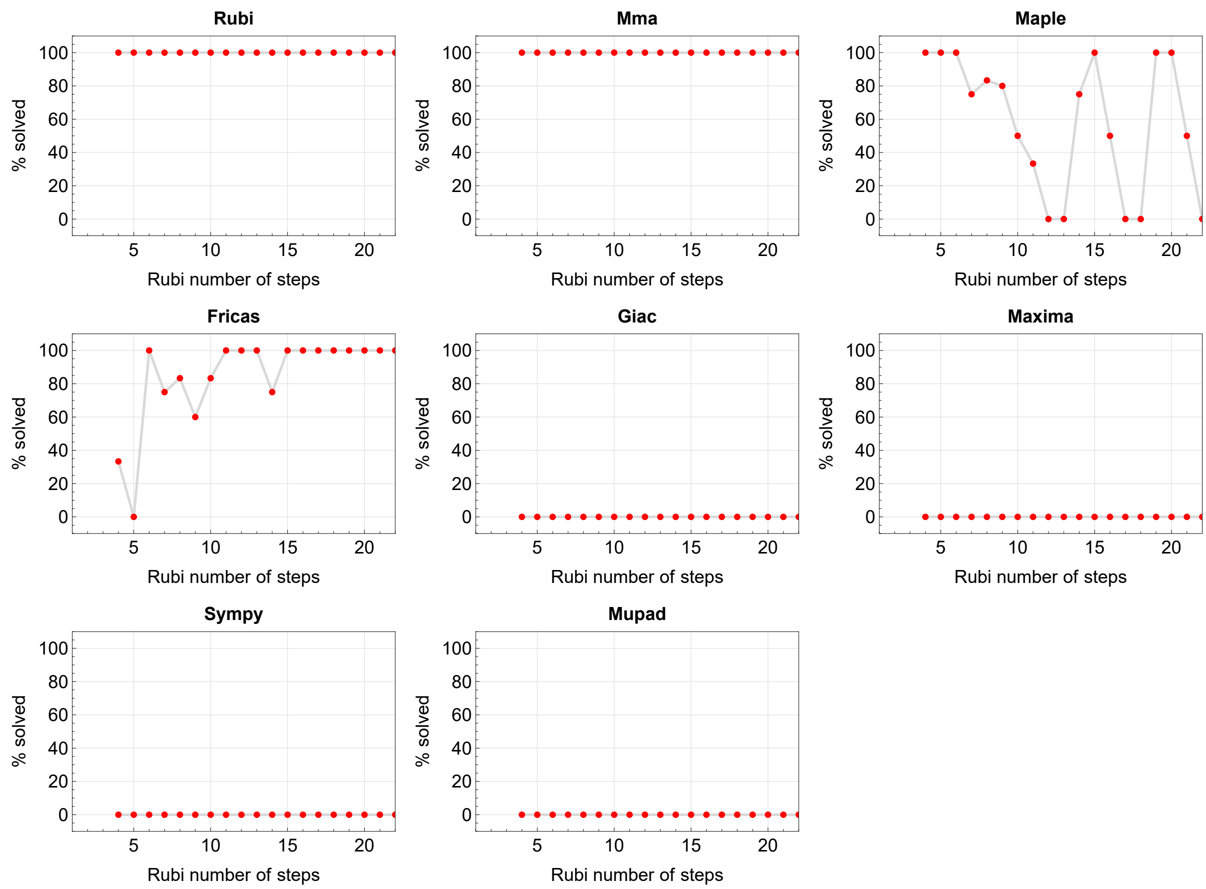


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

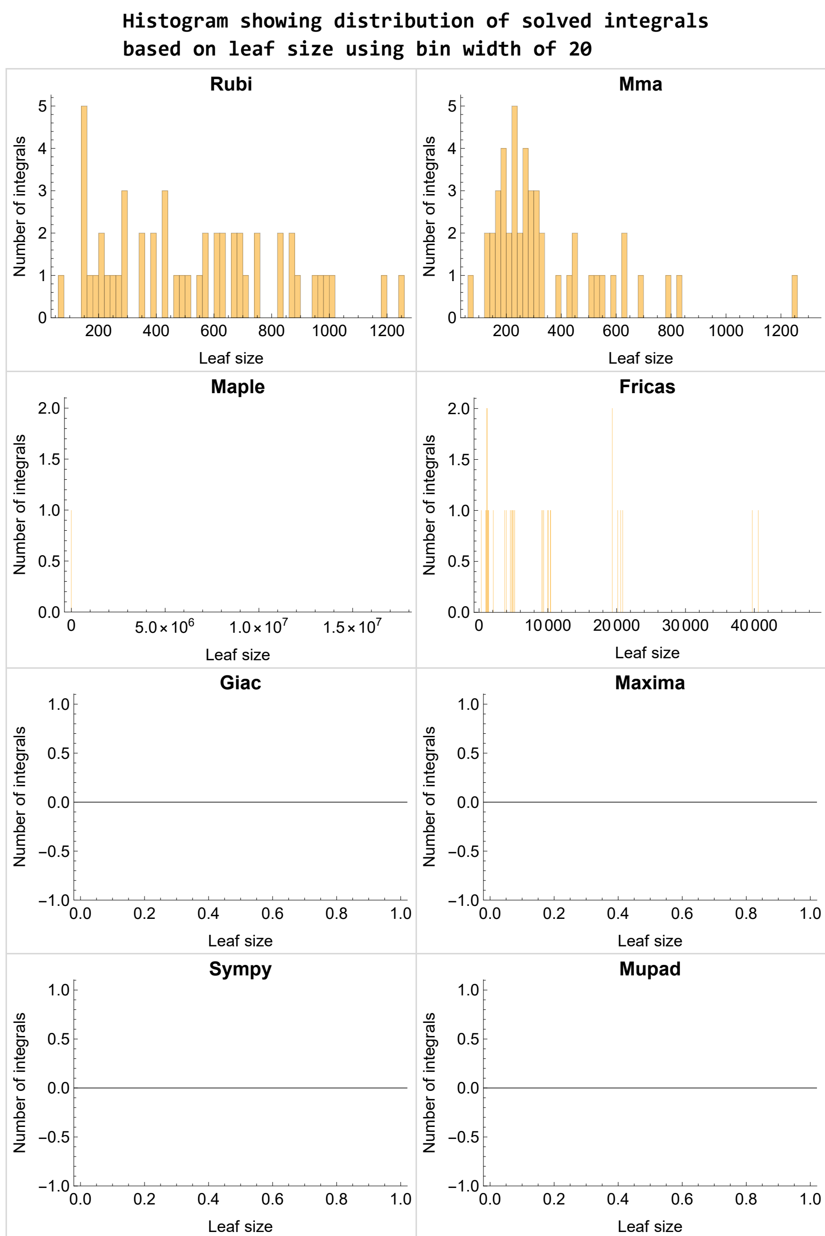


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

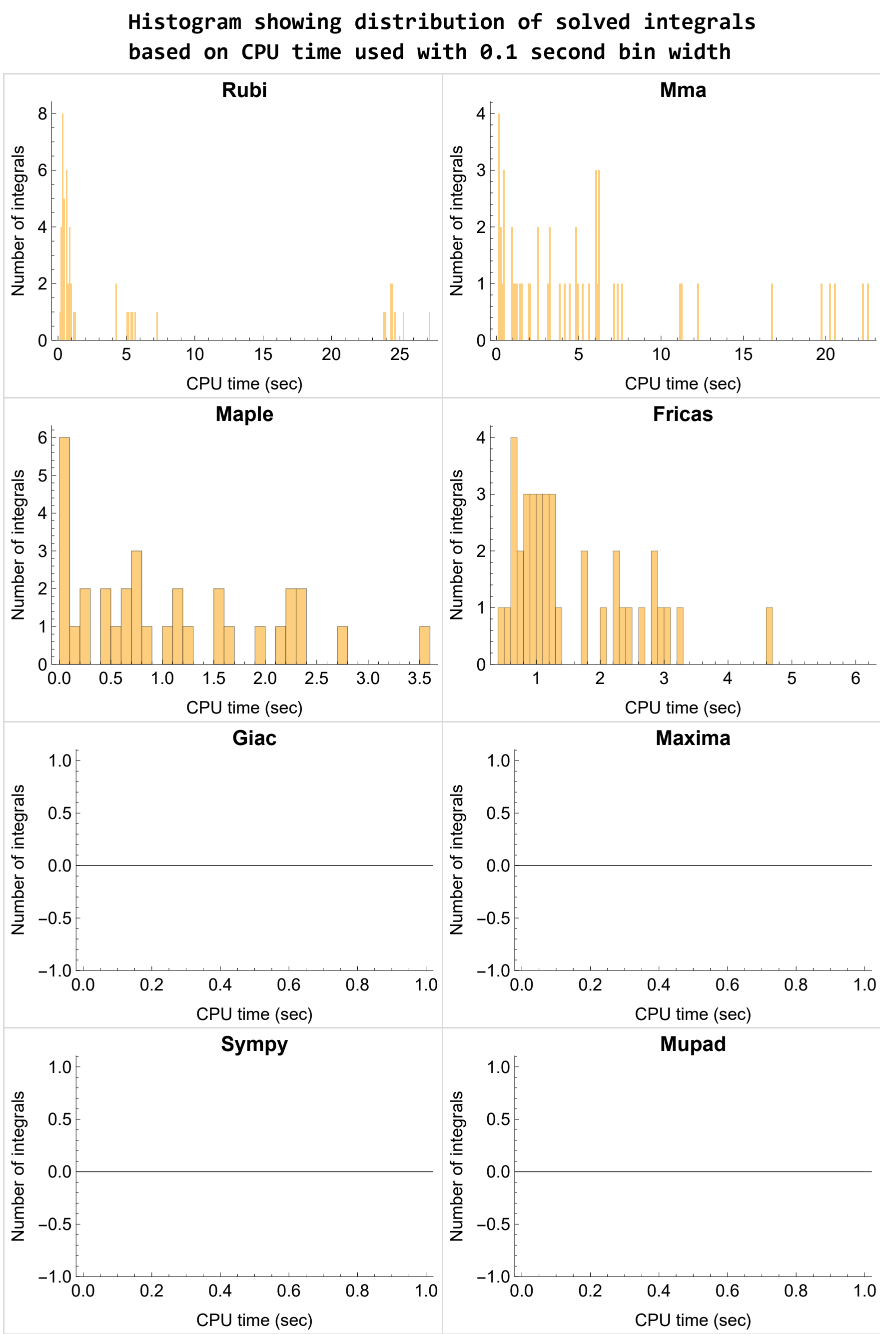


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

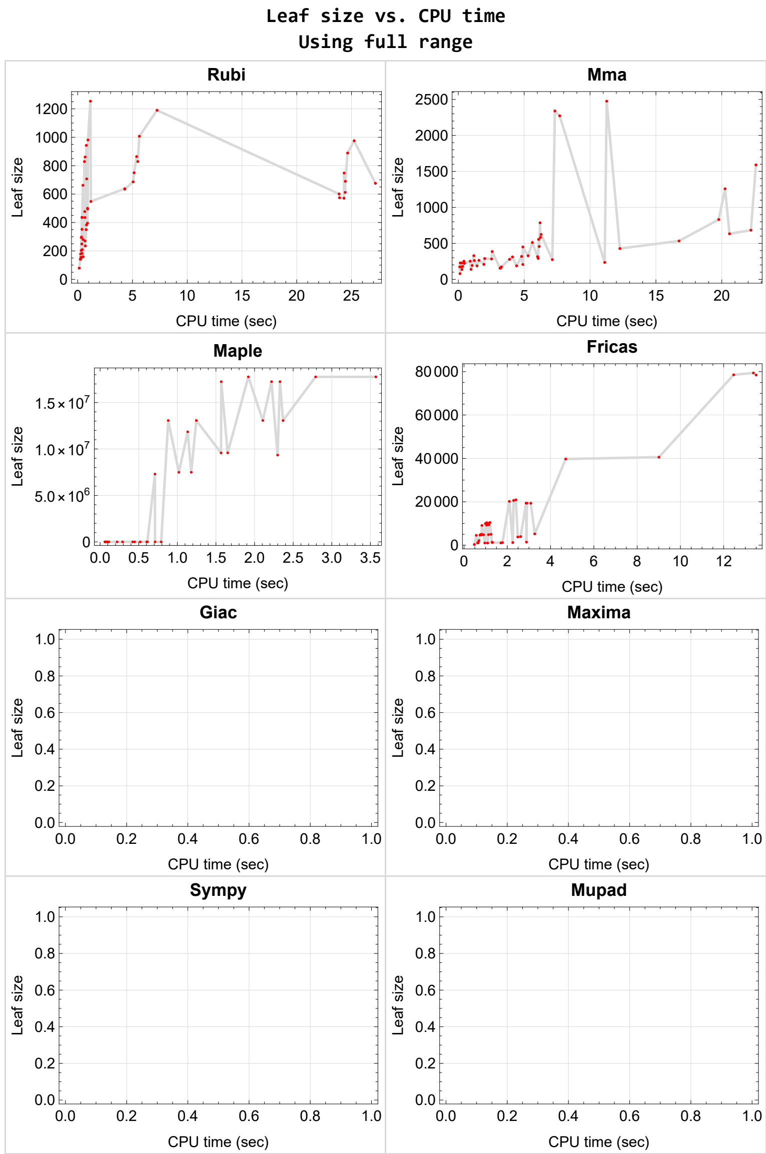


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{19, 20, 21, 51\}$

Maple $\{1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

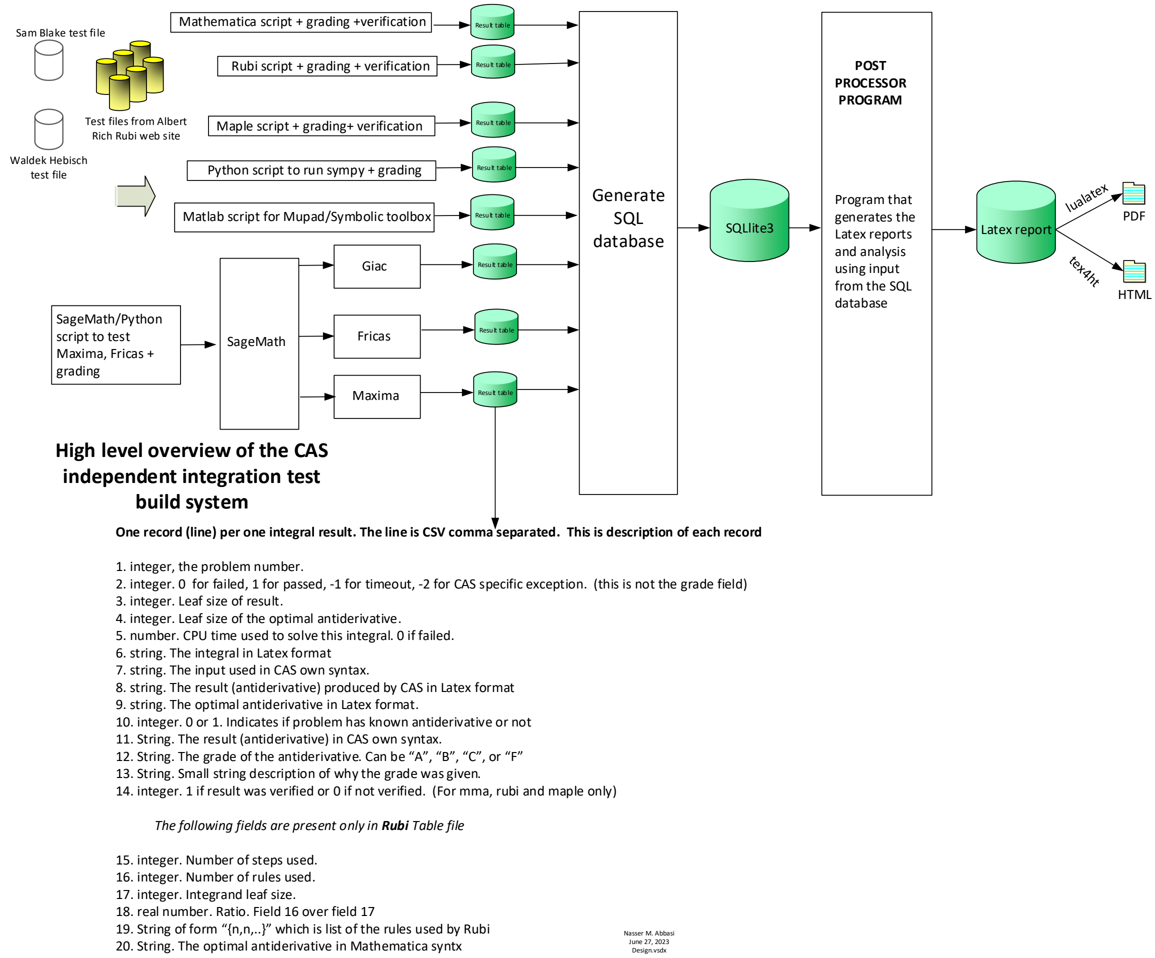
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 27, 28, 29, 30, 31, 36, 37, 38, 39, 40, 45, 46, 47, 48, 49, 50 }

B grade { }

C grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 41, 42, 43, 44, 51 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 27, 28, 29, 32, 33, 36, 37, 38, 41, 42, 43 }

B grade { 1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 45, 46, 47, 48, 51 }

C grade { }

F normal fail { 30, 31, 34, 35, 39, 40, 44, 49, 50 }

F(-1) timedout fail { 7, 8, 9, 16, 17, 18, 24, 25, 26 }

F(-2) exception fail { }

Fricas

A grade { 27, 28, 29, 30, 31, 36, 37, 38, 39, 40 }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 45, 46, 47, 48, 49, 50 }

C grade { }

F normal fail { 35, 41, 42 }

F(-1) timedout fail { 32, 33, 34, 43, 44, 51 }

F(-2) exception fail { }

Maxima

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 16, 17, 18, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 38, 39, 41, 42, 43, 44, 46, 47, 48, 51 }

F(-1) timedout fail { 19, 20, 24, 25, 26, 36, 40, 50 }

F(-2) exception fail { 6, 7, 14, 15, 21, 22, 23, 45, 49 }

Giac

A grade { }

B grade { }

C grade { }

F normal fail { 6, 7, 8, 9, 15, 16, 17, 18, 30, 31, 33, 34, 35, 39, 40, 43, 44 }

F(-1) timedout fail { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 36, 37, 38, 41, 42, 45, 46, 47, 48, 49, 50, 51 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	975	975	623	17768518	0	5023	0	0	0
N.S.	1	1.00	0.64	18224.12	0.00	5.15	0.00	0.00	0.00
time (sec)	N/A	25.239	6.283	3.570	0.000	1.251	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	889	889	511	17248526	0	4855	0	0	0
N.S.	1	1.00	0.57	19402.17	0.00	5.46	0.00	0.00	0.00
time (sec)	N/A	24.642	5.621	2.219	0.000	1.130	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	748	748	386	17766953	0	4793	0	0	0
N.S.	1	1.00	0.52	23752.61	0.00	6.41	0.00	0.00	0.00
time (sec)	N/A	24.326	2.571	2.790	0.000	0.903	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	676	676	329	17248163	0	4685	0	0	0
N.S.	1	1.00	0.49	25515.03	0.00	6.93	0.00	0.00	0.00
time (sec)	N/A	27.178	1.179	2.329	0.000	0.805	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	601	601	252	17767879	0	4660	0	0	0
N.S.	1	1.00	0.42	29563.86	0.00	7.75	0.00	0.00	0.00
time (sec)	N/A	23.865	0.426	1.919	0.000	0.738	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	574	574	228	17248000	0	4547	0	0	0
N.S.	1	1.00	0.40	30048.78	0.00	7.92	0.00	0.00	0.00
time (sec)	N/A	23.908	0.146	1.568	0.000	0.567	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F(-1)	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	571	571	223	0	0	9103	0	0	0
N.S.	1	1.00	0.39	0.00	0.00	15.94	0.00	0.00	0.00
time (sec)	N/A	24.313	0.464	180.000	0.000	0.827	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F(-1)	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	612	612	261	0	0	9221	0	0	0
N.S.	1	1.00	0.43	0.00	0.00	15.07	0.00	0.00	0.00
time (sec)	N/A	24.421	1.217	180.000	0.000	1.049	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F(-1)	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	690	690	289	0	0	9425	0	0	0
N.S.	1	1.00	0.42	0.00	0.00	13.66	0.00	0.00	0.00
time (sec)	N/A	24.436	2.003	180.000	0.000	1.153	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	548	548	456	9581953	0	10457	0	0	0
N.S.	1	1.00	0.83	17485.32	0.00	19.08	0.00	0.00	0.00
time (sec)	N/A	1.203	6.140	1.652	0.000	1.201	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	495	495	283	7492392	0	10089	0	0	0
N.S.	1	1.00	0.57	15136.15	0.00	20.38	0.00	0.00	0.00
time (sec)	N/A	0.911	2.521	1.180	0.000	1.132	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	383	383	252	9581103	0	10337	0	0	0
N.S.	1	1.00	0.66	25015.93	0.00	26.99	0.00	0.00	0.00
time (sec)	N/A	0.803	0.906	1.566	0.000	1.046	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	352	352	228	7491751	0	9961	0	0	0
N.S.	1	1.00	0.65	21283.38	0.00	28.30	0.00	0.00	0.00
time (sec)	N/A	0.389	0.252	1.020	0.000	0.988	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	294	294	173	9339203	0	5045	0	0	0
N.S.	1	1.00	0.59	31766.00	0.00	17.16	0.00	0.00	0.00
time (sec)	N/A	0.322	0.108	2.298	0.000	0.810	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	298	298	173	7300729	0	4891	0	0	0
N.S.	1	1.00	0.58	24499.09	0.00	16.41	0.00	0.00	0.00
time (sec)	N/A	0.351	0.195	0.710	0.000	0.762	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F(-1)	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	350	350	223	0	0	20605	0	0	0
N.S.	1	1.00	0.64	0.00	0.00	58.87	0.00	0.00	0.00
time (sec)	N/A	0.770	0.444	180.000	0.000	2.291	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F(-1)	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	395	395	264	0	0	20189	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	51.11	0.00	0.00	0.00
time (sec)	N/A	0.889	1.569	180.000	0.000	2.095	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F(-1)	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	500	500	315	0	0	20910	0	0	0
N.S.	1	1.00	0.63	0.00	0.00	41.82	0.00	0.00	0.00
time (sec)	N/A	0.895	6.014	180.000	0.000	2.397	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1190	1190	2476	13068421	0	40569	0	0	0
N.S.	1	1.00	2.08	10981.87	0.00	34.09	0.00	0.00	0.00
time (sec)	N/A	7.243	11.263	2.107	0.000	9.007	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	864	864	2272	13066870	0	39731	0	0	0
N.S.	1	1.00	2.63	15123.69	0.00	45.98	0.00	0.00	0.00
time (sec)	N/A	5.380	7.695	2.369	0.000	4.700	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	686	686	2339	13067312	0	19371	0	0	0
N.S.	1	1.00	3.41	19048.56	0.00	28.24	0.00	0.00	0.00
time (sec)	N/A	5.058	7.332	1.246	0.000	2.868	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	638	638	328	11848772	0	19326	0	0	0
N.S.	1	1.00	0.51	18571.74	0.00	30.29	0.00	0.00	0.00
time (sec)	N/A	4.297	5.299	1.134	0.000	3.086	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	635	635	318	13066372	0	19368	0	0	0
N.S.	1	1.00	0.50	20576.96	0.00	30.50	0.00	0.00	0.00
time (sec)	N/A	4.300	4.802	0.882	0.000	2.909	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F(-1)	F(-1)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	750	750	450	0	0	78431	0	0	0
N.S.	1	1.00	0.60	0.00	0.00	104.57	0.00	0.00	0.00
time (sec)	N/A	5.146	4.901	180.000	0.000	13.499	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F(-1)	F(-1)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	829	829	583	0	0	78535	0	0	0
N.S.	1	1.00	0.70	0.00	0.00	94.73	0.00	0.00	0.00
time (sec)	N/A	5.485	6.235	180.000	0.000	12.458	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F(-1)	F(-1)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1007	1007	786	0	0	79389	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	78.84	0.00	0.00	0.00
time (sec)	N/A	5.626	6.206	180.000	0.000	13.372	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	290	455	0	1405	0	0	0
N.S.	1	1.00	1.07	1.69	0.00	5.20	0.00	0.00	0.00
time (sec)	N/A	0.665	6.061	0.290	0.000	2.886	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	208	318	0	1199	0	0	0
N.S.	1	1.00	1.00	1.52	0.00	5.74	0.00	0.00	0.00
time (sec)	N/A	0.403	1.942	0.083	0.000	2.253	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	180	217	0	1057	0	0	0
N.S.	1	1.00	1.01	1.21	0.00	5.91	0.00	0.00	0.00
time (sec)	N/A	0.263	0.325	0.066	0.000	1.705	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	203	203	192	0	0	2097	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	10.33	0.00	0.00	0.00
time (sec)	N/A	0.316	1.053	0.000	0.000	0.690	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	435	435	187	0	0	1186	0	0	0
N.S.	1	1.00	0.43	0.00	0.00	2.73	0.00	0.00	0.00
time (sec)	N/A	0.658	1.418	0.000	0.000	1.776	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1254	1254	633	1945	0	0	0	0	0
N.S.	1	1.00	0.50	1.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.157	20.584	0.711	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	829	829	428	1497	0	0	0	0	0
N.S.	1	1.00	0.52	1.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.613	12.265	0.422	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	861	861	1258	0	0	0	0	0	0
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.673	20.249	0.000	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	943	943	1590	0	0	0	0	0	0
N.S.	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.783	22.592	0.000	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	173	232	0	1226	0	0	0
N.S.	1	1.00	0.95	1.27	0.00	6.74	0.00	0.00	0.00
time (sec)	N/A	0.389	3.270	0.113	0.000	1.317	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	136	153	0	993	0	0	0
N.S.	1	1.00	0.96	1.09	0.00	7.04	0.00	0.00	0.00
time (sec)	N/A	0.234	0.278	0.075	0.000	1.094	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	102	0	299	0	0	0
N.S.	1	1.00	1.00	1.29	0.00	3.78	0.00	0.00	0.00
time (sec)	N/A	0.136	0.129	0.220	0.000	0.478	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	139	0	0	1015	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	7.15	0.00	0.00	0.00
time (sec)	N/A	0.267	0.965	0.000	0.000	0.962	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	249	249	188	0	0	1350	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	5.42	0.00	0.00	0.00
time (sec)	N/A	0.372	4.422	0.000	0.000	1.291	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	662	662	533	646	0	0	0	0	0
N.S.	1	1.00	0.81	0.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.473	16.753	0.791	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	311	402	0	0	0	0	0
N.S.	1	1.00	0.71	0.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.409	4.111	0.612	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	235	231	0	0	0	0	0
N.S.	1	1.00	0.54	0.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.386	11.113	0.446	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	707	707	683	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.819	22.211	0.000	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	274	684	0	3773	0	0	0
N.S.	1	1.00	1.17	2.91	0.00	16.06	0.00	0.00	0.00
time (sec)	N/A	0.691	7.138	0.520	0.000	2.487	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	206	509	0	1095	0	0	0
N.S.	1	1.00	1.30	3.20	0.00	6.89	0.00	0.00	0.00
time (sec)	N/A	0.490	4.898	0.099	0.000	0.651	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	155	456	0	1077	0	0	0
N.S.	1	1.00	1.01	2.96	0.00	6.99	0.00	0.00	0.00
time (sec)	N/A	0.351	3.167	0.089	0.000	0.627	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	156	406	0	1099	0	0	0
N.S.	1	1.00	1.01	2.62	0.00	7.09	0.00	0.00	0.00
time (sec)	N/A	0.291	3.203	0.061	0.000	0.656	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	280	280	278	0	0	3951	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	14.11	0.00	0.00	0.00
time (sec)	N/A	0.484	3.888	0.000	0.000	2.617	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-1)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	477	477	555	0	0	5189	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	10.88	0.00	0.00	0.00
time (sec)	N/A	0.646	6.096	0.000	0.000	3.268	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	981	981	831	3598	0	0	0	0	0
N.S.	1	1.00	0.85	3.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.937	19.767	0.603	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [1] had the largest ratio of [.4242000000000000021]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	21	14	1.00	33	0.424
2	A	19	12	1.00	33	0.364
3	A	16	12	1.00	33	0.364
4	A	10	9	1.00	33	0.273
5	A	10	9	1.00	31	0.290
6	A	9	7	1.00	24	0.292
7	A	18	13	1.00	31	0.419
8	A	17	12	1.00	33	0.364
9	A	21	14	1.00	33	0.424
10	A	15	10	1.00	33	0.303
11	A	14	9	1.00	33	0.273
12	A	11	8	1.00	33	0.242
13	A	9	7	1.00	33	0.212
14	A	6	4	1.00	31	0.129
15	A	6	3	1.00	24	0.125
16	A	10	7	1.00	31	0.226
17	A	11	8	1.00	33	0.242
18	A	14	9	1.00	33	0.273
19	A	20	12	1.00	33	0.364
20	A	14	11	1.00	33	0.333
21	A	10	7	1.00	33	0.212
22	A	7	5	1.00	33	0.152
23	A	7	5	1.00	31	0.161
24	A	13	10	1.00	31	0.323

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	13	10	1.00	33	0.303
26	A	18	12	1.00	33	0.364
27	A	9	8	1.00	35	0.229
28	A	8	7	1.00	35	0.200
29	A	8	7	1.00	33	0.212
30	A	10	7	1.00	33	0.212
31	A	22	9	1.00	35	0.257
32	A	14	9	1.00	35	0.257
33	A	8	6	1.00	26	0.231
34	A	9	8	1.00	35	0.229
35	A	10	8	1.00	35	0.229
36	A	8	7	1.00	35	0.200
37	A	7	6	1.00	35	0.171
38	A	4	4	1.00	33	0.121
39	A	8	5	1.00	33	0.152
40	A	11	6	1.00	35	0.171
41	A	5	5	1.00	35	0.143
42	A	4	4	1.00	35	0.114
43	A	4	3	1.00	26	0.115
44	A	7	7	1.00	35	0.200
45	A	8	7	1.00	35	0.200
46	A	6	6	1.00	35	0.171
47	A	6	6	1.00	35	0.171
48	A	6	6	1.00	33	0.182
49	A	12	7	1.00	33	0.212
50	A	16	8	1.00	35	0.229
51	A	9	8	1.00	35	0.229

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \tan^5(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 975

$$\int \tan^5(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$

$$= \frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{b^2+\cdots}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}\sqrt{\cdots}}\right)}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}e}$$

$$+\frac{b\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{c}e}$$

$$-\frac{b(b^2-4ac)\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{16c^{5/2}e}$$

$$+\frac{b(7b^2-12ac)\,(b^2-4ac)\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{256c^{9/2}e}$$

$$-\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\text{arctanh}\left(\frac{\cdots}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}\cdots}\right)}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}e}$$

$$+\frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e}$$

$$+\frac{b(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{8c^2e}$$

$$-\frac{b(7b^2-12ac)\,(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{128c^4e}$$

$$-\frac{\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}{3ce}$$

$$+\frac{\tan^2(d+ex)\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}{5ce}$$

$$+\frac{(35b^2-32ac-42bc\tan(d+ex))\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}{240c^3e}$$

```
[Out] -1/16*b*(-4*a*c+b^2)*arctanh(1/2*(b+2*c*tan(e*x+d))/c^(1/2)/(a+b*tan(e*x+d)
+c*tan(e*x+d)^2)^(1/2))/c^(5/2)/e+1/256*b*(-12*a*c+7*b^2)*(-4*a*c+b^2)*arct
anh(1/2*(b+2*c*tan(e*x+d))/c^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))/c
^(9/2)/e+1/2*b*arctanh(1/2*(b+2*c*tan(e*x+d))/c^(1/2)/(a+b*tan(e*x+d)+c*tan
(e*x+d)^2)^(1/2))/e/c^(1/2)-1/2*arctanh(1/2*(b^2+(a-c)*(a-c+(a^2-2*a*c+b^2+
c^2)^(1/2))+b*(a^2-2*a*c+b^2+c^2)^(1/2)*tan(e*x+d))/(a^2-2*a*c+b^2+c^2)^(1/
4)*2^(1/2)/(a^2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c
^2)^(1/2)))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a^2+b^2+c*(c-(a^2
-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+
```

```
b^2+c^2)^(1/4)/e*2^(1/2)+1/2*arctan(1/2*(b^2+(a-c)*(a-c-(a^2-2*a*c+b^2+c^2)
^(1/2))-b*(a^2-2*a*c+b^2+c^2)^(1/2)*tan(e*x+d))/(a^2-2*a*c+b^2+c^2)^(1/4)*2
^(1/2)/(a^2+b^2+c*(c+(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^
(1/2)))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a^2+b^2+c*(c+(a^2-2*a
*c+b^2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+b^2+
c^2)^(1/4)/e*2^(1/2)+(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)/e+1/8*b*(a+b*tan
(e*x+d)+c*tan(e*x+d)^2)^(1/2)*(b+2*c*tan(e*x+d))/c^2/e-1/128*b*(-12*a*c+7*b
^2)*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*(b+2*c*tan(e*x+d))/c^4/e-1/3*(a+b
*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2)/c/e+1/5*tan(e*x+d)^2*(a+b*tan(e*x+d)+c*ta
n(e*x+d)^2)^(3/2)/c/e+1/240*(35*b^2-32*a*c-42*b*c*tan(e*x+d))*(a+b*tan(e*x+
d)+c*tan(e*x+d)^2)^(3/2)/c^3/e
```

Rubi [A] (verified)

Time = 25.24 (sec) , antiderivative size = 975, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules

used = {3781, 6857, 654, 626, 635, 212, 756, 793, 1035, 1092, 1050, 1044, 214, 211}

$$\int \tan^5(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$= \frac{\left(c\tan^2(d+ex)+b\tan(d+ex)+a\right)^{3/2}\tan^2(d+ex)}{5ce}$$
$$+ \frac{\left(35b^2-42c\tan(d+ex)b-32ac\right)\left(c\tan^2(d+ex)+b\tan(d+ex)+a\right)^{3/2}}{240c^3e}$$
$$- \frac{\left(c\tan^2(d+ex)+b\tan(d+ex)+a\right)^{3/2}}{3ce}$$
$$+ \frac{\sqrt{a^2-\left(2c+\sqrt{a^2-2ca+b^2+c^2}\right)a+b^2+c\left(c+\sqrt{a^2-2ca+b^2+c^2}\right)}\arctan\left(\frac{b}{\sqrt{2}\sqrt[4]{a^2-2ca+b^2+c}}\right.}{\sqrt{2}\sqrt[4]{a^2-2ca+b^2+c^2}e}$$
$$+ \frac{b(7b^2-12ac)\left(b^2-4ac\right)\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}\right)}{256c^{9/2}e}$$
$$- \frac{b(b^2-4ac)\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}\right)}{16c^{5/2}e}$$
$$+ \frac{b\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}\right)}{2\sqrt{c}e}$$
$$- \frac{\sqrt{a^2-\left(2c-\sqrt{a^2-2ca+b^2+c^2}\right)a+b^2+c\left(c-\sqrt{a^2-2ca+b^2+c^2}\right)}\operatorname{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2-2ca+b^2+c}}\right.}{\sqrt{2}\sqrt[4]{a^2-2ca+b^2+c^2}e}$$
$$- \frac{b(7b^2-12ac)\left(b+2c\tan(d+ex)\right)\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}{128c^4e}$$
$$+ \frac{b\left(b+2c\tan(d+ex)\right)\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}{8c^2e}$$
$$+ \frac{\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}{e}$$

```
[In] Int[Tan[d + e*x]^5*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2
 + b^2 - 2*a*c + c^2])]*ArcTan[(b^2 + (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a
*c + c^2]) - b*Sqrt[a^2 + b^2 - 2*a*c + c^2]*Tan[d + e*x])/(Sqrt[2]*(a^2 +
b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c
^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*Tan[d + e*x] + c
*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*e) + (b*ArcTan
h[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x
]^2])])/(2*Sqrt[c]*e) - (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*Tan[d + e*x])/(2*
Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(16*c^(5/2)*e) + (b*
```

```
(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sq
rt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(256*c^(9/2)*e) - (Sqrt[a^2 +
b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a^2 + b^2 - 2*a
*c + c^2])]*ArcTanh[(b^2 + (a - c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2])
+ b*Sqrt[a^2 + b^2 - 2*a*c + c^2]*Tan[d + e*x])/(Sqrt[2]*(a^2 + b^2 - 2*a*c
 + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2
*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*
x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*e) + Sqrt[a + b*Tan[d + e
*x] + c*Tan[d + e*x]^2]/e + (b*(b + 2*c*Tan[d + e*x])*Sqrt[a + b*Tan[d + e*
x] + c*Tan[d + e*x]^2])/(8*c^2*e) - (b*(7*b^2 - 12*a*c)*(b + 2*c*Tan[d + e*
x])*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/(128*c^4*e) - (a + b*Tan[d
 + e*x] + c*Tan[d + e*x]^2)^(3/2)/(3*c*e) + (Tan[d + e*x]^2*(a + b*Tan[d +
e*x] + c*Tan[d + e*x]^2)^(3/2))/(5*c*e) + ((35*b^2 - 32*a*c - 42*b*c*Tan[d
+ e*x])*(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2))/(240*c^3*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
```

```
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^5\sqrt{a+bx+cx^2}}{1+x^2}\,dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{\text{Subst}\left(\int \left(-x\sqrt{a+bx+cx^2}+x^3\sqrt{a+bx+cx^2}+\frac{x\sqrt{a+bx+cx^2}}{1+x^2}\right)dx, x, \tan(d+ex)\right)}{e} \\
&= -\frac{\text{Subst}\left(\int x\sqrt{a+bx+cx^2}\,dx, x, \tan(d+ex)\right)}{e} \\
&\quad + \frac{\text{Subst}\left(\int x^3\sqrt{a+bx+cx^2}\,dx, x, \tan(d+ex)\right)}{e} \\
&\quad + \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx+cx^2}}{1+x^2}\,dx, x, \tan(d+ex)\right)}{e}
\end{aligned}
$$

$$= \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e} - \frac{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{3ce}$$
$$+ \frac{\tan^2(d+ex)\,(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{5ce}$$
$$- \frac{\text{Subst}\left(\int \frac{\frac{b}{2}-(a-c)x-\frac{bx^2}{2}}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}$$
$$+ \frac{\text{Subst}\left(\int x\left(-2a-\frac{7bx}{2}\right)\sqrt{a+bx+cx^2}\,dx, x, \tan(d+ex)\right)}{5ce}$$
$$+ \frac{b\text{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx, x, \tan(d+ex)\right)}{2ce}$$
$$= \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e}$$
$$+ \frac{b(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{8c^2e}$$
$$- \frac{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{3ce}$$
$$+ \frac{\tan^2(d+ex)\,(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{5ce}$$
$$+ \frac{(35b^2-32ac-42bc\tan(d+ex))\,(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{240c^3e}$$
$$- \frac{\text{Subst}\left(\int \frac{b+(-a+c)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}$$
$$+ \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2e}$$
$$- \frac{(b(7b^2-12ac))\,\text{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx, x, \tan(d+ex)\right)}{32c^3e}$$
$$- \frac{(b(b^2-4ac))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{16c^2e}$$

$$
\begin{aligned}
&= \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e} \\
&\quad + \frac{b(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{8c^2e} \\
&\quad - \frac{b(7b^2-12ac)(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{128c^4e} \\
&\quad - \frac{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{3ce} \\
&\quad + \frac{\tan^2(d+ex)(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{5ce} \\
&\quad + \frac{(35b^2-32ac-42bc\tan(d+ex))(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{240c^3e} \\
&\quad + \frac{b\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e} \\
&\quad - \frac{(b(b^2-4ac))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8c^2e} \\
&\quad + \frac{(b(7b^2-12ac)(b^2-4ac))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{256c^4e} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-b\sqrt{a^2+b^2-2ac+c^2}+\left(-b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}\,e} \\
&\quad - \frac{\text{Subst}\left(\int \frac{b\sqrt{a^2+b^2-2ac+c^2}+\left(-b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}\,e}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b \operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{c}e} \\
&- \frac{b(b^2-4ac)\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{16c^{5/2}e} \\
&+ \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e} \\
&+ \frac{b(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{8c^2e} \\
&- \frac{b(7b^2-12ac)(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{128c^4e} \\
&- \frac{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{3ce} \\
&+ \frac{\tan^2(d+ex)(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{5ce} \\
&+ \frac{(35b^2-32ac-42bc\tan(d+ex))(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{240c^3e} \\
&+ \frac{(b(7b^2-12ac)(b^2-4ac))\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{128c^4e} \\
&- \frac{\left(b\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\operatorname{Subst}\left(\int \frac{1}{2b\sqrt{a^2+b^2-2ac+c^2}\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac\right.\right.}}\right.}{e} \\
&- \frac{\left(b\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\operatorname{Subst}\left(\int \frac{1}{-2b\sqrt{a^2+b^2-2ac+c^2}\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2\right.\right.}}\right.}{e}
\end{aligned}
$$

$$= \frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2-2}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+}}$$

$$+\frac{b\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{c}e}$$

$$-\frac{b(b^2-4ac)\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{16c^{5/2}e}$$

$$+\frac{b(7b^2-12ac)\left(b^2-4ac\right)\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{256c^{9/2}e}$$

$$-\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\operatorname{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac}}$$

$$+\frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e}$$

$$+\frac{b(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{8c^2e}$$

$$-\frac{b(7b^2-12ac)\left(b+2c\tan(d+ex)\right)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{128c^4e}$$

$$-\frac{\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}{3ce}$$

$$+\frac{\tan^2(d+ex)\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}{5ce}$$

$$+\frac{\left(35b^2-32ac-42bc\tan(d+ex)\right)\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}{240c^3e}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.28 (sec) , antiderivative size = 623, normalized size of antiderivative = 0.64

$$\int \tan^5(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$

$$= \frac{-\frac{1}{2}\sqrt{a-ib-c}\operatorname{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)-\frac{1}{2}\sqrt{a+ib-c}\operatorname{arctanh}\left(\frac{2a+ib+(b+2ic)\tan(}{2\sqrt{a+ib-c}\sqrt{a+b\tan(d+ex)+}}\right.}{}$$

```
[In] Integrate[Tan[d + e*x]^5*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (-1/2*(Sqrt[a - I*b - c]*ArcTanh[(2*a - I*b + (b - (2*I)*c)*Tan[d + e*x])/(
2*Sqrt[a - I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])]) - (Sqrt[
a + I*b - c]*ArcTanh[(2*a + I*b + (b + (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a + I
*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/2 + (b*ArcTanh[(b +
2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])
/(2*Sqrt[c]) + Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2] - (a + b*Tan[d +
 e*x] + c*Tan[d + e*x]^2)^(3/2)/(3*c) + (Tan[d + e*x]^2*(a + b*Tan[d + e*x]
 + c*Tan[d + e*x]^2)^(3/2))/(5*c) + (b*(-1/8*((b^2 - 4*a*c)*ArcTanh[(b + 2*
c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/c
^(3/2) + ((b + 2*c*Tan[d + e*x])*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2
])/(4*c)))/(2*c) + ((((35*b^2)/4 - 8*a*c - (21*b*c*Tan[d + e*x])/2)*(a + b*
Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2))/(12*c^2) + (((-35*b^3)/4 + 15*a*b*c
)*(-1/8*((b^2 - 4*a*c)*ArcTanh[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b
*Tan[d + e*x] + c*Tan[d + e*x]^2])])/c^(3/2) + ((b + 2*c*Tan[d + e*x])*Sqrt
[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/(4*c)))/(8*c^2))/(5*c))/e
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 3.57 (sec) , antiderivative size = 17768518, normalized size of antiderivative = 18224.12

output too large to display

```
[In] int((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)^5,x)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2511 vs. 2(876) = 1752.

Time = 1.25 (sec) , antiderivative size = 5023, normalized size of antiderivative = 5.15

$$\int \tan^5(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)^5,x, algorithm="
fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \tan^5(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$=\int \sqrt{a+b\tan{(d+ex)}+c\tan^2{(d+ex)}}\tan^5{(d+ex)}\,dx$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2)*tan(e*x+d)**5,x)
[Out] Integral(sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2)*tan(d + e*x)**5, x)
```

Maxima [F]

$$\int \tan^5(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$=\int \sqrt{c\tan{(ex+d)}^2+b\tan{(ex+d)}+a}\tan{(ex+d)}^5\,dx$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)^5,x, algorithm="
maxima")
[Out] integrate(sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a)*tan(e*x + d)^5, x)
```

Giac [F(-1)]

Timed out.

$$\int \tan^5(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)^5,x, algorithm="
giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \tan^5(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \text{Hanged}$$

```
[In] int(tan(d + e*x)^5*(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)
[Out] \text{Hanged}
```

3.2 $\int \tan^4(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 889

$$\int \tan^4(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx =$$

$$-\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{\cdots}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+\cdots}}\right)}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}e}$$

$$+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e}$$

$$+\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8c^{3/2}e}$$

$$-\frac{(b^2-4ac)(5b^2-4ac)\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{128c^{7/2}e}$$

$$-\frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\operatorname{arctanh}\left(\frac{\cdots}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+\cdots}}\right)}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}e}$$

$$-\frac{(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ce}$$

$$+\frac{(5b^2-4ac)(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{64c^3e}$$

$$-\frac{5b(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{24c^2e}$$

$$+\frac{\tan(d+ex)(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{4ce}$$

```
[Out] 1/8*(-4*a*c+b^2)*arctanh(1/2*(b+2*c*tan(e*x+d))/c^(1/2)/(a+b*tan(e*x+d)+c*t
an(e*x+d)^2)^(1/2))/c^(3/2)/e-1/128*(-4*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2
*(b+2*c*tan(e*x+d))/c^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))/c^(7/2)/
e+arctanh(1/2*(b+2*c*tan(e*x+d))/c^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1
/2))*c^(1/2)/e-1/2*arctan(1/2*(b*(a^2-2*a*c+b^2+c^2)^(1/2)-(b^2+(a-c)*(a-c+
(a^2-2*a*c+b^2+c^2)^(1/2)))*tan(e*x+d))/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)/(
a^2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)))^
(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a^2+b^2+c*(c-(a^2-2*a*c+b^2+c
^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/
4)/e*2^(1/2)-1/2*arctanh(1/2*(b*(a^2-2*a*c+b^2+c^2)^(1/2)+(b^2+(a-c)*(a-c-(
a^2-2*a*c+b^2+c^2)^(1/2)))*tan(e*x+d))/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)/(a
^2+b^2+c*(c+(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(
1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a^2+b^2+c*(c+(a^2-2*a*c+b^2+c^
2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/4
)/e*2^(1/2)-1/4*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*(b+2*c*tan(e*x+d))/c/
e+1/64*(-4*a*c+5*b^2)*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*(b+2*c*tan(e*x+
d))/c^3/e-5/24*b*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2)/c^2/e+1/4*tan(e*x+d)
*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2)/c/e
```

Rubi [A] (verified)

Time = 24.64 (sec) , antiderivative size = 889, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules

used = {3781, 6857, 626, 635, 212, 756, 654, 1004, 1050, 1044, 214, 211}

$$\int \tan^4(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$= \frac{\tan(d+ex)\left(c\tan^2(d+ex)+b\tan(d+ex)+a\right)^{3/2}}{4ce} - \frac{5b(c\tan^2(d+ex)+b\tan(d+ex)+a)^{3/2}}{24c^2e} + \frac{(5b^2-4ac)\left(b+2c\tan(d+ex)\right)\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}{64c^3e} - \frac{(b+2c\tan(d+ex))\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}{4ce} - \frac{\sqrt{a^2-\left(2c-\sqrt{a^2-2ca+b^2+c^2}\right)a+b^2+c\left(c-\sqrt{a^2-2ca+b^2+c^2}\right)}\arctan\left(\frac{}{\sqrt{2}\sqrt[4]{a^2-2ca+b^2+}}\right.}{\sqrt{2}\sqrt[4]{a^2-2ca+b^2+c^2}e} + \frac{(b^2-4ac)\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}\right)}{8c^{3/2}e} - \frac{(b^2-4ac)(5b^2-4ac)\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}\right)}{128c^{7/2}e} + \frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}\right)}{e} - \frac{\sqrt{a^2-\left(2c+\sqrt{a^2-2ca+b^2+c^2}\right)a+b^2+c\left(c+\sqrt{a^2-2ca+b^2+c^2}\right)}\,\text{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2-2ca+b^2+}}\right.}{\sqrt{2}\sqrt[4]{a^2-2ca+b^2+c^2}e}$$

```
[In] Int[Tan[d + e*x]^4*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] -((Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a
^2 + b^2 - 2*a*c + c^2])]*ArcTan[(b*Sqrt[a^2 + b^2 - 2*a*c + c^2] - (b^2 +
(a - c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]))*Tan[d + e*x])/(Sqrt[2]*(a^
2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c
 + c^2]) - a*(2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*Tan[d + e*x]
 + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*e)) + (Sqr
t[c]*ArcTanh[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*
Tan[d + e*x]^2])])/e + ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*Tan[d + e*x])/(2*Sqr
t[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(8*c^(3/2)*e) - ((b^2 -
 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a +
b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(128*c^(7/2)*e) - (Sqrt[a^2 + b^2 + c
*(c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^
2])]*ArcTanh[(b*Sqrt[a^2 + b^2 - 2*a*c + c^2] + (b^2 + (a - c)*(a - c - Sqr
```

```
t[a^2 + b^2 - 2*a*c + c^2]))*Tan[d + e*x])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^
2)^(1/4)*Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c +
Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2]
)])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*e) - ((b + 2*c*Tan[d + e*x])*S
qrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/(4*c*e) + ((5*b^2 - 4*a*c)*(b +
 2*c*Tan[d + e*x])*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/(64*c^3*e)
- (5*b*(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2))/(24*c^2*e) + (Tan[d +
 e*x]*(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2))/(4*c*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1004

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

`[n, 0]`

Rubi steps

$$\text{integral}=\frac{\text{Subst}\left(\int\frac{x^4\sqrt{a+bx+cx^2}}{1+x^2}\,dx,x,\tan(d+ex)\right)}{e}$$

$$=\frac{\text{Subst}\left(\int\left(-\sqrt{a+bx+cx^2}+x^2\sqrt{a+bx+cx^2}+\frac{\sqrt{a+bx+cx^2}}{1+x^2}\right)\,dx,x,\tan(d+ex)\right)}{e}$$

$$=-\frac{\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,\tan(d+ex)\right)}{e}+\frac{\text{Subst}\left(\int x^2\sqrt{a+bx+cx^2}\,dx,x,\tan(d+ex)\right)}{e}+\frac{\text{Subst}\left(\int\frac{\sqrt{a+bx+cx^2}}{1+x^2}\,dx,x,\tan(d+ex)\right)}{e}$$

$$=-\frac{(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ce}+\frac{\tan(d+ex)\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}{4ce}-\frac{\text{Subst}\left(\int\frac{-a+c-bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{e}+\frac{\text{Subst}\left(\int\left(-a-\frac{5bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,\tan(d+ex)\right)}{4ce}+\frac{c\text{Subst}\left(\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{e}+\frac{(b^2-4ac)\,\text{Subst}\left(\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{8ce}$$

$$
\begin{aligned}
&= -\frac{(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ce} \\
&\quad -\frac{5b(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{24c^2e} \\
&\quad +\frac{\tan(d+ex)\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}{4ce} \\
&\quad +\frac{(2c)\mathrm{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e} \\
&\quad +\frac{(b^2-4ac)\,\mathrm{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{4ce} \\
&\quad +\frac{(5b^2-4ac)\,\mathrm{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,\tan(d+ex)\right)}{16c^2e} \\
&\quad -\frac{\mathrm{Subst}\left(\int\frac{b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)-b\sqrt{a^2+b^2-2ac+c^2}\,x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}\,e} \\
&\quad +\frac{\mathrm{Subst}\left(\int\frac{b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)+b\sqrt{a^2+b^2-2ac+c^2}\,x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}\,e} \\
&= \frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e} \\
&\quad +\frac{(b^2-4ac)\,\mathrm{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8c^{3/2}e} \\
&\quad -\frac{(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ce} \\
&\quad +\frac{(5b^2-4ac)\,(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{64c^3e} \\
&\quad -\frac{5b(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{24c^2e} \\
&\quad +\frac{\tan(d+ex)\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}{4ce} \\
&\quad -\frac{((b^2-4ac)\,(5b^2-4ac))\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{128c^3e} \\
&\quad -\frac{\left(b\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\mathrm{Subst}\left(\int\frac{1}{-2b\sqrt{a^2+b^2-2ac+c^2}\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-\cdots}\right.\right.}\right.}{e} \\
&\quad -\frac{\left(b\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\mathrm{Subst}\left(\int\frac{1}{2b\sqrt{a^2+b^2-2ac+c^2}\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2\cdots}\right.\right.}\right.}{e}
\end{aligned}
$$

$$=$$

$$-\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2-}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+}}$$

$$+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e}$$

$$+\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8c^{3/2}e}$$

$$-\frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\operatorname{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2-}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+}}$$

$$-\frac{(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ce}$$

$$+\frac{(5b^2-4ac)(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{64c^3e}$$

$$-\frac{5b(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{24c^2e}$$

$$+\frac{\tan(d+ex)(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{4ce}$$

$$-\frac{((b^2-4ac)(5b^2-4ac))\operatorname{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{64c^3e}$$

$$= -\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac}}$$

$$+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e}$$

$$+\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8c^{3/2}e}$$

$$-\frac{(b^2-4ac)\,(5b^2-4ac)\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{128c^{7/2}e}$$

$$-\frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\,\text{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac}}$$

$$-\frac{(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ce}$$

$$+\frac{(5b^2-4ac)\,(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{64c^3e}$$

$$-\frac{5b(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{24c^2e}$$

$$+\frac{\tan(d+ex)\,(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{4ce}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.62 (sec) , antiderivative size = 511, normalized size of antiderivative = 0.57

$$\int \tan^4(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$

$$= \frac{-4i\sqrt{a-ib-c}\,\text{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)+4i\sqrt{a+ib-c}\,\text{arctanh}\left(\frac{2a+ib+(b+2ic)\tan}{2\sqrt{a+ib-c}\sqrt{a+b\tan(d+ex}}\right.}{}$$

```
[In] Integrate[Tan[d + e*x]^4*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] ((-4*I)*Sqrt[a - I*b - c]*ArcTanh[(2*a - I*b + (b - (2*I)*c)*Tan[d + e*x])/
(2*Sqrt[a - I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])] + (4*I)*
Sqrt[a + I*b - c]*ArcTanh[(2*a + I*b + (b + (2*I)*c)*Tan[d + e*x])/(2*Sqrt[
a + I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])] + 8*Sqrt[c]*ArcT
```

```
anh[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e
*x]^2])] + ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a
+ b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/c^(3/2) - (2*(b + 2*c*Tan[d + e*x])
*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/c - (5*b*(a + b*Tan[d + e*x]
+ c*Tan[d + e*x]^2)^(3/2))/(3*c^2) + (2*Tan[d + e*x]*(a + b*Tan[d + e*x] +
c*Tan[d + e*x]^2)^(3/2))/c - ((-5*b^2 + 4*a*c)*(-((b^2 - 4*a*c)*ArcTanh[(b
+ 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])
]) + 2*Sqrt[c]*(b + 2*c*Tan[d + e*x])*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e
*x]^2]))/(16*c^(7/2)))/(8*e)
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.22 (sec) , antiderivative size = 17248526, normalized size of antiderivative = 19402.17

output too large to display

```
[In] int((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)^4,x)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2427 vs. 2(795) = 1590.

Time = 1.13 (sec) , antiderivative size = 4855, normalized size of antiderivative = 5.46

$$\int \tan^4(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)^4,x, algorithm="
fricas")
```

```
[Out] [1/768*(192*c^4*e*sqrt((e^2*sqrt(-b^2/e^4) - a + c)/e^2)*log((2*(4*a^3*b^2
+ 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b -
 b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) + (2*(2*a^3*b + a*b^3 + b
^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^
2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2
+ b*tan(e*x + d) + a) + ((b^5 + 2*(4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 5*a*b^
3)*c)*e*tan(e*x + d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^4*c - 2*a*b^2*c^2)*e*tan(
e*x + d) - (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b + a*b^3)*c)*e - ((4*a^3*
b + 3*a*b^3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3*tan(e*x + d)^2 + 2*(4*a^
4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^3*tan(e*
x + d) - (4*a^3*b + a*b^3 + (4*a^2*b + b^3)*c)*e^3)*sqrt(-b^2/e^4))*sqrt((e
^2*sqrt(-b^2/e^4) - a + c)/e^2))/(tan(e*x + d)^2 + 1)) - 192*c^4*e*sqrt((e^
2*sqrt(-b^2/e^4) - a + c)/e^2)*log((2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*
```

```
b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3
*a*b^3)*c)*tan(e*x + d) + (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(
e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^
2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) - ((
b^5 + 2*(4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 5*a*b^3)*c)*e*tan(e*x + d)^2 - 4
*(2*a^3*b^2 + a*b^4 + b^4*c - 2*a*b^2*c^2)*e*tan(e*x + d) - (8*a^4*b + 6*a^
2*b^3 + b^5 - 2*(4*a^3*b + a*b^3)*c)*e - ((4*a^3*b + 3*a*b^3 - 8*a*b*c^2 -
(4*a^2*b - 3*b^3)*c)*e^3*tan(e*x + d)^2 + 2*(4*a^4 + 3*a^2*b^2 + b^4 + (4*a
^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^3*tan(e*x + d) - (4*a^3*b + a*b^3
+ (4*a^2*b + b^3)*c)*e^3)*sqrt(-b^2/e^4))*sqrt((e^2*sqrt(-b^2/e^4) - a + c)
/e^2))/(tan(e*x + d)^2 + 1)) + 192*c^4*e*sqrt(-(e^2*sqrt(-b^2/e^4) + a - c)
/e^2)*log((2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^
2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) -
 (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2
*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4)
)*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) + ((b^5 + 2*(4*a^2*b - b^3)*c
^2 - 2*(4*a^3*b + 5*a*b^3)*c)*e*tan(e*x + d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^4
*c - 2*a*b^2*c^2)*e*tan(e*x + d) - (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b
+ a*b^3)*c)*e + ((4*a^3*b + 3*a*b^3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3*
tan(e*x + d)^2 + 2*(4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3
+ 3*a*b^2)*c)*e^3*tan(e*x + d) - (4*a^3*b + a*b^3 + (4*a^2*b + b^3)*c)*e^3)
*sqrt(-b^2/e^4))*sqrt(-(e^2*sqrt(-b^2/e^4) + a - c)/e^2))/(tan(e*x + d)^2 +
 1)) - 192*c^4*e*sqrt(-(e^2*sqrt(-b^2/e^4) + a - c)/e^2)*log((2*(4*a^3*b^2
+ 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b -
 b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) - (2*(2*a^3*b + a*b^3 + b
^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^
2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2
+ b*tan(e*x + d) + a) - ((b^5 + 2*(4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 5*a*b^
3)*c)*e*tan(e*x + d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^4*c - 2*a*b^2*c^2)*e*tan(
e*x + d) - (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b + a*b^3)*c)*e + ((4*a^3*
b + 3*a*b^3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3*tan(e*x + d)^2 + 2*(4*a^
4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^3*tan(e*
x + d) - (4*a^3*b + a*b^3 + (4*a^2*b + b^3)*c)*e^3)*sqrt(-b^2/e^4))*sqrt(-(
e^2*sqrt(-b^2/e^4) + a - c)/e^2))/(tan(e*x + d)^2 + 1)) - 3*(5*b^4 - 24*a*b
^2*c + 64*a*c^3 - 128*c^4 + 16*(a^2 - b^2)*c^2)*sqrt(c)*log(8*c^2*tan(e*x +
 d)^2 + 8*b*c*tan(e*x + d) + b^2 + 4*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d)
 + a)*(2*c*tan(e*x + d) + b)*sqrt(c) + 4*a*c) + 4*(48*c^4*tan(e*x + d)^3 +
8*b*c^3*tan(e*x + d)^2 + 15*b^3*c - 52*a*b*c^2 - 48*b*c^3 - 2*(5*b^2*c^2 -
12*a*c^3 + 48*c^4)*tan(e*x + d))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a
))/(c^4*e), 1/384*(96*c^4*e*sqrt((e^2*sqrt(-b^2/e^4) - a + c)/e^2)*log((2*(
4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 +
(4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) + (2*(2*a^3*b +
 a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (
4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e
*x + d)^2 + b*tan(e*x + d) + a) + ((b^5 + 2*(4*a^2*b - b^3)*c^2 - 2*(4*a^3*
```

```
b + 5*a*b^3)*c)*e*tan(e*x + d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^4*c - 2*a*b^2*c
^2)*e*tan(e*x + d) - (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b + a*b^3)*c)*e
- ((4*a^3*b + 3*a*b^3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3*tan(e*x + d)^2
 + 2*(4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*
e^3*tan(e*x + d) - (4*a^3*b + a*b^3 + (4*a^2*b + b^3)*c)*e^3)*sqrt(-b^2/e^4
))*sqrt((e^2*sqrt(-b^2/e^4) - a + c)/e^2))/(tan(e*x + d)^2 + 1)) - 96*c^4*e
*sqrt((e^2*sqrt(-b^2/e^4) - a + c)/e^2)*log((2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4
*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*
a^3*b + 3*a*b^3)*c)*tan(e*x + d) + (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)
*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3
 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d)
+ a) - ((b^5 + 2*(4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 5*a*b^3)*c)*e*tan(e*x +
 d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^4*c - 2*a*b^2*c^2)*e*tan(e*x + d) - (8*a^4
*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b + a*b^3)*c)*e - ((4*a^3*b + 3*a*b^3 - 8*a
*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3*tan(e*x + d)^2 + 2*(4*a^4 + 3*a^2*b^2 + b
^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^3*tan(e*x + d) - (4*a^3*b
 + a*b^3 + (4*a^2*b + b^3)*c)*e^3)*sqrt(-b^2/e^4))*sqrt((e^2*sqrt(-b^2/e^4)
 - a + c)/e^2))/(tan(e*x + d)^2 + 1)) + 96*c^4*e*sqrt(-(e^2*sqrt(-b^2/e^4)
+ a - c)/e^2)*log((2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*
b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*
x + d) - (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4
 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-
b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) + ((b^5 + 2*(4*a^2*b
- b^3)*c^2 - 2*(4*a^3*b + 5*a*b^3)*c)*e*tan(e*x + d)^2 - 4*(2*a^3*b^2 + a*b
^4 + b^4*c - 2*a*b^2*c^2)*e*tan(e*x + d) - (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(
4*a^3*b + a*b^3)*c)*e + ((4*a^3*b + 3*a*b^3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)
*c)*e^3*tan(e*x + d)^2 + 2*(4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2
*(4*a^3 + 3*a*b^2)*c)*e^3*tan(e*x + d) - (4*a^3*b + a*b^3 + (4*a^2*b + b^3)
*c)*e^3)*sqrt(-b^2/e^4))*sqrt(-(e^2*sqrt(-b^2/e^4) + a - c)/e^2))/(tan(e*x
+ d)^2 + 1)) - 96*c^4*e*sqrt(-(e^2*sqrt(-b^2/e^4) + a - c)/e^2)*log((2*(4*a
^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*
a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) - (2*(2*a^3*b + a*
b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a
^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x
+ d)^2 + b*tan(e*x + d) + a) - ((b^5 + 2*(4*a^2*b - b^3)*c^2 - 2*(4*a^3*b +
 5*a*b^3)*c)*e*tan(e*x + d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^4*c - 2*a*b^2*c^2)
*e*tan(e*x + d) - (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b + a*b^3)*c)*e + (
(4*a^3*b + 3*a*b^3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3*tan(e*x + d)^2 +
2*(4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^3
*tan(e*x + d) - (4*a^3*b + a*b^3 + (4*a^2*b + b^3)*c)*e^3)*sqrt(-b^2/e^4))*
sqrt(-(e^2*sqrt(-b^2/e^4) + a - c)/e^2))/(tan(e*x + d)^2 + 1)) + 3*(5*b^4 -
 24*a*b^2*c + 64*a*c^3 - 128*c^4 + 16*(a^2 - b^2)*c^2)*sqrt(-c)*arctan(1/2*
sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a)*(2*c*tan(e*x + d) + b)*sqrt(-c)
/(c^2*tan(e*x + d)^2 + b*c*tan(e*x + d) + a*c)) + 2*(48*c^4*tan(e*x + d)^3
+ 8*b*c^3*tan(e*x + d)^2 + 15*b^3*c - 52*a*b*c^2 - 48*b*c^3 - 2*(5*b^2*c^2
```

```
- 12*a*c^3 + 48*c^4)*tan(e*x + d))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) +
 a))/(c^4*e)]
```

Sympy [F]

$$\int \tan^4(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \int \sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\tan^4(d+ex)\,dx$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2)*tan(e*x+d)**4,x)
[Out] Integral(sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2)*tan(d + e*x)**4, x)
```

Maxima [F]

$$\int \tan^4(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \int \sqrt{c\tan(ex+d)^2+b\tan(ex+d)+a}\tan(ex+d)^4\,dx$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)^4,x, algorithm="
maxima")
[Out] integrate(sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a)*tan(e*x + d)^4, x)
```

Giac [F(-1)]

Timed out.

$$\int \tan^4(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)^4,x, algorithm="
giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \tan^4(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx=\text{Hanged}$$

```
[In] int(tan(d + e*x)^4*(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)
[Out] \text{Hanged}
```

3.3 $\int \tan^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 748

$$\int \tan^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx =$$

$$-\frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}e}$$

$$-\frac{b\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{c}e}$$

$$+\frac{b(b^2-4ac)\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{16c^{5/2}e}$$

$$+\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\text{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}e}$$

$$-\frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e}$$

$$-\frac{b(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{8c^2e}$$

$$+\frac{\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}{3ce}$$

```
[Out] 1/16*b*(-4*a*c+b^2)*arctanh(1/2*(b+2*c*tan(e*x+d))/c^(1/2)/(a+b*tan(e*x+d)+
c*tan(e*x+d)^2)^(1/2))/c^(5/2)/e-1/2*b*arctanh(1/2*(b+2*c*tan(e*x+d))/c^(1/
2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))/e/c^(1/2)+1/2*arctanh(1/2*(b^2+(a
-c)*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))+b*(a^2-2*a*c+b^2+c^2)^(1/2)*tan(e*x+d))
```

```
/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)/(a^2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))
-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(
1/2))*(a^2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(
1/2)))^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/4)/e*2^(1/2)-1/2*arctan(1/2*(b^2+(a-c)*
(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))-b*(a^2-2*a*c+b^2+c^2)^(1/2)*tan(e*x+d))/(a^
2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)/(a^2+b^2+c*(c+(a^2-2*a*c+b^2+c^2)^(1/2))-a*(
2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)
)*(a^2+b^2+c*(c+(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)
))^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/4)/e*2^(1/2)-(a+b*tan(e*x+d)+c*tan(e*x+d)^2
)^(1/2)/e-1/8*b*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*(b+2*c*tan(e*x+d))/c^
2/e+1/3*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2)/c/e
```

Rubi [A] (verified)

Time = 24.33 (sec) , antiderivative size = 748, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3781, 6857, 654, 626, 635, 212, 1035, 1092, 1050, 1044, 214, 211}

$$\int \tan^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx =$$

$$-\frac{\sqrt{-a\left(\sqrt{a^2-2ac+b^2+c^2}+2c\right)+c\left(\sqrt{a^2-2ac+b^2+c^2}+c\right)+a^2+b^2}\arctan\left(\frac{\cdots}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+\cdots}}\right.}{\sqrt{2}e\sqrt[4]{a^2-2ac+b^2+c^2}}$$

$$+\frac{\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+a^2+b^2}\,\text{arctanh}\left(\frac{\cdots}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+\cdots}}\right.}{\sqrt{2}e\sqrt[4]{a^2-2ac+b^2+c^2}}$$

$$+\frac{b(b^2-4ac)\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{16c^{5/2}e}$$

$$-\frac{b\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{c}e}$$

$$-\frac{b(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{8c^2e}$$

$$+\frac{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{3ce}-\frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e}$$

```
[In] Int[Tan[d + e*x]^3*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] -((Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a
^2 + b^2 - 2*a*c + c^2])]*ArcTan[(b^2 + (a - c)*(a - c - Sqrt[a^2 + b^2 - 2
*a*c + c^2]) - b*Sqrt[a^2 + b^2 - 2*a*c + c^2]*Tan[d + e*x])/(Sqrt[2]*(a^2
+ b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c +
```

```
 c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*Tan[d + e*x] +
 c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*e)) - (b*Arc
Tanh[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d +
e*x]^2])])/(2*Sqrt[c]*e) + (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*Tan[d + e*x])/
(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(16*c^(5/2)*e) +
(Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a^2
 + b^2 - 2*a*c + c^2])]*ArcTanh[(b^2 + (a - c)*(a - c + Sqrt[a^2 + b^2 - 2*
a*c + c^2]) + b*Sqrt[a^2 + b^2 - 2*a*c + c^2]*Tan[d + e*x])/(Sqrt[2]*(a^2 +
 b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c +
c^2]) - a*(2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*Tan[d + e*x] +
c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*e) - Sqrt[a +
 b*Tan[d + e*x] + c*Tan[d + e*x]^2]/e - (b*(b + 2*c*Tan[d + e*x])*Sqrt[a +
b*Tan[d + e*x] + c*Tan[d + e*x]^2])/(8*c^2*e) + (a + b*Tan[d + e*x] + c*Tan
[d + e*x]^2)^(3/2)/(3*c*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^3\sqrt{a+bx+cx^2}}{1+x^2}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(x\sqrt{a+bx+cx^2} - \frac{x\sqrt{a+bx+cx^2}}{1+x^2}\right)dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int x\sqrt{a+bx+cx^2}\,dx, x, \tan(d+ex)\right)}{e} - \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx+cx^2}}{1+x^2}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= -\frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e} + \frac{\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}{3ce} + \frac{\text{Subst}\left(\int \frac{\frac{b}{2}-(a-c)x-\frac{bx^2}{2}}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e} - \frac{b\text{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx, x, \tan(d+ex)\right)}{2ce}$$

$$= -\frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e} - \frac{b(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{8c^2e} + \frac{\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}{3ce} + \frac{\text{Subst}\left(\int \frac{b+(-a+c)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2e} + \frac{(b(b^2-4ac))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{16c^2e}$$

$$
\begin{aligned}
&= -\frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e} \\
&\quad - \frac{b(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{8c^2e} \\
&\quad + \frac{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{3ce} \\
&\quad - \frac{b\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e} \\
&\quad + \frac{(b(b^2-4ac))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8c^2e} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-b\sqrt{a^2+b^2-2ac+c^2}+\left(-b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}\,e} \\
&\quad + \frac{\text{Subst}\left(\int \frac{b\sqrt{a^2+b^2-2ac+c^2}+\left(-b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}\,e}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{c}e} \\
&\quad + \frac{b(b^2-4ac)\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{16c^{5/2}e} \\
&\quad - \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e} \\
&\quad - \frac{b(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{8c^2e} \\
&\quad + \frac{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}{3ce} \\
&\quad + \frac{\left(b\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{1}{2b\sqrt{a^2+b^2-2ac+c^2}\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac\right.\right.}}\right.}{e} \\
&\quad + \frac{\left(b\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{1}{-2b\sqrt{a^2+b^2-2ac+c^2}\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2\right.\right.}}\right.}{e}
\end{aligned}
$$

$$= -\frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac}}$$

$$-\frac{b\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{c}e}$$

$$+\frac{b(b^2-4ac)\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{16c^{5/2}e}$$

$$+\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\text{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac}}$$

$$-\frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e}$$

$$-\frac{b(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{8c^2e}$$

$$+\frac{\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}{3ce}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.57 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.52

$$\int \tan^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$

$$=\frac{3\sqrt{a-ib-c}\text{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)+3\sqrt{a+ib-c}\text{arctanh}\left(\frac{2a+ib+(b+2ic)\tan(d+e}{2\sqrt{a+ib-c}\sqrt{a+b\tan(d+ex)+c\,t}}\right.}{}$$

```
[In] Integrate[Tan[d + e*x]^3*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (3*Sqrt[a - I*b - c]*ArcTanh[(2*a - I*b + (b - (2*I)*c)*Tan[d + e*x])/(2*Sq
rt[a - I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])] + 3*Sqrt[a +
I*b - c]*ArcTanh[(2*a + I*b + (b + (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a + I*b -
 c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])] - (3*b*ArcTanh[(b + 2*c*T
an[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/Sqrt
[c] + (3*b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a +
 b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(8*c^(5/2)) - 6*Sqrt[a + b*Tan[d + e
*x] + c*Tan[d + e*x]^2] - (3*b*(b + 2*c*Tan[d + e*x])*Sqrt[a + b*Tan[d + e*
x] + c*Tan[d + e*x]^2])/(4*c^2) + (2*(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2
)^(3/2))/c)/(6*e)
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.79 (sec) , antiderivative size = 17766953, normalized size of antiderivative = 23752.61

output too large to display

```
[In] int((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)^3,x)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2396 vs. 2(671) = 1342.

Time = 0.90 (sec) , antiderivative size = 4793, normalized size of antiderivative = 6.41

$$\int \tan^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)^3,x, algorithm="
fricas")
```

```
[Out] [-1/96*(24*c^3*e*sqrt(-(e^2*sqrt(-b^2/e^4) - a + c)/e^2)*log(-(2*(4*a^3*b^2
 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b
- b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) + (2*(2*a^3*b + a*b^3 +
b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b
^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2
 + b*tan(e*x + d) + a) + ((4*a^3*b^2 + 3*a*b^4 - 8*a*b^2*c^2 - (4*a^2*b^2 -
 3*b^4)*c)*e*tan(e*x + d)^2 + 2*(4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3
)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*e*tan(e*x + d) - (4*a^3*b^2 + a*b^4 + (4*a
^2*b^2 + b^4)*c)*e + ((b^4 + 2*(4*a^2 - b^2)*c^2 - 2*(4*a^3 + 5*a*b^2)*c)*e
^3*tan(e*x + d)^2 - 4*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^3*tan(e*x + d
) - (8*a^4 + 6*a^2*b^2 + b^4 - 2*(4*a^3 + a*b^2)*c)*e^3)*sqrt(-b^2/e^4))*sq
rt(-(e^2*sqrt(-b^2/e^4) - a + c)/e^2))/(tan(e*x + d)^2 + 1)) - 24*c^3*e*sqr
t(-(e^2*sqrt(-b^2/e^4) - a + c)/e^2)*log(-(2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c
 - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^
3*b + 3*a*b^3)*c)*tan(e*x + d) + (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e
^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 +
 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) +
a) - ((4*a^3*b^2 + 3*a*b^4 - 8*a*b^2*c^2 - (4*a^2*b^2 - 3*b^4)*c)*e*tan(e*x
 + d)^2 + 2*(4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b +
 3*a*b^3)*c)*e*tan(e*x + d) - (4*a^3*b^2 + a*b^4 + (4*a^2*b^2 + b^4)*c)*e +
 ((b^4 + 2*(4*a^2 - b^2)*c^2 - 2*(4*a^3 + 5*a*b^2)*c)*e^3*tan(e*x + d)^2 -
4*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^3*tan(e*x + d) - (8*a^4 + 6*a^2*b
^2 + b^4 - 2*(4*a^3 + a*b^2)*c)*e^3)*sqrt(-b^2/e^4))*sqrt(-(e^2*sqrt(-b^2/e
```

```
^4) - a + c)/e^2))/(tan(e*x + d)^2 + 1)) + 24*c^3*e*sqrt((e^2*sqrt(-b^2/e^4
) + a - c)/e^2)*log(-(2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a
^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan
(e*x + d) - (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*
a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqr
t(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) + ((4*a^3*b^2 + 3*
a*b^4 - 8*a*b^2*c^2 - (4*a^2*b^2 - 3*b^4)*c)*e*tan(e*x + d)^2 + 2*(4*a^4*b
+ 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*e*tan(e*
x + d) - (4*a^3*b^2 + a*b^4 + (4*a^2*b^2 + b^4)*c)*e - ((b^4 + 2*(4*a^2 - b
^2)*c^2 - 2*(4*a^3 + 5*a*b^2)*c)*e^3*tan(e*x + d)^2 - 4*(2*a^3*b + a*b^3 +
b^3*c - 2*a*b*c^2)*e^3*tan(e*x + d) - (8*a^4 + 6*a^2*b^2 + b^4 - 2*(4*a^3 +
 a*b^2)*c)*e^3)*sqrt(-b^2/e^4))*sqrt((e^2*sqrt(-b^2/e^4) + a - c)/e^2))/(ta
n(e*x + d)^2 + 1)) - 24*c^3*e*sqrt((e^2*sqrt(-b^2/e^4) + a - c)/e^2)*log(-(
2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5
 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) - (2*(2*a^3*
b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4
+ (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*ta
n(e*x + d)^2 + b*tan(e*x + d) + a) - ((4*a^3*b^2 + 3*a*b^4 - 8*a*b^2*c^2 -
(4*a^2*b^2 - 3*b^4)*c)*e*tan(e*x + d)^2 + 2*(4*a^4*b + 3*a^2*b^3 + b^5 + (4
*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*e*tan(e*x + d) - (4*a^3*b^2 +
a*b^4 + (4*a^2*b^2 + b^4)*c)*e - ((b^4 + 2*(4*a^2 - b^2)*c^2 - 2*(4*a^3 + 5
*a*b^2)*c)*e^3*tan(e*x + d)^2 - 4*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^3
*tan(e*x + d) - (8*a^4 + 6*a^2*b^2 + b^4 - 2*(4*a^3 + a*b^2)*c)*e^3)*sqrt(-
b^2/e^4))*sqrt((e^2*sqrt(-b^2/e^4) + a - c)/e^2))/(tan(e*x + d)^2 + 1)) + 3
*(b^3 - 4*a*b*c - 8*b*c^2)*sqrt(c)*log(8*c^2*tan(e*x + d)^2 + 8*b*c*tan(e*x
 + d) + b^2 - 4*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a)*(2*c*tan(e*x +
d) + b)*sqrt(c) + 4*a*c) - 4*(8*c^3*tan(e*x + d)^2 + 2*b*c^2*tan(e*x + d) -
 3*b^2*c + 8*a*c^2 - 24*c^3)*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a))/(
c^3*e), -1/48*(12*c^3*e*sqrt(-(e^2*sqrt(-b^2/e^4) - a + c)/e^2)*log(-(2*(4*
a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4
*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) + (2*(2*a^3*b + a
*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*
a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x
 + d)^2 + b*tan(e*x + d) + a) + ((4*a^3*b^2 + 3*a*b^4 - 8*a*b^2*c^2 - (4*a^
2*b^2 - 3*b^4)*c)*e*tan(e*x + d)^2 + 2*(4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*
b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*e*tan(e*x + d) - (4*a^3*b^2 + a*b^4
 + (4*a^2*b^2 + b^4)*c)*e + ((b^4 + 2*(4*a^2 - b^2)*c^2 - 2*(4*a^3 + 5*a*b^
2)*c)*e^3*tan(e*x + d)^2 - 4*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^3*tan(
e*x + d) - (8*a^4 + 6*a^2*b^2 + b^4 - 2*(4*a^3 + a*b^2)*c)*e^3)*sqrt(-b^2/e
^4))*sqrt(-(e^2*sqrt(-b^2/e^4) - a + c)/e^2))/(tan(e*x + d)^2 + 1)) - 12*c^
3*e*sqrt(-(e^2*sqrt(-b^2/e^4) - a + c)/e^2)*log(-(2*(4*a^3*b^2 + 2*a*b^4 +
2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 -
2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) + (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b
*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(
4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x
```

```
+ d) + a) - ((4*a^3*b^2 + 3*a*b^4 - 8*a*b^2*c^2 - (4*a^2*b^2 - 3*b^4)*c)*e*
tan(e*x + d)^2 + 2*(4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*
a^3*b + 3*a*b^3)*c)*e*tan(e*x + d) - (4*a^3*b^2 + a*b^4 + (4*a^2*b^2 + b^4)
*c)*e + ((b^4 + 2*(4*a^2 - b^2)*c^2 - 2*(4*a^3 + 5*a*b^2)*c)*e^3*tan(e*x +
d)^2 - 4*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^3*tan(e*x + d) - (8*a^4 +
6*a^2*b^2 + b^4 - 2*(4*a^3 + a*b^2)*c)*e^3)*sqrt(-b^2/e^4))*sqrt(-(e^2*sqrt
(-b^2/e^4) - a + c)/e^2))/(tan(e*x + d)^2 + 1)) + 12*c^3*e*sqrt((e^2*sqrt(-
b^2/e^4) + a - c)/e^2)*log(-(2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2
 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)
*c)*tan(e*x + d) - (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d
) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e
^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) + ((4*a^3*b
^2 + 3*a*b^4 - 8*a*b^2*c^2 - (4*a^2*b^2 - 3*b^4)*c)*e*tan(e*x + d)^2 + 2*(4
*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*e
*tan(e*x + d) - (4*a^3*b^2 + a*b^4 + (4*a^2*b^2 + b^4)*c)*e - ((b^4 + 2*(4*
a^2 - b^2)*c^2 - 2*(4*a^3 + 5*a*b^2)*c)*e^3*tan(e*x + d)^2 - 4*(2*a^3*b + a
*b^3 + b^3*c - 2*a*b*c^2)*e^3*tan(e*x + d) - (8*a^4 + 6*a^2*b^2 + b^4 - 2*(
4*a^3 + a*b^2)*c)*e^3)*sqrt(-b^2/e^4))*sqrt((e^2*sqrt(-b^2/e^4) + a - c)/e^
2))/(tan(e*x + d)^2 + 1)) - 12*c^3*e*sqrt((e^2*sqrt(-b^2/e^4) + a - c)/e^2)
*log(-(2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^
3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) - (2*
(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2
 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sq
rt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) - ((4*a^3*b^2 + 3*a*b^4 - 8*a*b^2
*c^2 - (4*a^2*b^2 - 3*b^4)*c)*e*tan(e*x + d)^2 + 2*(4*a^4*b + 3*a^2*b^3 + b
^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*e*tan(e*x + d) - (4*a^3
*b^2 + a*b^4 + (4*a^2*b^2 + b^4)*c)*e - ((b^4 + 2*(4*a^2 - b^2)*c^2 - 2*(4*
a^3 + 5*a*b^2)*c)*e^3*tan(e*x + d)^2 - 4*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c
^2)*e^3*tan(e*x + d) - (8*a^4 + 6*a^2*b^2 + b^4 - 2*(4*a^3 + a*b^2)*c)*e^3)
*sqrt(-b^2/e^4))*sqrt((e^2*sqrt(-b^2/e^4) + a - c)/e^2))/(tan(e*x + d)^2 +
1)) + 3*(b^3 - 4*a*b*c - 8*b*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*tan(e*x + d)^2
 + b*tan(e*x + d) + a)*(2*c*tan(e*x + d) + b)*sqrt(-c)/(c^2*tan(e*x + d)^2
+ b*c*tan(e*x + d) + a*c)) - 2*(8*c^3*tan(e*x + d)^2 + 2*b*c^2*tan(e*x + d)
 - 3*b^2*c + 8*a*c^2 - 24*c^3)*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a))
/(c^3*e)]
```

Sympy [F]

$$\int \tan^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \int \sqrt{a+b\tan{(d+ex)}+c\tan^2{(d+ex)}}\tan^3{(d+ex)}\,dx$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2)*tan(e*x+d)**3,x)
```

```
[Out] Integral(sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2)*tan(d + e*x)**3, x)
```

Maxima [F]

$$\int \tan^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$=\int \sqrt{c\tan(ex+d)^2+b\tan(ex+d)+a}\tan(ex+d)^3\,dx$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)^3,x, algorithm="
maxima")
[Out] integrate(sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a)*tan(e*x + d)^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \tan^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)^3,x, algorithm="
giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \tan^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$=\int \tan(d+e\,x)^3\,\sqrt{c\tan(d+e\,x)^2+b\tan(d+e\,x)+a}\,dx$$

```
[In] int(tan(d + e*x)^3*(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)
[Out] int(tan(d + e*x)^3*(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2), x)
```

3.4 $\int \tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 676

$$\int \tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$

$$= \frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{b\sqrt{a^2+b^2-2ac+c}}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}\sqrt{}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}e}$$

$$-\frac{(b^2-4(a-2c)c)\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8c^{3/2}e}$$

$$+\frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\,\text{arctanh}\left(\frac{b\sqrt{a^2+b^2-2a}}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}e}$$

$$+\frac{(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ce}$$

```
[Out] -1/8*(b^2-4*(a-2*c)*c)*arctanh(1/2*(b+2*c*tan(e*x+d))/c^(1/2)/(a+b*tan(e*x+
d)+c*tan(e*x+d)^2)^(1/2))/c^(3/2)/e+1/2*arctan(1/2*(b*(a^2-2*a*c+b^2+c^2)^(
1/2)-(a^2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1
/2)))*tan(e*x+d))/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)/(a^2+b^2+c*(c-(a^2-2*a*
c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a+b*tan(e*x+d)+
c*tan(e*x+d)^2)^(1/2))*(a^2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2
-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/4)/e*2^(1/2)+1/2*arcta
nh(1/2*(b*(a^2-2*a*c+b^2+c^2)^(1/2)+(a^2+b^2+c*(c+(a^2-2*a*c+b^2+c^2)^(1/2)
)-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))*tan(e*x+d))/(a^2-2*a*c+b^2+c^2)^(1/4)*
2^(1/2)/(a^2+b^2+c*(c+(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)
^(1/2)))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a^2+b^2+c*(c+(a^2-2*
```

```
a*c+b^2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+b^2
+c^2)^(1/4)/e*2^(1/2)+1/4*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*(b+2*c*tan(
e*x+d))/c/e
```

Rubi [A] (verified)

Time = 27.18 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3781, 1085, 1092, 635, 212, 1050, 1044, 214, 211}

$$\int \tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$

$$= \frac{\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+a^2+b^2}\arctan\left(\frac{b\sqrt{a^2-2ac+}}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+}}\right.}{\sqrt{2}e\sqrt[4]{a^2-2ac+b^2+c^2}}$$

$$+\frac{\sqrt{-a\left(\sqrt{a^2-2ac+b^2+c^2}+2c\right)+c\left(\sqrt{a^2-2ac+b^2+c^2}+c\right)+a^2+b^2}\operatorname{arctanh}\left(\frac{\left(-a\left(\sqrt{a}\right.\right.}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2}}\right.}{\sqrt{2}e\sqrt[4]{a^2-2ac+b^2+c^2}}$$

$$-\frac{(b^2-4c(a-2c))\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8c^{3/2}e}$$

$$+\frac{(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ce}$$

```
[In] Int[Tan[d + e*x]^2*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a^2
 + b^2 - 2*a*c + c^2])]*ArcTan[(b*Sqrt[a^2 + b^2 - 2*a*c + c^2] - (a^2 + b^
2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a^2 + b^2 - 2*a*c
 + c^2]))*Tan[d + e*x])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 +
 b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a^2 + b^2 - 2*
a*c + c^2])]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 +
 b^2 - 2*a*c + c^2)^(1/4)*e) - ((b^2 - 4*(a - 2*c)*c)*ArcTanh[(b + 2*c*Tan[
d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(8*c^(3
/2)*e) + (Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c +
 Sqrt[a^2 + b^2 - 2*a*c + c^2])]*ArcTanh[(b*Sqrt[a^2 + b^2 - 2*a*c + c^2] +
 (a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2 + b
^2 - 2*a*c + c^2]))*Tan[d + e*x])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*
Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2
+ b^2 - 2*a*c + c^2])]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt
[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*e) + ((b + 2*c*Tan[d + e*x])*Sqrt[a + b
*Tan[d + e*x] + c*Tan[d + e*x]^2])/(4*c*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 1085

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(
a + b*x + c*x^2)^p*((d + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3
))), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^
2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*((-b)*f)*(q + 1)) + (p + q + 1)*(b
^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*
```

```
f)*(C*((-b)*f)*(q + 1)) + (p + q + 1)*((-b)*c*(C*(-4*d*f)*(2*p + q + 2) + f
*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x + (p*((-b)*f)*(C*((-b)*f)*(q + 1)) +
(p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C
*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A,
C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2
*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx+cx^2}}{1+x^2}\,dx, x, \tan(d+ex)\right)}{e}\\
&= \frac{(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ce}\\
&\quad - \frac{\text{Subst}\left(\int \frac{\frac{1}{4}(b^2+4ac)+2bcx+\frac{1}{4}(b^2-4(a-2c)c)x^2}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2ce}\\
&= \frac{(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ce}\\
&\quad - \frac{\text{Subst}\left(\int \frac{\frac{1}{4}(b^2+4ac)+\frac{1}{4}(-b^2+4(a-2c)c)+2bcx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2ce}\\
&\quad - \frac{(b^2-4(a-2c)c)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{8ce}
\end{aligned}
$$

$$= \frac{(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ce} - \frac{(b^2-4(a-2c)c)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{4ce} + \frac{\text{Subst}\left(\int \frac{-2c\left(a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)-2bc\sqrt{a^2+b^2-2ac+c^2}x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{4c\sqrt{a^2+b^2-2ac+c^2}e} - \frac{\text{Subst}\left(\int \frac{-2c\left(a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)+2bc\sqrt{a^2+b^2-2ac+c^2}x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{4c\sqrt{a^2+b^2-2ac+c^2}e}$$

$$= -\frac{(b^2-4(a-2c)c)\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8c^{3/2}e} + \frac{(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ce} - \frac{\left(2bc\left(a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{}{8bc^2\sqrt{a^2+b^2}\cdots}\right.}{} - \frac{\left(2bc\left(a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{}{-8bc^2\sqrt{a^2+}\cdots}\right.}{}$$

$$= \frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{b\sqrt{a^2+}\cdots}{\sqrt{2}\sqrt[4]{a^2+b^2-2a}\cdots}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c}\cdots} - \frac{(b^2-4(a-2c)c)\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8c^{3/2}e} + \frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\,\text{arctanh}\left(\frac{b\sqrt{}\cdots}{\sqrt{2}\sqrt[4]{a^2+b^2-}\cdots}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+}\cdots} + \frac{(b+2c\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ce}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.49

$$\int \tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$

$$= \frac{4i\sqrt{a-ib-c}\operatorname{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)-4i\sqrt{a+ib-c}\operatorname{arctanh}\left(\frac{2a+ib+(b+2ic)\tan(d}{2\sqrt{a+ib-c}\sqrt{a+b\tan(d+ex)+}}\right.}{}$$

```
[In] Integrate[Tan[d + e*x]^2*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] ((4*I)*Sqrt[a - I*b - c]*ArcTanh[(2*a - I*b + (b - (2*I)*c)*Tan[d + e*x])/(
2*Sqrt[a - I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])] - (4*I)*S
qrt[a + I*b - c]*ArcTanh[(2*a + I*b + (b + (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a
 + I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])] - 8*Sqrt[c]*ArcTa
nh[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*
x]^2])] + ((-b^2 + 4*a*c)*ArcTanh[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a
+ b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/c^(3/2) + (2*(b + 2*c*Tan[d + e*x])
*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/c)/(8*e)
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.33 (sec) , antiderivative size = 17248163, normalized size of antiderivative = 25515.03

output too large to display

```
[In] int((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)^2,x)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2342 vs. $2(610) = 1220$.

Time = 0.81 (sec) , antiderivative size = 4685, normalized size of antiderivative = 6.93

$$\int \tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)^2,x, algorithm="
fricas")
```

```
[Out] [-1/16*(4*c^2*e*sqrt((e^2*sqrt(-b^2/e^4) - a + c)/e^2)*log((2*(4*a^3*b^2 +
2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b
^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) + (2*(2*a^3*b + a*b^3 + b^3
*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)
*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 +
b*tan(e*x + d) + a) + ((b^5 + 2*(4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 5*a*b^3)
*c)*e*tan(e*x + d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^4*c - 2*a*b^2*c^2)*e*tan(e*
x + d) - (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b + a*b^3)*c)*e - ((4*a^3*b
+ 3*a*b^3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3*tan(e*x + d)^2 + 2*(4*a^4
+ 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^3*tan(e*x
+ d) - (4*a^3*b + a*b^3 + (4*a^2*b + b^3)*c)*e^3)*sqrt(-b^2/e^4))*sqrt((e^2
*sqrt(-b^2/e^4) - a + c)/e^2))/(tan(e*x + d)^2 + 1)) - 4*c^2*e*sqrt((e^2*sq
rt(-b^2/e^4) - a + c)/e^2)*log((2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*
c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b
^3)*c)*tan(e*x + d) + (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x
+ d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c
)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) - ((b^5
+ 2*(4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 5*a*b^3)*c)*e*tan(e*x + d)^2 - 4*(2*
a^3*b^2 + a*b^4 + b^4*c - 2*a*b^2*c^2)*e*tan(e*x + d) - (8*a^4*b + 6*a^2*b^
3 + b^5 - 2*(4*a^3*b + a*b^3)*c)*e - ((4*a^3*b + 3*a*b^3 - 8*a*b*c^2 - (4*a
^2*b - 3*b^3)*c)*e^3*tan(e*x + d)^2 + 2*(4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 -
 b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^3*tan(e*x + d) - (4*a^3*b + a*b^3 + (4
*a^2*b + b^3)*c)*e^3)*sqrt(-b^2/e^4))*sqrt((e^2*sqrt(-b^2/e^4) - a + c)/e^2
))/(tan(e*x + d)^2 + 1)) + 4*c^2*e*sqrt(-(e^2*sqrt(-b^2/e^4) + a - c)/e^2)*
log((2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3
+ b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) - (2*(2
*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 +
 b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt
(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) + ((b^5 + 2*(4*a^2*b - b^3)*c^2 - 2
*(4*a^3*b + 5*a*b^3)*c)*e*tan(e*x + d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^4*c - 2
*a*b^2*c^2)*e*tan(e*x + d) - (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b + a*b^
3)*c)*e + ((4*a^3*b + 3*a*b^3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3*tan(e*
x + d)^2 + 2*(4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*
b^2)*c)*e^3*tan(e*x + d) - (4*a^3*b + a*b^3 + (4*a^2*b + b^3)*c)*e^3)*sqrt(
-b^2/e^4))*sqrt(-(e^2*sqrt(-b^2/e^4) + a - c)/e^2))/(tan(e*x + d)^2 + 1)) -
 4*c^2*e*sqrt(-(e^2*sqrt(-b^2/e^4) + a - c)/e^2)*log((2*(4*a^3*b^2 + 2*a*b^
4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^
2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) - (2*(2*a^3*b + a*b^3 + b^3*c - 2
*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 -
 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(
e*x + d) + a) - ((b^5 + 2*(4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 5*a*b^3)*c)*e*
tan(e*x + d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^4*c - 2*a*b^2*c^2)*e*tan(e*x + d)
 - (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b + a*b^3)*c)*e + ((4*a^3*b + 3*a*
b^3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3*tan(e*x + d)^2 + 2*(4*a^4 + 3*a^
2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^3*tan(e*x + d) -
```

```
 (4*a^3*b + a*b^3 + (4*a^2*b + b^3)*c)*e^3)*sqrt(-b^2/e^4))*sqrt(-(e^2*sqrt
(-b^2/e^4) + a - c)/e^2))/(tan(e*x + d)^2 + 1)) - (b^2 - 4*a*c + 8*c^2)*sqr
t(c)*log(8*c^2*tan(e*x + d)^2 + 8*b*c*tan(e*x + d) + b^2 - 4*sqrt(c*tan(e*x
 + d)^2 + b*tan(e*x + d) + a)*(2*c*tan(e*x + d) + b)*sqrt(c) + 4*a*c) - 4*(
2*c^2*tan(e*x + d) + b*c)*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a))/(c^2
*e), -1/8*(2*c^2*e*sqrt((e^2*sqrt(-b^2/e^4) - a + c)/e^2)*log((2*(4*a^3*b^2
 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b
- b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) + (2*(2*a^3*b + a*b^3 +
b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b
^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2
 + b*tan(e*x + d) + a) + ((b^5 + 2*(4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 5*a*b
^3)*c)*e*tan(e*x + d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^4*c - 2*a*b^2*c^2)*e*tan
(e*x + d) - (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b + a*b^3)*c)*e - ((4*a^3
*b + 3*a*b^3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3*tan(e*x + d)^2 + 2*(4*a
^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^3*tan(e
*x + d) - (4*a^3*b + a*b^3 + (4*a^2*b + b^3)*c)*e^3)*sqrt(-b^2/e^4))*sqrt((
e^2*sqrt(-b^2/e^4) - a + c)/e^2))/(tan(e*x + d)^2 + 1)) - 2*c^2*e*sqrt((e^2
*sqrt(-b^2/e^4) - a + c)/e^2)*log((2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b
^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*
a*b^3)*c)*tan(e*x + d) + (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e
*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2
)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) - ((b
^5 + 2*(4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 5*a*b^3)*c)*e*tan(e*x + d)^2 - 4*
(2*a^3*b^2 + a*b^4 + b^4*c - 2*a*b^2*c^2)*e*tan(e*x + d) - (8*a^4*b + 6*a^2
*b^3 + b^5 - 2*(4*a^3*b + a*b^3)*c)*e - ((4*a^3*b + 3*a*b^3 - 8*a*b*c^2 - (
4*a^2*b - 3*b^3)*c)*e^3*tan(e*x + d)^2 + 2*(4*a^4 + 3*a^2*b^2 + b^4 + (4*a^
2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^3*tan(e*x + d) - (4*a^3*b + a*b^3 +
 (4*a^2*b + b^3)*c)*e^3)*sqrt(-b^2/e^4))*sqrt((e^2*sqrt(-b^2/e^4) - a + c)/
e^2))/(tan(e*x + d)^2 + 1)) + 2*c^2*e*sqrt(-(e^2*sqrt(-b^2/e^4) + a - c)/e^
2)*log((2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b
^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) - (2
*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^
2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*s
qrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) + ((b^5 + 2*(4*a^2*b - b^3)*c^2
- 2*(4*a^3*b + 5*a*b^3)*c)*e*tan(e*x + d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^4*c
- 2*a*b^2*c^2)*e*tan(e*x + d) - (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b + a
*b^3)*c)*e + ((4*a^3*b + 3*a*b^3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3*tan
(e*x + d)^2 + 2*(4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3
*a*b^2)*c)*e^3*tan(e*x + d) - (4*a^3*b + a*b^3 + (4*a^2*b + b^3)*c)*e^3)*sq
rt(-b^2/e^4))*sqrt(-(e^2*sqrt(-b^2/e^4) + a - c)/e^2))/(tan(e*x + d)^2 + 1)
) - 2*c^2*e*sqrt(-(e^2*sqrt(-b^2/e^4) + a - c)/e^2)*log((2*(4*a^3*b^2 + 2*a
*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)
*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) - (2*(2*a^3*b + a*b^3 + b^3*c
- 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^
2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*t
```

```
an(e*x + d) + a) - ((b^5 + 2*(4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 5*a*b^3)*c)
*e*tan(e*x + d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^4*c - 2*a*b^2*c^2)*e*tan(e*x +
 d) - (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b + a*b^3)*c)*e + ((4*a^3*b + 3
*a*b^3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3*tan(e*x + d)^2 + 2*(4*a^4 + 3
*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^3*tan(e*x + d
) - (4*a^3*b + a*b^3 + (4*a^2*b + b^3)*c)*e^3)*sqrt(-b^2/e^4))*sqrt(-(e^2*s
qrt(-b^2/e^4) + a - c)/e^2))/(tan(e*x + d)^2 + 1)) - (b^2 - 4*a*c + 8*c^2)*
sqrt(-c)*arctan(1/2*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a)*(2*c*tan(e*
x + d) + b)*sqrt(-c)/(c^2*tan(e*x + d)^2 + b*c*tan(e*x + d) + a*c)) - 2*(2*
c^2*tan(e*x + d) + b*c)*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a))/(c^2*e
)]
```

Sympy [F]

$$\int \tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$=\int \sqrt{a+b\tan{(d+ex)}+c\tan^2{(d+ex)}}\tan^2{(d+ex)}\,dx$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2)*tan(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2)*tan(d + e*x)**2, x)
```

Maxima [F]

$$\int \tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$=\int \sqrt{c\tan{(ex+d)}^2+b\tan{(ex+d)}+a}\tan{(ex+d)}^2\,dx$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)^2,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a)*tan(e*x + d)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)^2,x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$= \int \tan(d+e\,x)^2\,\sqrt{c\tan\left(d+e\,x\right)^2+b\tan\left(d+e\,x\right)+a}\,dx$$

```
[In] int(tan(d + e*x)^2*(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)
```

```
[Out] int(tan(d + e*x)^2*(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2), x)
```

3.5 $\int \tan(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 601

$$\int \tan(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$

$$= \frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{b^2+}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}\sqrt{}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}e}$$

$$+\frac{b\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{c}e}$$

$$-\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\operatorname{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}e}$$

$$+\frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e}$$

```
[Out] 1/2*b*arctanh(1/2*(b+2*c*tan(e*x+d))/c^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2
)^(1/2))/e/c^(1/2)-1/2*arctanh(1/2*(b^2+(a-c)*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2
))+b*(a^2-2*a*c+b^2+c^2)^(1/2)*tan(e*x+d))/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2
)/(a^2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)
))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a^2+b^2+c*(c-(a^2-2*a*c+b^
2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+b^2+c^2)^
(1/4)/e*2^(1/2)+1/2*arctan(1/2*(b^2+(a-c)*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))-b
*(a^2-2*a*c+b^2+c^2)^(1/2)*tan(e*x+d))/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)/(a
^2+b^2+c*(c+(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(
1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a^2+b^2+c*(c+(a^2-2*a*c+b^2+c^
2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/4
)/e*2^(1/2)+(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)/e
```

Rubi [A] (verified)

Time = 23.87 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3781, 1035, 1092, 635, 212, 1050, 1044, 214, 211}

$$\int \tan(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$

$$= \frac{\sqrt{-a\left(\sqrt{a^2-2ac+b^2+c^2}+2c\right)+c\left(\sqrt{a^2-2ac+b^2+c^2}+c\right)+a^2+b^2}\,\arctan\left(\frac{}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+}}\right.}{\sqrt{2}e\sqrt[4]{a^2-2ac+b^2+c^2}}$$

$$-\frac{\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+a^2+b^2}\,\text{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2}}\right.}{\sqrt{2}e\sqrt[4]{a^2-2ac+b^2+c^2}}$$

$$+\frac{b\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{c}e}+\frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e}$$

```
[In] Int[Tan[d + e*x]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2
 + b^2 - 2*a*c + c^2])]*ArcTan[(b^2 + (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a
*c + c^2]) - b*Sqrt[a^2 + b^2 - 2*a*c + c^2]*Tan[d + e*x])/(Sqrt[2]*(a^2 +
b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c
^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*Tan[d + e*x] + c
*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*e) + (b*ArcTan
h[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x
]^2])])/(2*Sqrt[c]*e) - (Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c
^2]) - a*(2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*ArcTanh[(b^2 + (a - c)*(a -
 c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) + b*Sqrt[a^2 + b^2 - 2*a*c + c^2]*Tan[d
 + e*x])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c - S
qrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sq
rt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c +
c^2)^(1/4)*e) + Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2]/e
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
```

```
, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx+cx^2}}{1+x^2}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e} - \frac{\text{Subst}\left(\int \frac{\frac{b}{2}-(a-c)x-\frac{bx^2}{2}}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e} - \frac{\text{Subst}\left(\int \frac{b+(-a+c)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2e}$$

$$= \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e} + \frac{b\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e} + \frac{\text{Subst}\left(\int \frac{-b\sqrt{a^2+b^2-2ac+c^2}+\left(-b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e} - \frac{\text{Subst}\left(\int \frac{b\sqrt{a^2+b^2-2ac+c^2}+\left(-b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e}$$

$$= \frac{b\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{c}e} + \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e} - \frac{\left(b\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{1}{2b\sqrt{a^2+b^2-2ac+c^2}\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2}\right)\right)}\right)}{e} - \frac{\left(b\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{1}{-2b\sqrt{a^2+b^2-2ac+c^2}\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-}\right)\right)}\right)}{e}$$

$$= \frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\,\arctan\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2-2a}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c}}$$

$$+ \frac{b\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{c}e}$$

$$- \frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\text{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2-}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+}}$$

$$+ \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.42

$$\int \tan(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$

$$= \frac{-\frac{1}{2}\sqrt{a-ib-c}\text{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)-\frac{1}{2}\sqrt{a+ib-c}\text{arctanh}\left(\frac{2a+ib+(b+2ic)\tan(d+}{2\sqrt{a+ib-c}\sqrt{a+b\tan(d+ex)+c}}\right.}{e}$$

```
[In] Integrate[Tan[d + e*x]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (-1/2*(Sqrt[a - I*b - c]*ArcTanh[(2*a - I*b + (b - (2*I)*c)*Tan[d + e*x])/(
2*Sqrt[a - I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])]) - (Sqrt[
a + I*b - c]*ArcTanh[(2*a + I*b + (b + (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a + I
*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/2 + (b*ArcTanh[(b +
2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])
/(2*Sqrt[c]) + Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/e
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.92 (sec) , antiderivative size = 17767879, normalized size of antiderivative = 29563.86

output too large to display

```
[In] int((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d),x)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2330 vs. 2(542) = 1084.

Time = 0.74 (sec) , antiderivative size = 4660, normalized size of antiderivative = 7.75

$$\int \tan(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d),x, algorithm="fr
icas")
```

```
[Out] [1/4*(c*e*sqrt(-(e^2*sqrt(-b^2/e^4) - a + c)/e^2)*log(-(2*(4*a^3*b^2 + 2*a*
b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*
c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) + (2*(2*a^3*b + a*b^3 + b^3*c -
 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2
 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*ta
n(e*x + d) + a) + ((4*a^3*b^2 + 3*a*b^4 - 8*a*b^2*c^2 - (4*a^2*b^2 - 3*b^4)
*c)*e*tan(e*x + d)^2 + 2*(4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 -
 2*(4*a^3*b + 3*a*b^3)*c)*e*tan(e*x + d) - (4*a^3*b^2 + a*b^4 + (4*a^2*b^2
+ b^4)*c)*e + ((b^4 + 2*(4*a^2 - b^2)*c^2 - 2*(4*a^3 + 5*a*b^2)*c)*e^3*tan(
e*x + d)^2 - 4*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^3*tan(e*x + d) - (8*
a^4 + 6*a^2*b^2 + b^4 - 2*(4*a^3 + a*b^2)*c)*e^3)*sqrt(-b^2/e^4))*sqrt(-(e^
2*sqrt(-b^2/e^4) - a + c)/e^2))/(tan(e*x + d)^2 + 1)) - c*e*sqrt(-(e^2*sqrt
(-b^2/e^4) - a + c)/e^2)*log(-(2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c
^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^
3)*c)*tan(e*x + d) + (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x +
 d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)
*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) - ((4*a^3
*b^2 + 3*a*b^4 - 8*a*b^2*c^2 - (4*a^2*b^2 - 3*b^4)*c)*e*tan(e*x + d)^2 + 2*
(4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)
*e*tan(e*x + d) - (4*a^3*b^2 + a*b^4 + (4*a^2*b^2 + b^4)*c)*e + ((b^4 + 2*(
4*a^2 - b^2)*c^2 - 2*(4*a^3 + 5*a*b^2)*c)*e^3*tan(e*x + d)^2 - 4*(2*a^3*b +
 a*b^3 + b^3*c - 2*a*b*c^2)*e^3*tan(e*x + d) - (8*a^4 + 6*a^2*b^2 + b^4 - 2
*(4*a^3 + a*b^2)*c)*e^3)*sqrt(-b^2/e^4))*sqrt(-(e^2*sqrt(-b^2/e^4) - a + c)
```

```
/e^2))/(tan(e*x + d)^2 + 1)) + c*e*sqrt((e^2*sqrt(-b^2/e^4) + a - c)/e^2)*l
og(-(2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3
+ b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) - (2*(2
*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 +
 b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt
(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) + ((4*a^3*b^2 + 3*a*b^4 - 8*a*b^2*c
^2 - (4*a^2*b^2 - 3*b^4)*c)*e*tan(e*x + d)^2 + 2*(4*a^4*b + 3*a^2*b^3 + b^5
 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*e*tan(e*x + d) - (4*a^3*b
^2 + a*b^4 + (4*a^2*b^2 + b^4)*c)*e - ((b^4 + 2*(4*a^2 - b^2)*c^2 - 2*(4*a^
3 + 5*a*b^2)*c)*e^3*tan(e*x + d)^2 - 4*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2
)*e^3*tan(e*x + d) - (8*a^4 + 6*a^2*b^2 + b^4 - 2*(4*a^3 + a*b^2)*c)*e^3)*s
qrt(-b^2/e^4))*sqrt((e^2*sqrt(-b^2/e^4) + a - c)/e^2))/(tan(e*x + d)^2 + 1)
) - c*e*sqrt((e^2*sqrt(-b^2/e^4) + a - c)/e^2)*log(-(2*(4*a^3*b^2 + 2*a*b^4
 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2
 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) - (2*(2*a^3*b + a*b^3 + b^3*c - 2*
a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 -
2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(e
*x + d) + a) - ((4*a^3*b^2 + 3*a*b^4 - 8*a*b^2*c^2 - (4*a^2*b^2 - 3*b^4)*c)
*e*tan(e*x + d)^2 + 2*(4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*
(4*a^3*b + 3*a*b^3)*c)*e*tan(e*x + d) - (4*a^3*b^2 + a*b^4 + (4*a^2*b^2 + b
^4)*c)*e - ((b^4 + 2*(4*a^2 - b^2)*c^2 - 2*(4*a^3 + 5*a*b^2)*c)*e^3*tan(e*x
 + d)^2 - 4*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^3*tan(e*x + d) - (8*a^4
 + 6*a^2*b^2 + b^4 - 2*(4*a^3 + a*b^2)*c)*e^3)*sqrt(-b^2/e^4))*sqrt((e^2*sq
rt(-b^2/e^4) + a - c)/e^2))/(tan(e*x + d)^2 + 1)) + b*sqrt(c)*log(8*c^2*tan
(e*x + d)^2 + 8*b*c*tan(e*x + d) + b^2 + 4*sqrt(c*tan(e*x + d)^2 + b*tan(e*
x + d) + a)*(2*c*tan(e*x + d) + b)*sqrt(c) + 4*a*c) + 4*sqrt(c*tan(e*x + d)
^2 + b*tan(e*x + d) + a)*c)/(c*e), 1/4*(c*e*sqrt(-(e^2*sqrt(-b^2/e^4) - a +
 c)/e^2)*log(-(2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b +
3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x +
d) + (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3
*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/
e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) + ((4*a^3*b^2 + 3*a*b^4 -
 8*a*b^2*c^2 - (4*a^2*b^2 - 3*b^4)*c)*e*tan(e*x + d)^2 + 2*(4*a^4*b + 3*a^2
*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*e*tan(e*x + d)
- (4*a^3*b^2 + a*b^4 + (4*a^2*b^2 + b^4)*c)*e + ((b^4 + 2*(4*a^2 - b^2)*c^2
 - 2*(4*a^3 + 5*a*b^2)*c)*e^3*tan(e*x + d)^2 - 4*(2*a^3*b + a*b^3 + b^3*c -
 2*a*b*c^2)*e^3*tan(e*x + d) - (8*a^4 + 6*a^2*b^2 + b^4 - 2*(4*a^3 + a*b^2)
*c)*e^3)*sqrt(-b^2/e^4))*sqrt(-(e^2*sqrt(-b^2/e^4) - a + c)/e^2))/(tan(e*x
+ d)^2 + 1)) - c*e*sqrt(-(e^2*sqrt(-b^2/e^4) - a + c)/e^2)*log(-(2*(4*a^3*b
^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*
b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) + (2*(2*a^3*b + a*b^3
+ b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 -
 b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)
^2 + b*tan(e*x + d) + a) - ((4*a^3*b^2 + 3*a*b^4 - 8*a*b^2*c^2 - (4*a^2*b^2
 - 3*b^4)*c)*e*tan(e*x + d)^2 + 2*(4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b
```

```
^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*e*tan(e*x + d) - (4*a^3*b^2 + a*b^4 + (4
*a^2*b^2 + b^4)*c)*e + ((b^4 + 2*(4*a^2 - b^2)*c^2 - 2*(4*a^3 + 5*a*b^2)*c)
*e^3*tan(e*x + d)^2 - 4*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^3*tan(e*x +
 d) - (8*a^4 + 6*a^2*b^2 + b^4 - 2*(4*a^3 + a*b^2)*c)*e^3)*sqrt(-b^2/e^4))*
sqrt(-(e^2*sqrt(-b^2/e^4) - a + c)/e^2))/(tan(e*x + d)^2 + 1)) + c*e*sqrt((
e^2*sqrt(-b^2/e^4) + a - c)/e^2)*log(-(2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4
*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b
+ 3*a*b^3)*c)*tan(e*x + d) - (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*t
an(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a
*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) +
 ((4*a^3*b^2 + 3*a*b^4 - 8*a*b^2*c^2 - (4*a^2*b^2 - 3*b^4)*c)*e*tan(e*x + d
)^2 + 2*(4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a
*b^3)*c)*e*tan(e*x + d) - (4*a^3*b^2 + a*b^4 + (4*a^2*b^2 + b^4)*c)*e - ((b
^4 + 2*(4*a^2 - b^2)*c^2 - 2*(4*a^3 + 5*a*b^2)*c)*e^3*tan(e*x + d)^2 - 4*(2
*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^3*tan(e*x + d) - (8*a^4 + 6*a^2*b^2 +
 b^4 - 2*(4*a^3 + a*b^2)*c)*e^3)*sqrt(-b^2/e^4))*sqrt((e^2*sqrt(-b^2/e^4) +
 a - c)/e^2))/(tan(e*x + d)^2 + 1)) - c*e*sqrt((e^2*sqrt(-b^2/e^4) + a - c)
/e^2)*log(-(2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a
^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d)
- (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^
2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4
))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) - ((4*a^3*b^2 + 3*a*b^4 - 8*
a*b^2*c^2 - (4*a^2*b^2 - 3*b^4)*c)*e*tan(e*x + d)^2 + 2*(4*a^4*b + 3*a^2*b^
3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*e*tan(e*x + d) - (
4*a^3*b^2 + a*b^4 + (4*a^2*b^2 + b^4)*c)*e - ((b^4 + 2*(4*a^2 - b^2)*c^2 -
2*(4*a^3 + 5*a*b^2)*c)*e^3*tan(e*x + d)^2 - 4*(2*a^3*b + a*b^3 + b^3*c - 2*
a*b*c^2)*e^3*tan(e*x + d) - (8*a^4 + 6*a^2*b^2 + b^4 - 2*(4*a^3 + a*b^2)*c)
*e^3)*sqrt(-b^2/e^4))*sqrt((e^2*sqrt(-b^2/e^4) + a - c)/e^2))/(tan(e*x + d)
^2 + 1)) - 2*b*sqrt(-c)*arctan(1/2*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) +
 a)*(2*c*tan(e*x + d) + b)*sqrt(-c)/(c^2*tan(e*x + d)^2 + b*c*tan(e*x + d)
+ a*c)) + 4*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a)*c)/(c*e)]
```

Sympy [F]

$$\int \tan(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$=\int \sqrt{a+b\tan\left(d+ex\right)+c\tan^2\left(d+ex\right)}\tan\left(d+ex\right)dx$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2)*tan(e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2)*tan(d + e*x), x)
```

Maxima [F]

$$\int \tan(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \int \sqrt{c\tan{(ex+d)}^2+b\tan{(ex+d)}+a}\tan{(ex+d)}\,dx$$

[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d),x, algorithm="maxima")

[Out] integrate(sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a)*tan(e*x + d), x)

Giac [F(-1)]

Timed out.

$$\int \tan(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \text{Timed out}$$

[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \tan(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \int \tan(d+e\,x)\,\sqrt{c\,{\tan\left(d+e\,x\right)}^2+b\tan\left(d+e\,x\right)+a}\,dx$$

[In] int(tan(d + e*x)*(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)

[Out] int(tan(d + e*x)*(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2), x)

3.6 $\int \sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 574

$$\int \sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx =$$

$$-\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{\cdots}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+\cdots}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}e}$$

$$+\frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e}$$

$$-\frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\,\mathrm{arctanh}\left(\frac{\cdots}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+\cdots}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}e}$$

```
[Out] arctanh(1/2*(b+2*c*tan(e*x+d))/c^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2
))*c^(1/2)/e-1/2*arctan(1/2*(b*(a^2-2*a*c+b^2+c^2)^(1/2)-(b^2+(a-c)*(a-c+(a
^2-2*a*c+b^2+c^2)^(1/2)))*tan(e*x+d))/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)/(a^
2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)))^(1
/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a^2+b^2+c*(c-(a^2-2*a*c+b^2+c^2
)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/4)
/e*2^(1/2)-1/2*arctanh(1/2*(b*(a^2-2*a*c+b^2+c^2)^(1/2)+(b^2+(a-c)*(a-c-(a^
2-2*a*c+b^2+c^2)^(1/2)))*tan(e*x+d))/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)/(a^2
+b^2+c*(c+(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/
2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a^2+b^2+c*(c+(a^2-2*a*c+b^2+c^2)
^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/4)/
e*2^(1/2)
```

Rubi [A] (verified)

Time = 23.91 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1004, 635, 212, 1050, 1044, 214, 211}

$$\int \sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx =$$

$$-\frac{\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+a^2+b^2}\arctan\left(\frac{}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+}}\right.}{\sqrt{2}e\sqrt[4]{a^2-2ac+b^2+c^2}}$$

$$-\frac{\sqrt{-a\left(\sqrt{a^2-2ac+b^2+c^2}+2c\right)+c\left(\sqrt{a^2-2ac+b^2+c^2}+c\right)+a^2+b^2}\text{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+}}\right.}{\sqrt{2}e\sqrt[4]{a^2-2ac+b^2+c^2}}$$

$$+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e}$$

```
[In] Int[Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] -((Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a
^2 + b^2 - 2*a*c + c^2])]*ArcTan[(b*Sqrt[a^2 + b^2 - 2*a*c + c^2] - (b^2 +
(a - c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]))*Tan[d + e*x])/(Sqrt[2]*(a^
2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c
 + c^2]) - a*(2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*Tan[d + e*x]
 + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*e)) + (Sqr
t[c]*ArcTanh[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*
Tan[d + e*x]^2])])/e - (Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c^
2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*ArcTanh[(b*Sqrt[a^2 + b^2 -
2*a*c + c^2] + (b^2 + (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2]))*Tan[
d + e*x])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c +
Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*S
qrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c +
 c^2)^(1/4)*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1004

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{1+x^2}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= -\frac{\text{Subst}\left(\int \frac{-a+c-bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e} + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{(2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e} - \frac{\text{Subst}\left(\int \frac{b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)-b\sqrt{a^2+b^2-2ac+c^2}x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e} + \frac{\text{Subst}\left(\int \frac{b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)+b\sqrt{a^2+b^2-2ac+c^2}x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e}$$

$$= \frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e} - \frac{\left(b\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{1}{-2b\sqrt{a^2+b^2-2ac+c^2}\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2a\right.\right.}}\right.}{e} - \frac{\left(b\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{1}{2b\sqrt{a^2+b^2-2ac+c^2}\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2ac\right.\right.}}\right.}{e}$$

$$= -\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\,\text{arctan}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2-}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+}} + \frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e} - \frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\,\text{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2-}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.40

$$\int \sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$

$$= \frac{-i\sqrt{a-ib-c}\,\text{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)+i\sqrt{a+ib-c}\,\text{arctanh}\left(\frac{2a+ib+(b+2ic)\tan(d+e}{2\sqrt{a+ib-c}\sqrt{a+b\tan(d+ex)+c\,\text{t}}}\right.}{2e}$$

```
[In] Integrate[Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] ((-I)*Sqrt[a - I*b - c]*ArcTanh[(2*a - I*b + (b - (2*I)*c)*Tan[d + e*x])/(2
*Sqrt[a - I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])] + I*Sqrt[a
 + I*b - c]*ArcTanh[(2*a + I*b + (b + (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a + I*
b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])] + 2*Sqrt[c]*ArcTanh[(b
 + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2]
)])/(2*e)
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.57 (sec) , antiderivative size = 17248000, normalized size of antiderivative = 30048.78

output too large to display

```
[In] int((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2273 vs. 2(516) = 1032.

Time = 0.57 (sec) , antiderivative size = 4547, normalized size of antiderivative = 7.92

$$\int \sqrt{a + b\tan(d + ex) + c\tan^2(d + ex)}\, dx = \text{Too large to display}$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(e*sqrt((e^2*sqrt(-b^2/e^4) - a + c)/e^2)*log((2*(4*a^3*b^2 + 2*a*b^4
+ 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2
- 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) + (2*(2*a^3*b + a*b^3 + b^3*c - 2*a
*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2
*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(e*
x + d) + a) + ((b^5 + 2*(4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 5*a*b^3)*c)*e*ta
n(e*x + d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^4*c - 2*a*b^2*c^2)*e*tan(e*x + d) -
 (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b + a*b^3)*c)*e - ((4*a^3*b + 3*a*b^
3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3*tan(e*x + d)^2 + 2*(4*a^4 + 3*a^2*
b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^3*tan(e*x + d) - (
4*a^3*b + a*b^3 + (4*a^2*b + b^3)*c)*e^3)*sqrt(-b^2/e^4))*sqrt((e^2*sqrt(-b
^2/e^4) - a + c)/e^2))/(tan(e*x + d)^2 + 1)) - e*sqrt((e^2*sqrt(-b^2/e^4) -
 a + c)/e^2)*log((2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b
 + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x
 + d) + (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4
+ 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b
^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) - ((b^5 + 2*(4*a^2*b -
```

b^3)*c^2 - 2*(4*a^3*b + 5*a*b^3)*c)*e*tan(e*x + d)^2 - 4*(2*a^3*b^2 + a*b^
4 + b^4*c - 2*a*b^2*c^2)*e*tan(e*x + d) - (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(4
*a^3*b + a*b^3)*c)*e - ((4*a^3*b + 3*a*b^3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)*
c)*e^3*tan(e*x + d)^2 + 2*(4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*
(4*a^3 + 3*a*b^2)*c)*e^3*tan(e*x + d) - (4*a^3*b + a*b^3 + (4*a^2*b + b^3)*
c)*e^3)*sqrt(-b^2/e^4))*sqrt((e^2*sqrt(-b^2/e^4) - a + c)/e^2))/(tan(e*x +
d)^2 + 1)) + e*sqrt(-(e^2*sqrt(-b^2/e^4) + a - c)/e^2)*log((2*(4*a^3*b^2 +
2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b
^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) - (2*(2*a^3*b + a*b^3 + b^3
*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)
c^2 - 2(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 +
b*tan(e*x + d) + a) + ((b^5 + 2*(4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 5*a*b^3)
*c)*e*tan(e*x + d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^4*c - 2*a*b^2*c^2)*e*tan(e*
x + d) - (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b + a*b^3)*c)*e + ((4*a^3*b
+ 3*a*b^3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3*tan(e*x + d)^2 + 2*(4*a^4
+ 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^3*tan(e*x
+ d) - (4*a^3*b + a*b^3 + (4*a^2*b + b^3)*c)*e^3)*sqrt(-b^2/e^4))*sqrt(-(e^
2*sqrt(-b^2/e^4) + a - c)/e^2))/(tan(e*x + d)^2 + 1)) - e*sqrt(-(e^2*sqrt(-
b^2/e^4) + a - c)/e^2)*log((2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2
- (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*
c)*tan(e*x + d) - (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d)
+ (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^
2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) - ((b^5 + 2*
(4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 5*a*b^3)*c)*e*tan(e*x + d)^2 - 4*(2*a^3*
b^2 + a*b^4 + b^4*c - 2*a*b^2*c^2)*e*tan(e*x + d) - (8*a^4*b + 6*a^2*b^3 +
b^5 - 2*(4*a^3*b + a*b^3)*c)*e + ((4*a^3*b + 3*a*b^3 - 8*a*b*c^2 - (4*a^2*b
- 3*b^3)*c)*e^3*tan(e*x + d)^2 + 2*(4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2
)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^3*tan(e*x + d) - (4*a^3*b + a*b^3 + (4*a^2
*b + b^3)*c)*e^3)*sqrt(-b^2/e^4))*sqrt(-(e^2*sqrt(-b^2/e^4) + a - c)/e^2))/
(tan(e*x + d)^2 + 1)) + 2*sqrt(c)*log(8*c^2*tan(e*x + d)^2 + 8*b*c*tan(e*x
+ d) + b^2 + 4*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a)*(2*c*tan(e*x + d
) + b)*sqrt(c) + 4*a*c))/e, 1/4*(e*sqrt((e^2*sqrt(-b^2/e^4) - a + c)/e^2)*l
og((2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 +
b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) + (2*(2*
a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 +
b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(
c*tan(e*x + d)^2 + b*tan(e*x + d) + a) + ((b^5 + 2*(4*a^2*b - b^3)*c^2 - 2*
(4*a^3*b + 5*a*b^3)*c)*e*tan(e*x + d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^4*c - 2*
a*b^2*c^2)*e*tan(e*x + d) - (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b + a*b^3
)*c)*e - ((4*a^3*b + 3*a*b^3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3*tan(e*x
+ d)^2 + 2*(4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b
^2)*c)*e^3*tan(e*x + d) - (4*a^3*b + a*b^3 + (4*a^2*b + b^3)*c)*e^3)*sqrt(-
b^2/e^4))*sqrt((e^2*sqrt(-b^2/e^4) - a + c)/e^2))/(tan(e*x + d)^2 + 1)) - e
*sqrt((e^2*sqrt(-b^2/e^4) - a + c)/e^2)*log((2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4
*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*

```
a^3*b + 3*a*b^3)*c)*tan(e*x + d) + (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)
*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3
 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d)
+ a) - ((b^5 + 2*(4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 5*a*b^3)*c)*e*tan(e*x +
 d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^4*c - 2*a*b^2*c^2)*e*tan(e*x + d) - (8*a^4
*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b + a*b^3)*c)*e - ((4*a^3*b + 3*a*b^3 - 8*a
*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3*tan(e*x + d)^2 + 2*(4*a^4 + 3*a^2*b^2 + b
^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^3*tan(e*x + d) - (4*a^3*b
 + a*b^3 + (4*a^2*b + b^3)*c)*e^3)*sqrt(-b^2/e^4))*sqrt((e^2*sqrt(-b^2/e^4)
 - a + c)/e^2))/(tan(e*x + d)^2 + 1)) + e*sqrt(-(e^2*sqrt(-b^2/e^4) + a - c
)/e^2)*log((2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a
^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d)
- (2*(2*a^3*b + a*b^3 + b^3*c - 2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^
2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4
))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) + ((b^5 + 2*(4*a^2*b - b^3)*
c^2 - 2*(4*a^3*b + 5*a*b^3)*c)*e*tan(e*x + d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^
4*c - 2*a*b^2*c^2)*e*tan(e*x + d) - (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b
 + a*b^3)*c)*e + ((4*a^3*b + 3*a*b^3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3
*tan(e*x + d)^2 + 2*(4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3
 + 3*a*b^2)*c)*e^3*tan(e*x + d) - (4*a^3*b + a*b^3 + (4*a^2*b + b^3)*c)*e^3
)*sqrt(-b^2/e^4))*sqrt(-(e^2*sqrt(-b^2/e^4) + a - c)/e^2))/(tan(e*x + d)^2
+ 1)) - e*sqrt(-(e^2*sqrt(-b^2/e^4) + a - c)/e^2)*log((2*(4*a^3*b^2 + 2*a*b
^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c
^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) - (2*(2*a^3*b + a*b^3 + b^3*c -
2*a*b*c^2)*e^2*tan(e*x + d) + (4*a^4 + 3*a^2*b^2 + b^4 + (4*a^2 - b^2)*c^2
- 2*(4*a^3 + 3*a*b^2)*c)*e^2)*sqrt(-b^2/e^4))*sqrt(c*tan(e*x + d)^2 + b*tan
(e*x + d) + a) - ((b^5 + 2*(4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 5*a*b^3)*c)*e
*tan(e*x + d)^2 - 4*(2*a^3*b^2 + a*b^4 + b^4*c - 2*a*b^2*c^2)*e*tan(e*x + d
) - (8*a^4*b + 6*a^2*b^3 + b^5 - 2*(4*a^3*b + a*b^3)*c)*e + ((4*a^3*b + 3*a
*b^3 - 8*a*b*c^2 - (4*a^2*b - 3*b^3)*c)*e^3*tan(e*x + d)^2 + 2*(4*a^4 + 3*a
^2*b^2 + b^4 + (4*a^2 - b^2)*c^2 - 2*(4*a^3 + 3*a*b^2)*c)*e^3*tan(e*x + d)
- (4*a^3*b + a*b^3 + (4*a^2*b + b^3)*c)*e^3)*sqrt(-b^2/e^4))*sqrt(-(e^2*sqr
t(-b^2/e^4) + a - c)/e^2))/(tan(e*x + d)^2 + 1)) - 4*sqrt(-c)*arctan(1/2*sq
rt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a)*(2*c*tan(e*x + d) + b)*sqrt(-c)/(
c^2*tan(e*x + d)^2 + b*c*tan(e*x + d) + a*c)))/e]
```

Sympy [F]

$$\int \sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx=\int \sqrt{a+b\tan\left(d+ex\right)+c\tan^2\left(d+ex\right)}\,dx$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((c-b-a)*(c+b-a)>0)', see `assume?`
for mor
```

Giac [F]

$$\int \sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \int \sqrt{c\tan(ex+d)^2+b\tan(ex+d)+a}\,dx$$

```
[In] integrate((a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \int \sqrt{c\tan(d+e\,x)^2+b\tan(d+e\,x)+a}\,dx$$

```
[In] int((a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)
```

```
[Out] int((a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2), x)
```

3.7 $\int \cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 571

$$\int \cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx =$$

$$-\frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}e}$$

$$-\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e}$$

$$+\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\text{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}e}$$

```
[Out] -arctanh(1/2*(2*a+b*tan(e*x+d))/a^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/
2))*a^(1/2)/e+1/2*arctanh(1/2*(b^2+(a-c)*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))+b*
(a^2-2*a*c+b^2+c^2)^(1/2)*tan(e*x+d))/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)/(a^
2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)))^(1
/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a^2+b^2+c*(c-(a^2-2*a*c+b^2+c^2
)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/4)
/e*2^(1/2)-1/2*arctan(1/2*(b^2+(a-c)*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))-b*(a^2
-2*a*c+b^2+c^2)^(1/2)*tan(e*x+d))/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)/(a^2+b^
2+c*(c+(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/
(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a^2+b^2+c*(c+(a^2-2*a*c+b^2+c^2)^(1
/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/4)/e*2
^(1/2)
```

Rubi [A] (verified)

Time = 24.31 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3781, 6857, 748, 857, 635, 212, 738, 1035, 1092, 1050, 1044, 214, 211}

$$\int \cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx =$$

$$-\frac{\sqrt{-a\left(\sqrt{a^2-2ac+b^2+c^2}+2c\right)+c\left(\sqrt{a^2-2ac+b^2+c^2}+c\right)+a^2+b^2}\arctan\left(\frac{}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+}}\right.}{\sqrt{2}e\sqrt[4]{a^2-2ac+b^2+c^2}}$$

$$+\frac{\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+a^2+b^2}\text{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+}}\right.}{\sqrt{2}e\sqrt[4]{a^2-2ac+b^2+c^2}}$$

$$-\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e}$$

```
[In] Int[Cot[d + e*x]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] -((Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a
^2 + b^2 - 2*a*c + c^2])]*ArcTan[(b^2 + (a - c)*(a - c - Sqrt[a^2 + b^2 - 2
*a*c + c^2]) - b*Sqrt[a^2 + b^2 - 2*a*c + c^2]*Tan[d + e*x])/(Sqrt[2]*(a^2
+ b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c +
 c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*Tan[d + e*x] +
 c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*e)) - (Sqrt[
a]*ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x] + c*Ta
n[d + e*x]^2])])/e + (Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]
) - a*(2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*ArcTanh[(b^2 + (a - c)*(a - c
+ Sqrt[a^2 + b^2 - 2*a*c + c^2]) + b*Sqrt[a^2 + b^2 - 2*a*c + c^2]*Tan[d +
e*x])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c - Sqrt
[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[
a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2
)^(1/4)*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
```

```
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x(1+x^2)}\,dx, x, \tan(d+ex)\right)}{e}$$

$$=\frac{\text{Subst}\left(\int \left(\frac{\sqrt{a+bx+cx^2}}{x}-\frac{x\sqrt{a+bx+cx^2}}{1+x^2}\right)\,dx,x,\tan(d+ex)\right)}{e}$$

$$=\frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx,x,\tan(d+ex)\right)}{e}-\frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx+cx^2}}{1+x^2}\,dx,x,\tan(d+ex)\right)}{e}$$

$$=-\frac{\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{2e}+\frac{\text{Subst}\left(\int \frac{\frac{b}{2}-(a-c)x-\frac{bx^2}{2}}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{e}$$

$$=\frac{\text{Subst}\left(\int \frac{b+(-a+c)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{e}+\frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{e}$$

$$=-\frac{(2a)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+b\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e}$$
$$-\frac{\text{Subst}\left(\int \frac{-b\sqrt{a^2+b^2-2ac+c^2}+\left(-b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e}$$
$$+\frac{\text{Subst}\left(\int \frac{b\sqrt{a^2+b^2-2ac+c^2}+\left(-b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e}$$

$$=-\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e}$$
$$+\frac{\left(b\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{1}{2b\sqrt{a^2+b^2-2ac+c^2}\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2\right.\right.}}\right.}{e}$$
$$+\frac{\left(b\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{1}{-2b\sqrt{a^2+b^2-2ac+c^2}\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-\right.\right.}}\right.}{e}$$

$$=$$
$$-\frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac}}$$
$$-\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e}$$
$$+\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\text{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.39

$$\int \cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$= \frac{-2\sqrt{a}\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)+\sqrt{a-ib-c}\operatorname{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)+\sqrt{}}{2e}$$

```
[In] Integrate[Cot[d + e*x]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*
x] + c*Tan[d + e*x]^2])] + Sqrt[a - I*b - c]*ArcTanh[(2*a - I*b + (b - (2*I
)*c)*Tan[d + e*x])/(2*Sqrt[a - I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d +
 e*x]^2])] + Sqrt[a + I*b - c]*ArcTanh[(2*a + I*b + (b + (2*I)*c)*Tan[d + e
*x])/(2*Sqrt[a + I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(2
*e)
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int(cot(e*x+d)*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
```

```
[Out] int(cot(e*x+d)*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4533 vs. 2(516) = 1032.

Time = 0.83 (sec) , antiderivative size = 9103, normalized size of antiderivative = 15.94

$$\int \cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(e*x+d)*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="fr
icas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$=\int \sqrt{a+b\tan{(d+ex)}+c\tan^2{(d+ex)}}\cot{(d+ex)}\,dx$$

```
[In] integrate(cot(e*x+d)*(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2)*cot(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cot(e*x+d)*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="ma
xima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((-16*a*(a/4-c/4))>0)', see `assume?
` for m
```

Giac [F]

$$\int \cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$=\int \sqrt{c\tan{(ex+d)}^2+b\tan{(ex+d)}+a}\cot{(ex+d)}\,dx$$

```
[In] integrate(cot(e*x+d)*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="gi
ac")
[Out] integrate(sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a)*cot(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$=\int \cot(d+e\,x)\,\sqrt{c\tan\left(d+e\,x\right)^2+b\tan\left(d+e\,x\right)+a}\,dx$$

```
[In] int(cot(d + e*x)*(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)
[Out] int(cot(d + e*x)*(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2), x)
```

3.8 $\int \cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 612

$$\int \cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$

$$= \frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{b\sqrt{\cdots}}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}\cdots}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}\,e\cdots}$$

$$-\frac{b\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{a}e}$$

$$+\frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\operatorname{arctanh}\left(\frac{\cdots}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+\cdots}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}\,e\cdots}$$

$$-\frac{\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e}$$

```
[Out] -1/2*b*arctanh(1/2*(2*a+b*tan(e*x+d))/a^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^
2)^(1/2))/e/a^(1/2)+1/2*arctan(1/2*(b*(a^2-2*a*c+b^2+c^2)^(1/2)-(b^2+(a-c)*
(a-c+(a^2-2*a*c+b^2+c^2)^(1/2)))*tan(e*x+d))/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1
/2)/(a^2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/
2)))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a^2+b^2+c*(c-(a^2-2*a*c+
b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+b^2+c^2
)^(1/4)/e*2^(1/2)+1/2*arctanh(1/2*(b*(a^2-2*a*c+b^2+c^2)^(1/2)+(b^2+(a-c)*(
a-c-(a^2-2*a*c+b^2+c^2)^(1/2)))*tan(e*x+d))/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/
2)/(a^2+b^2+c*(c+(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2
)))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a^2+b^2+c*(c+(a^2-2*a*c+b
^2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+b^2+c^2)
^(1/4)/e*2^(1/2)-cot(e*x+d)*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)/e
```

Rubi [A] (verified)

Time = 24.42 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3781, 6857, 746, 857, 635, 212, 738, 1004, 1050, 1044, 211, 214}

$$\int \cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$

$$= \frac{\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+a^2+b^2}\arctan\left(\frac{b\sqrt{}}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}}\right.}{\sqrt{2}e\sqrt[4]{a^2-2ac+b^2+c^2}}$$

$$+\frac{\sqrt{-a\left(\sqrt{a^2-2ac+b^2+c^2}+2c\right)+c\left(\sqrt{a^2-2ac+b^2+c^2}+c\right)+a^2+b^2}\text{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+}}\right.}{\sqrt{2}e\sqrt[4]{a^2-2ac+b^2+c^2}}$$

$$-\frac{b\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{a}e}$$

$$-\frac{\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e}$$

```
[In] Int[Cot[d + e*x]^2*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a^2
 + b^2 - 2*a*c + c^2])]*ArcTan[(b*Sqrt[a^2 + b^2 - 2*a*c + c^2] - (b^2 + (a
 - c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]))*Tan[d + e*x])/(Sqrt[2]*(a^2
+ b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c +
 c^2]) - a*(2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*Tan[d + e*x] +
 c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*e) - (b*ArcT
anh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e
*x]^2])])/(2*Sqrt[a]*e) + (Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c +
 c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*ArcTanh[(b*Sqrt[a^2 + b^2
 - 2*a*c + c^2] + (b^2 + (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2]))*T
an[d + e*x])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c
 + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])
]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*
c + c^2)^(1/4)*e) - (Cot[d + e*x]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^
2])/e
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1004

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2(1+x^2)}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\sqrt{a+bx+cx^2}}{x^2} + \frac{\sqrt{a+bx+cx^2}}{-1-x^2}\right)\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx, x, \tan(d+ex)\right)}{e} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{-1-x^2}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= -\frac{\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e} + \frac{\text{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2e} + \frac{\text{Subst}\left(\int \frac{a-c+bx}{(-1-x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e} - \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= -\frac{\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e} + \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2e} + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e} - \frac{(2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e} - \frac{\text{Subst}\left(\int \frac{b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)-b\sqrt{a^2+b^2-2ac+c^2}x}{(-1-x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e} + \frac{\text{Subst}\left(\int \frac{b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)+b\sqrt{a^2+b^2-2ac+c^2}x}{(-1-x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e}$$

$$= -\frac{\sqrt{c}\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e} - \frac{\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e} - \frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e} + \frac{(2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e} + \frac{\left(b\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{1}{2b\sqrt{a^2+b^2-2ac+c^2}\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2\right.\right.}}\right.}{e} + \frac{\left(b\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{1}{-2b\sqrt{a^2+b^2-2ac+c^2}\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-\right.\right.}}\right.}{e}$$

$$= \frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2-2a}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c}}$$

$$- \frac{b\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{a}e}$$

$$+ \frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\text{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2-}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+}}$$

$$- \frac{\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{e}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.43

$$\int \cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx =$$

$$-\frac{\frac{b\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a}} - i\sqrt{a-ib-c}\,\text{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right) + i\sqrt{a+}}{2e}$$

```
[In] Integrate[Cot[d + e*x]^2*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] -1/2*((b*ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x]
+ c*Tan[d + e*x]^2])])/Sqrt[a] - I*Sqrt[a - I*b - c]*ArcTanh[(2*a - I*b + (
b - (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a - I*b - c]*Sqrt[a + b*Tan[d + e*x] + c
*Tan[d + e*x]^2])] + I*Sqrt[a + I*b - c]*ArcTanh[(2*a + I*b + (b + (2*I)*c)
*Tan[d + e*x])/(2*Sqrt[a + I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x
]^2])] + 2*Cot[d + e*x]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/e
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int(cot(e*x+d)^2*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
[Out] int(cot(e*x+d)^2*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4592 vs. $2(552) = 1104$.

Time = 1.05 (sec) , antiderivative size = 9221, normalized size of antiderivative = 15.07

$$\int \cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(e*x+d)^2*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \int \sqrt{a+b\tan{(d+ex)}+c\tan^2{(d+ex)}}\cot^2{(d+ex)}\,dx$$

```
[In] integrate(cot(e*x+d)**2*(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2)*cot(d + e*x)**2, x)
```

Maxima [F]

$$\int \cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \int \sqrt{c\tan{(ex+d)}^2+b\tan{(ex+d)}+a}\cot{(ex+d)}^2\,dx$$

```
[In] integrate(cot(e*x+d)^2*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
maxima")
[Out] integrate(sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a)*cot(e*x + d)^2, x)
```

Giac [F]

$$\int \cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$= \int \sqrt{c\tan\left(ex+d\right)^2+b\tan\left(ex+d\right)+a}\cot\left(ex+d\right)^2\,dx$$

```
[In] integrate(cot(e*x+d)^2*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
giac")
[Out] integrate(sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a)*cot(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$= \int \cot(d+e\,x)^2\,\sqrt{c\tan\left(d+e\,x\right)^2+b\tan\left(d+e\,x\right)+a}\,dx$$

```
[In] int(cot(d + e*x)^2*(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)
[Out] int(cot(d + e*x)^2*(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2), x)
```

3.9 $\int \cot^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 690

$$\int \cot^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$

$$= \frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{b^2 \cdots}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2} \cdots}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}e \cdots}$$

$$+ \frac{\sqrt{a}\,\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e}$$

$$+ \frac{(b^2-4ac)\,\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8a^{3/2}e}$$

$$- \frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\,\text{arctanh}\left(\frac{\cdots}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+ \cdots}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}e \cdots}$$

$$- \frac{\cot^2(d+ex)(2a+b\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ae}$$

```
[Out] 1/8*(-4*a*c+b^2)*arctanh(1/2*(2*a+b*tan(e*x+d))/a^(1/2)/(a+b*tan(e*x+d)+c*t
an(e*x+d)^2)^(1/2))/a^(3/2)/e+arctanh(1/2*(2*a+b*tan(e*x+d))/a^(1/2)/(a+b*t
an(e*x+d)+c*tan(e*x+d)^2)^(1/2))*a^(1/2)/e-1/2*arctanh(1/2*(b^2+(a-c)*(a-c+
(a^2-2*a*c+b^2+c^2)^(1/2))+b*(a^2-2*a*c+b^2+c^2)^(1/2)*tan(e*x+d))/(a^2-2*a
*c+b^2+c^2)^(1/4)*2^(1/2)/(a^2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(
a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a^
2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)))^(1
/2)/(a^2-2*a*c+b^2+c^2)^(1/4)/e*2^(1/2)+1/2*arctan(1/2*(b^2+(a-c)*(a-c-(a^2
```

```
-2*a*c+b^2+c^2)^(1/2))-b*(a^2-2*a*c+b^2+c^2)^(1/2)*tan(e*x+d))/(a^2-2*a*c+b
^2+c^2)^(1/4)*2^(1/2)/(a^2+b^2+c*(c+(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c+(a^2-
2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a^2+b^
2+c*(c+(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/
(a^2-2*a*c+b^2+c^2)^(1/4)/e*2^(1/2)-1/4*cot(e*x+d)^2*(a+b*tan(e*x+d)+c*tan(
e*x+d)^2)^(1/2)*(2*a+b*tan(e*x+d))/a/e
```

Rubi [A] (verified)

Time = 24.44 (sec) , antiderivative size = 690, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3781, 6857, 734, 738, 212, 748, 857, 635, 1035, 1092, 1050, 1044, 214, 211}

$$\int \cot^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$

$$= \frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8a^{3/2}e}$$

$$+ \frac{\sqrt{-a\left(\sqrt{a^2-2ac+b^2+c^2}+2c\right)+c\left(\sqrt{a^2-2ac+b^2+c^2}+c\right)+a^2+b^2}\arctan\left(\frac{\cdots}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+\cdots}}\right.}{\sqrt{2}e\sqrt[4]{a^2-2ac+b^2+c^2}}$$

$$- \frac{\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+a^2+b^2}\operatorname{arctanh}\left(\frac{\cdots}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+\cdots}}\right.}{\sqrt{2}e\sqrt[4]{a^2-2ac+b^2+c^2}}$$

$$+ \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e}$$

$$- \frac{\cot^2(d+ex)(2a+b\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ae}$$

```
[In] Int[Cot[d + e*x]^3*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2
 + b^2 - 2*a*c + c^2])]*ArcTan[(b^2 + (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a
*c + c^2]) - b*Sqrt[a^2 + b^2 - 2*a*c + c^2]*Tan[d + e*x])/(Sqrt[2]*(a^2 +
b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c
^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*Tan[d + e*x] + c
*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*e) + (Sqrt[a]*
ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d
 + e*x]^2])])/e + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*
Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(8*a^(3/2)*e) - (Sqrt[a^2 +
b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a^2 + b^2 - 2*a
*c + c^2])]*ArcTanh[(b^2 + (a - c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2])
```

```
+ b*Sqrt[a^2 + b^2 - 2*a*c + c^2]*Tan[d + e*x])/(Sqrt[2]*(a^2 + b^2 - 2*a*c
 + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2
*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*
x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*e) - (Cot[d + e*x]^2*(2*a
 + b*Tan[d + e*x])*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/(4*a*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
```

```
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3(1+x^2)}\,dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\sqrt{a+bx+cx^2}}{x^3} - \frac{\sqrt{a+bx+cx^2}}{x} + \frac{x\sqrt{a+bx+cx^2}}{1+x^2}\right)dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, \tan(d+ex)\right)}{e} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, \tan(d+ex)\right)}{e} \\
&\quad + \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx+cx^2}}{1+x^2}\,dx, x, \tan(d+ex)\right)}{e} \\
&= -\frac{\cot^2(d+ex)(2a+b\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ae} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2e} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\frac{b}{2}-(a-c)x-\frac{bx^2}{2}}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e} \\
&\quad - \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{8ae}
\end{aligned}
$$

$$= -\frac{\cot^2(d+ex)(2a+b\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ae} - \frac{\text{Subst}\left(\int \frac{b+(-a+c)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e} - \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e} + \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{4ae}$$

$$= \frac{(b^2-4ac)\,\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8a^{3/2}e} - \frac{\cot^2(d+ex)(2a+b\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ae} + \frac{(2a)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e} + \frac{\text{Subst}\left(\int \frac{-b\sqrt{a^2+b^2-2ac+c^2}+\left(-b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e} - \frac{\text{Subst}\left(\int \frac{b\sqrt{a^2+b^2-2ac+c^2}+\left(-b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e}$$

$$= \frac{\sqrt{a}\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e} + \frac{(b^2-4ac)\,\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8a^{3/2}e} - \frac{\cot^2(d+ex)(2a+b\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ae} - \frac{\left(b\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{1}{2b\sqrt{a^2+b^2-2ac+c^2}\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac}\right.\right.}\right.}{e} - \frac{\left(b\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{1}{-2b\sqrt{a^2+b^2-2ac+c^2}\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2\right.\right.}\right.}{e}$$

$$= \frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2-2}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+}}$$

$$+ \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e}$$

$$+ \frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8a^{3/2}e}$$

$$- \frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\operatorname{arctanh}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac}}$$

$$- \frac{\cot^2(d+ex)(2a+b\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4ae}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.00 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.42

$$\int \cot^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$

$$= \frac{(8a^2+b^2-4ac)\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)-2\sqrt{a}\left(2a\sqrt{a-ib-c}\operatorname{arctanh}\left(\frac{2a-ib+(b-2ic)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d-}}\right.\right.}{}$$

```
[In] Integrate[Cot[d + e*x]^3*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] ((8*a^2 + b^2 - 4*a*c)*ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqrt[a + b
*Tan[d + e*x] + c*Tan[d + e*x]^2])] - 2*Sqrt[a]*(2*a*Sqrt[a - I*b - c]*ArcT
anh[(2*a - I*b + (b - (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a - I*b - c]*Sqrt[a +
b*Tan[d + e*x] + c*Tan[d + e*x]^2])] + 2*a*Sqrt[a + I*b - c]*ArcTanh[(2*a +
 I*b + (b + (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a + I*b - c]*Sqrt[a + b*Tan[d +
e*x] + c*Tan[d + e*x]^2])] + Cot[d + e*x]*(b + 2*a*Cot[d + e*x])*Sqrt[a + b
*Tan[d + e*x] + c*Tan[d + e*x]^2]))/(8*a^(3/2)*e)
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int(cot(e*x+d)^3*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
```

```
[Out] int(cot(e*x+d)^3*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4692 vs. $2(621) = 1242$.

Time = 1.15 (sec) , antiderivative size = 9425, normalized size of antiderivative = 13.66

$$\int \cot^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(e*x+d)^3*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \cot^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \int \sqrt{a+b\tan{(d+ex)}+c\tan^2{(d+ex)}}\cot^3{(d+ex)}\,dx$$

```
[In] integrate(cot(e*x+d)**3*(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2)*cot(d + e*x)**3, x)
```

Maxima [F]

$$\int \cot^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx = \int \sqrt{c\tan(ex+d)^2+b\tan(ex+d)+a}\cot(ex+d)^3\,dx$$

```
[In] integrate(cot(e*x+d)^3*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a)*cot(e*x + d)^3, x)
```

Giac [F]

$$\int \cot^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$=\int \sqrt{c\tan\left(ex+d\right)^2+b\tan\left(ex+d\right)+a}\cot\left(ex+d\right)^3\,dx$$

```
[In] integrate(cot(e*x+d)^3*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
giac")
[Out] integrate(sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a)*cot(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}\,dx$$
$$=\int \cot(d+e\,x)^3\,\sqrt{c\tan\left(d+e\,x\right)^2+b\tan\left(d+e\,x\right)+a}\,dx$$

```
[In] int(cot(d + e*x)^3*(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)
[Out] int(cot(d + e*x)^3*(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2), x)
```

3.10 $\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 548

$$\begin{aligned}
\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx
&= \frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\,\text{arctanh}\left(\frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}\,e} \\
&\quad - \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\,\text{arctanh}\left(\frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}\,e} \\
&\quad + \frac{b\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2c^{3/2}e} \\
&\quad - \frac{b(5b^2-12ac)\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{16c^{7/2}e} \\
&\quad - \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{ce} \\
&\quad + \frac{\tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{3ce} \\
&\quad + \frac{(15b^2-16ac-10bc\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{24c^3e}
\end{aligned}$$

```
[Out] 1/2*b*arctanh(1/2*(b+2*c*tan(e*x+d))/c^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2
)^(1/2))/c^(3/2)/e-1/16*b*(-12*a*c+5*b^2)*arctanh(1/2*(b+2*c*tan(e*x+d))/c^
(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))/c^(7/2)/e+1/2*arctanh(1/2*(a-c
-(a^2-2*a*c+b^2+c^2)^(1/2)+b*tan(e*x+d))*2^(1/2)/(a-c-(a^2-2*a*c+b^2+c^2)^(
1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a-c-(a^2-2*a*c+b^2+c^2)
^(1/2))^(1/2)/e*2^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/2)-1/2*arctanh(1/2*(a-c+(a^2
```

```
-2*a*c+b^2+c^2)^(1/2)+b*tan(e*x+d))*2^(1/2)/(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))
^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2
))^(1/2)/e*2^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/2)-(a+b*tan(e*x+d)+c*tan(e*x+d)^2
)^(1/2)/c/e+1/3*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)^2/c/e+1/24
*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*(15*b^2-16*a*c-10*b*c*tan(e*x+d))/c^
3/e
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3781, 6857, 654, 635, 212, 756, 793, 1050, 1044, 214}

$$\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$$
$$= \frac{\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\,\text{arctanh}\left(\frac{-\sqrt{a^2-2ac+b^2+c^2}+a+b\tan(d+ex)-c}{\sqrt{2}\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}}$$
$$-\frac{\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\,\text{arctanh}\left(\frac{\sqrt{a^2-2ac+b^2+c^2}+a+b\tan(d+ex)-c}{\sqrt{2}\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}}$$
$$-\frac{b(5b^2-12ac)\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{16c^{7/2}e}$$
$$+\frac{b\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2c^{3/2}e}$$
$$+\frac{(-16ac+15b^2-10bc\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{24c^3e}$$
$$+\frac{\tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{3ce}$$
$$-\frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{ce}$$

```
[In] Int[Tan[d + e*x]^5/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (Sqrt[a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTanh[(a - c - Sqrt[a^2 + b^
2 - 2*a*c + c^2] + b*Tan[d + e*x])/(Sqrt[2]*Sqrt[a - c - Sqrt[a^2 + b^2 - 2
*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*Sqrt[a
^2 + b^2 - 2*a*c + c^2]*e) - (Sqrt[a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*A
rcTanh[(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2] + b*Tan[d + e*x])/(Sqrt[2]*Sq
rt[a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d
 + e*x]^2])])/(Sqrt[2]*Sqrt[a^2 + b^2 - 2*a*c + c^2]*e) + (b*ArcTanh[(b + 2
*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/
(2*c^(3/2)*e) - (b*(5*b^2 - 12*a*c)*ArcTanh[(b + 2*c*Tan[d + e*x])/(2*Sqrt[
```

```
c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(16*c^(7/2)*e) - Sqrt[a +
 b*Tan[d + e*x] + c*Tan[d + e*x]^2]/(c*e) + (Tan[d + e*x]^2*Sqrt[a + b*Tan[
d + e*x] + c*Tan[d + e*x]^2])/(3*c*e) + ((15*b^2 - 16*a*c - 10*b*c*Tan[d +
e*x])*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/(24*c^3*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
```

```
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{x}{\sqrt{a+bx+cx^2}}+\frac{x^3}{\sqrt{a+bx+cx^2}}+\frac{x}{(1+x^2)\sqrt{a+bx+cx^2}}\right)\,dx, x, \tan(d+ex)\right)}{e}\\
&= -\frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}+\frac{\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}\\
&\quad+\frac{\text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{ce} \\
&\quad + \frac{\tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{3ce} \\
&\quad + \frac{\text{Subst}\left(\int \frac{x\left(-2a-\frac{5bx}{2}\right)}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{3ce} \\
&\quad + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2ce} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-b+\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-b+\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{ce} \\
&\quad + \frac{\tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{3ce} \\
&\quad + \frac{(15b^2-16ac-10bc\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{24c^3e} \\
&\quad + \frac{b\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{ce} \\
&\quad - \frac{(b(5b^2-12ac))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{16c^3e} \\
&\quad - \frac{\left(b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{-2b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)+bx^2}\,dx, x, \frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+b}{\sqrt{a+b\tan(d+ex)+c\tan}}\right.}{\sqrt{a^2+b^2-2ac+c^2}e} \\
&\quad + \frac{\left(b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{-2b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)+bx^2}\,dx, x, \frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+b}{\sqrt{a+b\tan(d+ex)+c\tan}}\right.}{\sqrt{a^2+b^2-2ac+c^2}e}
\end{aligned}
$$

$$= \frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e}$$
$$- \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e}$$
$$+ \frac{b\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2c^{3/2}e} - \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{ce}$$
$$+ \frac{\tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{3ce}$$
$$+ \frac{(15b^2-16ac-10bc\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{24c^3e}$$
$$- \frac{(b(5b^2-12ac))\operatorname{Subst}\left(\int\frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8c^3e}$$

$$= \frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e}$$
$$- \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e}$$
$$+ \frac{b\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2c^{3/2}e}$$
$$- \frac{b(5b^2-12ac)\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{16c^{7/2}e}$$
$$- \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{ce}$$
$$+ \frac{\tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{3ce}$$
$$+ \frac{(15b^2-16ac-10bc\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{24c^3e}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.14 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.83

$$\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$$

$$= \frac{-\frac{2\sqrt{a+ib-c}\operatorname{arctanh}\left(\frac{2a+ib-(-b-2ic)\tan(d+ex)}{2\sqrt{a+ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{4a+4ib-4c} - \frac{2\sqrt{a-ib-c}\operatorname{arctanh}\left(\frac{2a-ib-(-b+2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{4a-4ib-4c} + \frac{b\operatorname{arct}}{}}{}$$

```
[In] Integrate[Tan[d + e*x]^5/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] ((-2*Sqrt[a + I*b - c]*ArcTanh[(2*a + I*b - (-b - (2*I)*c)*Tan[d + e*x])/(2
*Sqrt[a + I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(4*a + (4
*I)*b - 4*c) - (2*Sqrt[a - I*b - c]*ArcTanh[(2*a - I*b - (-b + (2*I)*c)*Tan
[d + e*x])/(2*Sqrt[a - I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2]
)])/(4*a - (4*I)*b - 4*c) + (b*ArcTanh[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sq
rt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(2*c^(3/2)) - Sqrt[a + b*Tan[d
 + e*x] + c*Tan[d + e*x]^2]/c + (Tan[d + e*x]^2*Sqrt[a + b*Tan[d + e*x] + c
*Tan[d + e*x]^2])/(3*c) + ((((-15*b^3)/4 + 9*a*b*c)*ArcTanh[(b + 2*c*Tan[d
+ e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(4*c^(5/2
)) + (((15*b^2)/4 - 4*a*c - (5*b*c*Tan[d + e*x])/2)*Sqrt[a + b*Tan[d + e*x]
 + c*Tan[d + e*x]^2])/(2*c^2))/(3*c))/e
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.65 (sec) , antiderivative size = 9581953, normalized size of antiderivative = 17485.32

output too large to display

```
[In] int(tan(e*x+d)^5/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5228 vs. $2(485) = 970$.

Time = 1.20 (sec) , antiderivative size = 10457, normalized size of antiderivative = 19.08

$$\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(e*x+d)^5/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$$

```
[In] integrate(tan(e*x+d)**5/(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2),x)
```

```
[Out] Integral(tan(d + e*x)**5/sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\tan(ex+d)^5}{\sqrt{c\tan(ex+d)^2+b\tan(ex+d)+a}}\,dx$$

```
[In] integrate(tan(e*x+d)^5/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(tan(e*x + d)^5/sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^5/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx=\int \frac{\tan(d+e\,x)^5}{\sqrt{c\tan\left(d+e\,x\right)^2+b\tan\left(d+e\,x\right)+a}}\,dx$$

```
[In] int(tan(d + e*x)^5/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)
[Out] int(tan(d + e*x)^5/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2), x)
```

3.11 $\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 495

$$\begin{aligned}
&\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx \\
&= \frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} \\
&\quad - \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} \\
&\quad - \frac{\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{c}e} \\
&\quad + \frac{(3b^2-4ac)\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8c^{5/2}e} \\
&\quad - \frac{3b\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4c^2e} \\
&\quad + \frac{\tan(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{2ce}
\end{aligned}$$

```
[Out] 1/8*(-4*a*c+3*b^2)*arctanh(1/2*(b+2*c*tan(e*x+d))/c^(1/2)/(a+b*tan(e*x+d)+c
*tan(e*x+d)^2)^(1/2))/c^(5/2)/e-arctanh(1/2*(b+2*c*tan(e*x+d))/c^(1/2)/(a+b
*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))/e/c^(1/2)+1/2*arctan(1/2*(b-(a-c-(a^2-2*
a*c+b^2+c^2)^(1/2))*tan(e*x+d))*2^(1/2)/(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/
2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))^(
1/2)/e*2^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/2)-1/2*arctan(1/2*(b-(a-c+(a^2-2*a*c+
b^2+c^2)^(1/2))*tan(e*x+d))*2^(1/2)/(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(
```

```
a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)
/e*2^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/2)-3/4*b*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^
(1/2)/c^2/e+1/2*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(e*x+d)/c/e
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3781, 6857, 635, 212, 756, 654, 1001, 1044, 211}

$$\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$$
$$= \frac{\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\arctan\left(\frac{b-\left(-\sqrt{a^2-2ac+b^2+c^2}+a-c\right)\tan(d+ex)}{\sqrt{2}\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}}$$
$$-\frac{\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\arctan\left(\frac{b-\left(\sqrt{a^2-2ac+b^2+c^2}+a-c\right)\tan(d+ex)}{\sqrt{2}\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}}$$
$$+\frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8c^{5/2}e}$$
$$-\frac{\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{c}e}-\frac{3b\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4c^2e}$$
$$+\frac{\tan(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{2ce}$$

```
[In] Int[Tan[d + e*x]^4/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (Sqrt[a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTan[(b - (a - c - Sqrt[a^2
+ b^2 - 2*a*c + c^2])*Tan[d + e*x])/(Sqrt[2]*Sqrt[a - c - Sqrt[a^2 + b^2 -
2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*Sqrt[
a^2 + b^2 - 2*a*c + c^2]*e) - (Sqrt[a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*
ArcTan[(b - (a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2])*Tan[d + e*x])/(Sqrt[2]*
Sqrt[a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan
[d + e*x]^2])])/(Sqrt[2]*Sqrt[a^2 + b^2 - 2*a*c + c^2]*e) - ArcTanh[(b + 2*
c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])]/(S
qrt[c]*e) + ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt
[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(8*c^(5/2)*e) - (3*b*Sqrt[a + b*
Tan[d + e*x] + c*Tan[d + e*x]^2])/(4*c^2*e) + (Tan[d + e*x]*Sqrt[a + b*Tan[
d + e*x] + c*Tan[d + e*x]^2])/(2*c*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1001

```
Int[1/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_S
ymbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist[1/(2*q), Int[(c*d
 - a*f + q + c*e*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2
*q), Int[(c*d - a*f - q + c*e*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x
]] /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{\sqrt{a+bx+cx^2}}+\frac{x^2}{\sqrt{a+bx+cx^2}}+\frac{1}{(1+x^2)\sqrt{a+bx+cx^2}}\right)\,dx, x, \tan(d+ex)\right)}{e}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}+\frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}\\
&= \frac{\tan(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{2ce}\\
&\quad-\frac{2\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e}\\
&\quad+\frac{\text{Subst}\left(\int \frac{-a-\frac{3bx}{2}}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2ce}\\
&\quad-\frac{\text{Subst}\left(\int \frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}\,e}\\
&\quad+\frac{\text{Subst}\left(\int \frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}\,e}
\end{aligned}$$

$$= -\frac{\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{c}e} - \frac{3b\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4c^2e}$$
$$+ \frac{\tan(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{2ce}$$
$$+ \frac{(3b^2-4ac)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{8c^2e}$$
$$+ \frac{\left(b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\operatorname{Subst}\left(\int \frac{1}{2b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)+bx^2}\,dx, x, \frac{b-\left(a-c-\sqrt{a^2+b^2-2ac+c}{\sqrt{a+b\tan(d+ex)+c\,ta}}}{\sqrt{a^2+b^2-2ac+c^2}e}$$
$$- \frac{\left(b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\operatorname{Subst}\left(\int \frac{1}{2b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)+bx^2}\,dx, x, \frac{b-\left(a-c+\sqrt{a^2+b^2-2ac+c}{\sqrt{a+b\tan(d+ex)+c\,ta}}}{\sqrt{a^2+b^2-2ac+c^2}e}$$

$$= \frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e}$$
$$- \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e}$$
$$- \frac{\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{c}e} - \frac{3b\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4c^2e}$$
$$+ \frac{\tan(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{2ce}$$
$$+ \frac{(3b^2-4ac)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{4c^2e}$$

$$= \frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e}$$
$$- \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e}$$
$$- \frac{\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{c}e}$$
$$+ \frac{(3b^2-4ac)\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8c^{5/2}e}$$
$$- \frac{3b\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4c^2e}$$
$$+ \frac{\tan(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{2ce}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.52 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.57

$$\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$$
$$= \frac{-\frac{4i\operatorname{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a-ib-c}} + \frac{4i\operatorname{arctanh}\left(\frac{2a+ib+(b+2ic)\tan(d+ex)}{2\sqrt{a+ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a+ib-c}} + \frac{(3b^2-4c(a+2c))\operatorname{arcta}}{}}{8e}$$

```
[In] Integrate[Tan[d + e*x]^4/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (((-4*I)*ArcTanh[(2*a - I*b + (b - (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a - I*b -
 c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/Sqrt[a - I*b - c] + ((4*
I)*ArcTanh[(2*a + I*b + (b + (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a + I*b - c]*Sq
rt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/Sqrt[a + I*b - c] + ((3*b^2 -
4*c*(a + 2*c))*ArcTanh[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d +
 e*x] + c*Tan[d + e*x]^2])])/c^(5/2) + (2*(-3*b + 2*c*Tan[d + e*x])*Sqrt[a
+ b*Tan[d + e*x] + c*Tan[d + e*x]^2])/c^2)/(8*e)
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.18 (sec) , antiderivative size = 7492392, normalized size of antiderivative = 15136.15

output too large to display

```
[In] int(tan(e*x+d)^4/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5044 vs. $2(438) = 876$.

Time = 1.13 (sec) , antiderivative size = 10089, normalized size of antiderivative = 20.38

$$\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(e*x+d)^4/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\tan^4{(d+ex)}}{\sqrt{a+b\tan{(d+ex)}+c\tan^2{(d+ex)}}}\,dx$$

```
[In] integrate(tan(e*x+d)**4/(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2),x)
```

```
[Out] Integral(tan(d + e*x)**4/sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\tan{(ex+d)}^4}{\sqrt{c\tan{(ex+d)}^2+b\tan{(ex+d)}+a}}\,dx$$

```
[In] integrate(tan(e*x+d)^4/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(tan(e*x + d)^4/sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^4/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\tan(d+e\,x)^4}{\sqrt{c\tan\left(d+e\,x\right)^2+b\tan\left(d+e\,x\right)+a}}\,dx$$

```
[In] int(tan(d + e*x)^4/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)
```

```
[Out] int(tan(d + e*x)^4/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2), x)
```

3.12 $\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 383

$$\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = -\frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\,\operatorname{arctanh}\left(\frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}\,e} + \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\,\operatorname{arctanh}\left(\frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}\,e} - \frac{b\,\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2c^{3/2}e} + \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{ce}$$

```
[Out] -1/2*b*arctanh(1/2*(b+2*c*tan(e*x+d))/c^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^
2)^(1/2))/c^(3/2)/e-1/2*arctanh(1/2*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2)+b*tan(e*
x+d))*2^(1/2)/(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e
*x+d)^2)^(1/2))*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/e*2^(1/2)/(a^2-2*a*c+
b^2+c^2)^(1/2)+1/2*arctanh(1/2*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2)+b*tan(e*x+d))
*2^(1/2)/(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)
^2)^(1/2))*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/e*2^(1/2)/(a^2-2*a*c+b^2+c
^2)^(1/2)+(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)/c/e
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3781, 6857, 654, 635, 212, 1050, 1044, 214}

$$\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = -\frac{\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\,\text{arctanh}\left(\frac{-\sqrt{a^2-2ac+b^2+c^2}+a+b\tan(d+ex)-c}{\sqrt{2}\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}} + \frac{\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\,\text{arctanh}\left(\frac{\sqrt{a^2-2ac+b^2+c^2}+a+b\tan(d+ex)-c}{\sqrt{2}\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}} - \frac{b\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2c^{3/2}e} + \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{ce}$$

```
[In] Int[Tan[d + e*x]^3/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] -((Sqrt[a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTanh[(a - c - Sqrt[a^2 +
b^2 - 2*a*c + c^2] + b*Tan[d + e*x])/(Sqrt[2]*Sqrt[a - c - Sqrt[a^2 + b^2 -
 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*Sqrt
[a^2 + b^2 - 2*a*c + c^2]*e)) + (Sqrt[a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]
]*ArcTanh[(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2] + b*Tan[d + e*x])/(Sqrt[2]
*Sqrt[a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Ta
n[d + e*x]^2])])/(Sqrt[2]*Sqrt[a^2 + b^2 - 2*a*c + c^2]*e) - (b*ArcTanh[(b
+ 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])
])/(2*c^(3/2)*e) + Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2]/(c*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}\\ &= \frac{\text{Subst}\left(\int \left(\frac{x}{\sqrt{a+bx+cx^2}} - \frac{x}{(1+x^2)\sqrt{a+bx+cx^2}}\right)\,dx, x, \tan(d+ex)\right)}{e}\end{aligned}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e} - \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{ce} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2ce} + \frac{\text{Subst}\left(\int \frac{-b+\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e} - \frac{\text{Subst}\left(\int \frac{-b+\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e}$$

$$= \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{ce} - \frac{b\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{ce} + \frac{\left(b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{-2b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)+bx^2}\,dx, x, \frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+b}{\sqrt{a+b\tan(d+ex)+c\tan}}\right.}{\sqrt{a^2+b^2-2ac+c^2}e} - \frac{\left(b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{-2b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)+bx^2}\,dx, x, \frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+b}{\sqrt{a+b\tan(d+ex)+c\tan}}\right.}{\sqrt{a^2+b^2-2ac+c^2}e}$$

$$= -\frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\text{arctanh}\left(\frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} + \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\text{arctanh}\left(\frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} - \frac{b\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2c^{3/2}e} + \frac{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{ce}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.66

$$\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \frac{\frac{\text{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a-ib-c}} + \frac{\text{arctanh}\left(\frac{2a+ib+(b+2ic)\tan(d+ex)}{2\sqrt{a+ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a+ib-c}} - \frac{b\text{arctanh}\left(\frac{b+2c\tan}{2\sqrt{c}\sqrt{a+b\tan(d+e}}\right.}{c^{3/2}}}{2e}$$

```
[In] Integrate[Tan[d + e*x]^3/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (ArcTanh[(2*a - I*b + (b - (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a - I*b - c]*Sqrt
[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])]/Sqrt[a - I*b - c] + ArcTanh[(2*a
+ I*b + (b + (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a + I*b - c]*Sqrt[a + b*Tan[d +
 e*x] + c*Tan[d + e*x]^2])]/Sqrt[a + I*b - c] - (b*ArcTanh[(b + 2*c*Tan[d +
 e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/c^(3/2) +
(2*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/c)/(2*e)
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.57 (sec) , antiderivative size = 9581103, normalized size of antiderivative = 25015.93

output too large to display

```
[In] int(tan(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5168 vs. 2(338) = 676.

Time = 1.05 (sec) , antiderivative size = 10337, normalized size of antiderivative = 26.99

$$\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\tan^3{(d+ex)}}{\sqrt{a+b\tan{(d+ex)}+c\tan^2{(d+ex)}}}\,dx$$

```
[In] integrate(tan(e*x+d)**3/(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2),x)
```

```
[Out] Integral(tan(d + e*x)**3/sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\tan\left(ex+d\right)^3}{\sqrt{c\tan\left(ex+d\right)^2+b\tan\left(ex+d\right)+a}}\,dx$$

```
[In] integrate(tan(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(tan(e*x + d)^3/sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\tan(d+e\,x)^3}{\sqrt{c\tan\left(d+e\,x\right)^2+b\tan\left(d+e\,x\right)+a}}\,dx$$

```
[In] int(tan(d + e*x)^3/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)
```

```
[Out] int(tan(d + e*x)^3/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2), x)
```

3.13 $\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 352

$$\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = -\frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} + \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} + \frac{\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{c}e}$$

```
[Out] arctanh(1/2*(b+2*c*tan(e*x+d))/c^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2
))/e/c^(1/2)-1/2*arctan(1/2*(b-(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))*tan(e*x+d))*
2^(1/2)/(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^
2)^(1/2))*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/e*2^(1/2)/(a^2-2*a*c+b^2+c^
2)^(1/2)+1/2*arctan(1/2*(b-(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))*tan(e*x+d))*2^(1
/2)/(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(
1/2))*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/e*2^(1/2)/(a^2-2*a*c+b^2+c^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3781, 1093, 635, 212, 1001, 1044, 211}

$$\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = -\frac{\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\arctan\left(\frac{b-\left(-\sqrt{a^2-2ac+b^2+c^2}+a-c\right)\tan(d+ex)}{\sqrt{2}\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}} + \frac{\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\arctan\left(\frac{b-\left(\sqrt{a^2-2ac+b^2+c^2}+a-c\right)\tan(d+ex)}{\sqrt{2}\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}} + \frac{\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{c}e}$$

```
[In] Int[Tan[d + e*x]^2/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] -((Sqrt[a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTan[(b - (a - c - Sqrt[a^
2 + b^2 - 2*a*c + c^2])*Tan[d + e*x])/(Sqrt[2]*Sqrt[a - c - Sqrt[a^2 + b^2
- 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*Sqr
t[a^2 + b^2 - 2*a*c + c^2]*e)) + (Sqrt[a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2
]]*ArcTan[(b - (a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2])*Tan[d + e*x])/(Sqrt[
2]*Sqrt[a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*
Tan[d + e*x]^2])])/(Sqrt[2]*Sqrt[a^2 + b^2 - 2*a*c + c^2]*e) + ArcTanh[(b +
 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])]
/(Sqrt[c]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1001

```
Int[1/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_S
ymbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist[1/(2*q), Int[(c*d
 - a*f + q + c*e*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2
*q), Int[(c*d - a*f - q + c*e*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x
]] /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1093

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) +
 (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x]
 + Dist[(A*c - a*C)/c, Int[1/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /;
 FreeQ[{a, c, d, e, f, A, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{2\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e} \\
&\quad + \frac{\text{Subst}\left(\int \frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e} \\
&\quad - \frac{\text{Subst}\left(\int \frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e}
\end{aligned}
$$

$$= \frac{\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{c}e} - \frac{\left(b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{2b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)+bx^2}dx, x, \frac{b-\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)}{\sqrt{a+b\tan(d+ex)+c\tan^2}}\right.}{\sqrt{a^2+b^2-2ac+c^2}e} + \frac{\left(b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{2b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)+bx^2}dx, x, \frac{b-\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)}{\sqrt{a+b\tan(d+ex)+c\tan^2}}\right.}{\sqrt{a^2+b^2-2ac+c^2}e}$$

$$= -\frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} + \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} + \frac{\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{c}e}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.65

$$\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \frac{\frac{i\operatorname{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{a-ib-c}} - \frac{i\operatorname{arctanh}\left(\frac{2a+ib+(b+2ic)\tan(d+ex)}{2\sqrt{a+ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{a+ib-c}} + \frac{\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{c}}}{e}$$

```
[In] Integrate[Tan[d + e*x]^2/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (((I/2)*ArcTanh[(2*a - I*b + (b - (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a - I*b -
c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/Sqrt[a - I*b - c] - ((I/2
)*ArcTanh[(2*a + I*b + (b + (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a + I*b - c]*Sqr
t[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/Sqrt[a + I*b - c] + ArcTanh[(b
+ 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])
]/Sqrt[c])/e
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.02 (sec) , antiderivative size = 7491751, normalized size of antiderivative = 21283.38

output too large to display

```
[In] int(tan(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4980 vs. $2(313) = 626$.

Time = 0.99 (sec) , antiderivative size = 9961, normalized size of antiderivative = 28.30

$$\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$$

```
[In] integrate(tan(e*x+d)**2/(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2),x)
[Out] Integral(tan(d + e*x)**2/sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\tan(ex+d)^2}{\sqrt{c\tan(ex+d)^2+b\tan(ex+d)+a}}\,dx$$

```
[In] integrate(tan(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
maxima")
[Out] integrate(tan(e*x + d)^2/sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\tan(d+e\,x)^2}{\sqrt{c\tan\left(d+e\,x\right)^2+b\tan\left(d+e\,x\right)+a}}\,dx$$

```
[In] int(tan(d + e*x)^2/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)
```

```
[Out] int(tan(d + e*x)^2/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2), x)
```

3.14 $\int \frac{\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 294

$$\int \frac{\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\,\text{arctanh}\left(\frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} - \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\,\text{arctanh}\left(\frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e}$$

```
[Out] 1/2*arctanh(1/2*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2)+b*tan(e*x+d))*2^(1/2)/(a-c-(
a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a-c
-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/e*2^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/2)-1/2*a
rctanh(1/2*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2)+b*tan(e*x+d))*2^(1/2)/(a-c+(a^2-2
*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a-c+(a^2
-2*a*c+b^2+c^2)^(1/2))^(1/2)/e*2^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used

= {3781, 1050, 1044, 214}

$$\int \frac{\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \frac{\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\operatorname{arctanh}\left(\frac{-\sqrt{a^2-2ac+b^2+c^2}+a+b\tan(d+ex)-c}{\sqrt{2}\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}} - \frac{\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\operatorname{arctanh}\left(\frac{\sqrt{a^2-2ac+b^2+c^2}+a+b\tan(d+ex)-c}{\sqrt{2}\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}}$$

```
[In] Int[Tan[d + e*x]/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (Sqrt[a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTanh[(a - c - Sqrt[a^2 + b^
2 - 2*a*c + c^2] + b*Tan[d + e*x])/(Sqrt[2]*Sqrt[a - c - Sqrt[a^2 + b^2 - 2
*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*Sqrt[a
^2 + b^2 - 2*a*c + c^2]*e) - (Sqrt[a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*A
rcTanh[(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2] + b*Tan[d + e*x])/(Sqrt[2]*Sq
rt[a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d
 + e*x]^2])])/(Sqrt[2]*Sqrt[a^2 + b^2 - 2*a*c + c^2]*e)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
```

```
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= -\frac{\text{Subst}\left(\int \frac{-b+\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}\,e} + \frac{\text{Subst}\left(\int \frac{-b+\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}\,e}$$

$$= -\frac{\left(b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{-2b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)+bx^2}\,dx, x, \frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+}{\sqrt{a+b\tan(d+ex)+c\tan}}\right.}{\sqrt{a^2+b^2-2ac+c^2}\,e} + \frac{\left(b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{-2b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)+bx^2}\,dx, x, \frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+}{\sqrt{a+b\tan(d+ex)+c\tan}}\right.}{\sqrt{a^2+b^2-2ac+c^2}\,e}$$

$$= \frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\,\text{arctanh}\left(\frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}\,e} - \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\,\text{arctanh}\left(\frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}\,e}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.59

$$\int \frac{\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \frac{-\frac{\text{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{a-ib-c}} - \frac{\text{arctanh}\left(\frac{2a+ib+(b+2ic)\tan(d+ex)}{2\sqrt{a+ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2\sqrt{a+ib-c}}}{e}$$

```
[In] Integrate[Tan[d + e*x]/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (-1/2*ArcTanh[(2*a - I*b + (b - (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a - I*b - c]
*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])]/Sqrt[a - I*b - c] - ArcTanh[
(2*a + I*b + (b + (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a + I*b - c]*Sqrt[a + b*Ta
n[d + e*x] + c*Tan[d + e*x]^2])]/(2*Sqrt[a + I*b - c]))/e
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.30 (sec) , antiderivative size = 9339203, normalized size of antiderivative = 31766.00

output too large to display

```
[In] int(tan(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5045 vs. 2(261) = 522.

Time = 0.81 (sec) , antiderivative size = 5045, normalized size of antiderivative = 17.16

$$\int \frac{\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="fr
icas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\tan{(d+ex)}}{\sqrt{a+b\tan{(d+ex)}+c\tan^2{(d+ex)}}}\,dx$$

```
[In] integrate(tan(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2),x)
```

```
[Out] Integral(tan(d + e*x)/sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(tan(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\tan(d+e\,x)}{\sqrt{c\tan\left(d+e\,x\right)^2+b\tan\left(d+e\,x\right)+a}}\,dx$$

```
[In] int(tan(d + e*x)/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)
```

```
[Out] int(tan(d + e*x)/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2), x)
```

3.15 $\int \frac{1}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 298

$$\int \frac{1}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} - \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e}$$

```
[Out] 1/2*arctan(1/2*(b-(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))*tan(e*x+d))*2^(1/2)/(a-c-
(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a-
c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/e*2^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/2)-1/2*
arctan(1/2*(b-(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))*tan(e*x+d))*2^(1/2)/(a-c+(a^2
-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a-c+(a
^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/e*2^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used

$= \{1001, 1044, 211\}$

$$\int \frac{1}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$$
$$= \frac{\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\arctan\left(\frac{b-\left(-\sqrt{a^2-2ac+b^2+c^2}+a-c\right)\tan(d+ex)}{\sqrt{2}\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}}$$
$$- \frac{\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\arctan\left(\frac{b-\left(\sqrt{a^2-2ac+b^2+c^2}+a-c\right)\tan(d+ex)}{\sqrt{2}\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}}$$

```
[In] Int[1/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (Sqrt[a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTan[(b - (a - c - Sqrt[a^2
+ b^2 - 2*a*c + c^2])*Tan[d + e*x])/(Sqrt[2]*Sqrt[a - c - Sqrt[a^2 + b^2 -
2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*Sqrt[
a^2 + b^2 - 2*a*c + c^2]*e) - (Sqrt[a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*
ArcTan[(b - (a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2])*Tan[d + e*x])/(Sqrt[2]*
Sqrt[a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan
[d + e*x]^2])])/(Sqrt[2]*Sqrt[a^2 + b^2 - 2*a*c + c^2]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1001

```
Int[1/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_S
ymbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist[1/(2*q), Int[(c*d
 - a*f + q + c*e*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2
*q), Int[(c*d - a*f - q + c*e*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x
]] /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= -\frac{\text{Subst}\left(\int \frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e} + \frac{\text{Subst}\left(\int \frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e}$$

$$= \frac{\left(b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{2b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)+bx^2}\,dx, x, \frac{b-\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\text{ta}}{\sqrt{a+b\tan(d+ex)+c\tan^2(d}}\right.}{\sqrt{a^2+b^2-2ac+c^2}e} - \frac{\left(b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{2b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)+bx^2}\,dx, x, \frac{b-\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)}{\sqrt{a+b\tan(d+ex)+c\tan^2}}\right.}{\sqrt{a^2+b^2-2ac+c^2}e}$$

$$= \frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} - \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = -\frac{i\left(\frac{\text{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a-ib-c}} - \frac{\text{arctanh}\left(\frac{2a+ib+(b+2ic)\tan(d+ex)}{2\sqrt{a+ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a+ib-c}}\right)}{2e}$$

```
[In] Integrate[1/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] ((-1/2*I)*(ArcTanh[(2*a - I*b + (b - (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a - I*b
 - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])]/Sqrt[a - I*b - c] - Arc
Tanh[(2*a + I*b + (b + (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a + I*b - c]*Sqrt[a +
 b*Tan[d + e*x] + c*Tan[d + e*x]^2])]/Sqrt[a + I*b - c]))/e
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 0.71 (sec) , antiderivative size = 7300729, normalized size of antiderivative = 24499.09

output too large to display

```
[In] int(1/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4891 vs. 2(267) = 534.

Time = 0.76 (sec) , antiderivative size = 4891, normalized size of antiderivative = 16.41

$$\int \frac{1}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-((a^2 + b^2 - 2*a*c + c^2)*e^2*sqrt(-b^2/((a^4 + 2*a^2*b^2 + b^4
 - 4*a*c^3 + c^4 + 2*(3*a^2 + b^2)*c^2 - 4*(a^3 + a*b^2)*c)*e^4)) + a - c)/
((a^2 + b^2 - 2*a*c + c^2)*e^2))*log(-1/2*(2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c
 - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^
3*b + 3*a*b^3)*c)*tan(e*x + d) + (2*(2*a^5*b + 3*a^3*b^3 + a*b^5 - 3*a*b^3*
c^2 - 2*a*b*c^4 + (4*a^2*b + b^3)*c^3 - (4*a^4*b + a^2*b^3 - b^5)*c)*e^2*ta
n(e*x + d) + (4*a^6 + 7*a^4*b^2 + 4*a^2*b^4 + b^6 + (4*a^2 - b^2)*c^4 - 4*(
4*a^3 + a*b^2)*c^3 + 6*(4*a^4 + 3*a^2*b^2)*c^2 - 4*(4*a^5 + 5*a^3*b^2 + 2*a
*b^4)*c)*e^2)*sqrt(-b^2/((a^4 + 2*a^2*b^2 + b^4 - 4*a*c^3 + c^4 + 2*(3*a^2
+ b^2)*c^2 - 4*(a^3 + a*b^2)*c)*e^4)))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x +
d) + a) + (2*(2*a^3*b^3 + a*b^5 + (4*a^2*b - b^3)*c^3 - 8*(a^3*b + a*b^3)*c
^2 + (4*a^4*b + 3*a^2*b^3 + 2*b^5)*c)*e*tan(e*x + d)^2 + 2*(8*a^4*b^2 + 5*a
^2*b^4 + b^6 - 3*b^4*c^2 + 4*a*b^2*c^3 - 6*(2*a^3*b^2 + a*b^4)*c)*e*tan(e*x
 + d) + 2*(4*a^5*b + a^3*b^3 + (4*a^3*b + a*b^3)*c^2 - (8*a^4*b + 6*a^2*b^3
 + b^5)*c)*e + ((4*a^6*b + 7*a^4*b^3 + 4*a^2*b^5 + b^7 + 8*a*b*c^5 - (12*a^
2*b + 5*b^3)*c^4 - 8*(2*a^3*b - a*b^3)*c^3 + 2*(20*a^4*b + 11*a^2*b^3 - 2*b
^5)*c^2 - 4*(6*a^5*b + 8*a^3*b^3 + 3*a*b^5)*c)*e^3*tan(e*x + d)^2 + 2*(4*a^
7 + 3*a^5*b^2 - 2*a^3*b^4 - a*b^6 - (4*a^2 - b^2)*c^5 + (20*a^3 + 7*a*b^2)*
c^4 - 2*(20*a^4 + 15*a^2*b^2 + b^4)*c^3 + 2*(20*a^5 + 19*a^3*b^2 + 7*a*b^4)
*c^2 - (20*a^6 + 19*a^4*b^2 + 10*a^2*b^4 + 3*b^6)*c)*e^3*tan(e*x + d) - (12
*a^6*b + 19*a^4*b^3 + 8*a^2*b^5 + b^7 - (4*a^2*b + b^3)*c^4 + 6*(4*a^4*b +
a^2*b^3)*c^2 - 4*(8*a^5*b + 6*a^3*b^3 + a*b^5)*c)*e^3)*sqrt(-b^2/((a^4 + 2*
a^2*b^2 + b^4 - 4*a*c^3 + c^4 + 2*(3*a^2 + b^2)*c^2 - 4*(a^3 + a*b^2)*c)*e^
4)))*sqrt(-((a^2 + b^2 - 2*a*c + c^2)*e^2*sqrt(-b^2/((a^4 + 2*a^2*b^2 + b^4
```

```
- 4*a*c^3 + c^4 + 2*(3*a^2 + b^2)*c^2 - 4*(a^3 + a*b^2)*c)*e^4)) + a - c)/
((a^2 + b^2 - 2*a*c + c^2)*e^2)))/(tan(e*x + d)^2 + 1)) + 1/4*sqrt(-((a^2 +
 b^2 - 2*a*c + c^2)*e^2*sqrt(-b^2/((a^4 + 2*a^2*b^2 + b^4 - 4*a*c^3 + c^4 +
 2*(3*a^2 + b^2)*c^2 - 4*(a^3 + a*b^2)*c)*e^4)) + a - c)/((a^2 + b^2 - 2*a*
c + c^2)*e^2))*log(-1/2*(2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (
4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*
tan(e*x + d) + (2*(2*a^5*b + 3*a^3*b^3 + a*b^5 - 3*a*b^3*c^2 - 2*a*b*c^4 +
(4*a^2*b + b^3)*c^3 - (4*a^4*b + a^2*b^3 - b^5)*c)*e^2*tan(e*x + d) + (4*a^
6 + 7*a^4*b^2 + 4*a^2*b^4 + b^6 + (4*a^2 - b^2)*c^4 - 4*(4*a^3 + a*b^2)*c^3
 + 6*(4*a^4 + 3*a^2*b^2)*c^2 - 4*(4*a^5 + 5*a^3*b^2 + 2*a*b^4)*c)*e^2)*sqrt
(-b^2/((a^4 + 2*a^2*b^2 + b^4 - 4*a*c^3 + c^4 + 2*(3*a^2 + b^2)*c^2 - 4*(a^
3 + a*b^2)*c)*e^4)))*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) - (2*(2*a^
3*b^3 + a*b^5 + (4*a^2*b - b^3)*c^3 - 8*(a^3*b + a*b^3)*c^2 + (4*a^4*b + 3*
a^2*b^3 + 2*b^5)*c)*e*tan(e*x + d)^2 + 2*(8*a^4*b^2 + 5*a^2*b^4 + b^6 - 3*b
^4*c^2 + 4*a*b^2*c^3 - 6*(2*a^3*b^2 + a*b^4)*c)*e*tan(e*x + d) + 2*(4*a^5*b
 + a^3*b^3 + (4*a^3*b + a*b^3)*c^2 - (8*a^4*b + 6*a^2*b^3 + b^5)*c)*e + ((4
*a^6*b + 7*a^4*b^3 + 4*a^2*b^5 + b^7 + 8*a*b*c^5 - (12*a^2*b + 5*b^3)*c^4 -
 8*(2*a^3*b - a*b^3)*c^3 + 2*(20*a^4*b + 11*a^2*b^3 - 2*b^5)*c^2 - 4*(6*a^5
*b + 8*a^3*b^3 + 3*a*b^5)*c)*e^3*tan(e*x + d)^2 + 2*(4*a^7 + 3*a^5*b^2 - 2*
a^3*b^4 - a*b^6 - (4*a^2 - b^2)*c^5 + (20*a^3 + 7*a*b^2)*c^4 - 2*(20*a^4 +
15*a^2*b^2 + b^4)*c^3 + 2*(20*a^5 + 19*a^3*b^2 + 7*a*b^4)*c^2 - (20*a^6 + 1
9*a^4*b^2 + 10*a^2*b^4 + 3*b^6)*c)*e^3*tan(e*x + d) - (12*a^6*b + 19*a^4*b^
3 + 8*a^2*b^5 + b^7 - (4*a^2*b + b^3)*c^4 + 6*(4*a^4*b + a^2*b^3)*c^2 - 4*(
8*a^5*b + 6*a^3*b^3 + a*b^5)*c)*e^3)*sqrt(-b^2/((a^4 + 2*a^2*b^2 + b^4 - 4*
a*c^3 + c^4 + 2*(3*a^2 + b^2)*c^2 - 4*(a^3 + a*b^2)*c)*e^4)))*sqrt(-((a^2 +
 b^2 - 2*a*c + c^2)*e^2*sqrt(-b^2/((a^4 + 2*a^2*b^2 + b^4 - 4*a*c^3 + c^4 +
 2*(3*a^2 + b^2)*c^2 - 4*(a^3 + a*b^2)*c)*e^4)) + a - c)/((a^2 + b^2 - 2*a*
c + c^2)*e^2)))/(tan(e*x + d)^2 + 1)) - 1/4*sqrt(((a^2 + b^2 - 2*a*c + c^2)
*e^2*sqrt(-b^2/((a^4 + 2*a^2*b^2 + b^4 - 4*a*c^3 + c^4 + 2*(3*a^2 + b^2)*c^
2 - 4*(a^3 + a*b^2)*c)*e^4)) - a + c)/((a^2 + b^2 - 2*a*c + c^2)*e^2))*log(
-1/2*(2*(4*a^3*b^2 + 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3
 + b^5 + (4*a^2*b - b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) - (2*(
2*a^5*b + 3*a^3*b^3 + a*b^5 - 3*a*b^3*c^2 - 2*a*b*c^4 + (4*a^2*b + b^3)*c^3
 - (4*a^4*b + a^2*b^3 - b^5)*c)*e^2*tan(e*x + d) + (4*a^6 + 7*a^4*b^2 + 4*a
^2*b^4 + b^6 + (4*a^2 - b^2)*c^4 - 4*(4*a^3 + a*b^2)*c^3 + 6*(4*a^4 + 3*a^2
*b^2)*c^2 - 4*(4*a^5 + 5*a^3*b^2 + 2*a*b^4)*c)*e^2)*sqrt(-b^2/((a^4 + 2*a^2
*b^2 + b^4 - 4*a*c^3 + c^4 + 2*(3*a^2 + b^2)*c^2 - 4*(a^3 + a*b^2)*c)*e^4))
)*sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a) + (2*(2*a^3*b^3 + a*b^5 + (4*
a^2*b - b^3)*c^3 - 8*(a^3*b + a*b^3)*c^2 + (4*a^4*b + 3*a^2*b^3 + 2*b^5)*c)
*e*tan(e*x + d)^2 + 2*(8*a^4*b^2 + 5*a^2*b^4 + b^6 - 3*b^4*c^2 + 4*a*b^2*c^
3 - 6*(2*a^3*b^2 + a*b^4)*c)*e*tan(e*x + d) + 2*(4*a^5*b + a^3*b^3 + (4*a^3
*b + a*b^3)*c^2 - (8*a^4*b + 6*a^2*b^3 + b^5)*c)*e - ((4*a^6*b + 7*a^4*b^3
+ 4*a^2*b^5 + b^7 + 8*a*b*c^5 - (12*a^2*b + 5*b^3)*c^4 - 8*(2*a^3*b - a*b^3
)*c^3 + 2*(20*a^4*b + 11*a^2*b^3 - 2*b^5)*c^2 - 4*(6*a^5*b + 8*a^3*b^3 + 3*
a*b^5)*c)*e^3*tan(e*x + d)^2 + 2*(4*a^7 + 3*a^5*b^2 - 2*a^3*b^4 - a*b^6 - (
```

```
4*a^2 - b^2)*c^5 + (20*a^3 + 7*a*b^2)*c^4 - 2*(20*a^4 + 15*a^2*b^2 + b^4)*c
^3 + 2*(20*a^5 + 19*a^3*b^2 + 7*a*b^4)*c^2 - (20*a^6 + 19*a^4*b^2 + 10*a^2*
b^4 + 3*b^6)*c)*e^3*tan(e*x + d) - (12*a^6*b + 19*a^4*b^3 + 8*a^2*b^5 + b^7
 - (4*a^2*b + b^3)*c^4 + 6*(4*a^4*b + a^2*b^3)*c^2 - 4*(8*a^5*b + 6*a^3*b^3
 + a*b^5)*c)*e^3)*sqrt(-b^2/((a^4 + 2*a^2*b^2 + b^4 - 4*a*c^3 + c^4 + 2*(3*
a^2 + b^2)*c^2 - 4*(a^3 + a*b^2)*c)*e^4)))*sqrt(((a^2 + b^2 - 2*a*c + c^2)*
e^2*sqrt(-b^2/((a^4 + 2*a^2*b^2 + b^4 - 4*a*c^3 + c^4 + 2*(3*a^2 + b^2)*c^2
 - 4*(a^3 + a*b^2)*c)*e^4)) - a + c)/((a^2 + b^2 - 2*a*c + c^2)*e^2)))/(tan
(e*x + d)^2 + 1)) + 1/4*sqrt(((a^2 + b^2 - 2*a*c + c^2)*e^2*sqrt(-b^2/((a^4
 + 2*a^2*b^2 + b^4 - 4*a*c^3 + c^4 + 2*(3*a^2 + b^2)*c^2 - 4*(a^3 + a*b^2)*
c)*e^4)) - a + c)/((a^2 + b^2 - 2*a*c + c^2)*e^2))*log(-1/2*(2*(4*a^3*b^2 +
 2*a*b^4 + 2*b^4*c - 4*a*b^2*c^2 - (4*a^4*b + 3*a^2*b^3 + b^5 + (4*a^2*b -
b^3)*c^2 - 2*(4*a^3*b + 3*a*b^3)*c)*tan(e*x + d) - (2*(2*a^5*b + 3*a^3*b^3
+ a*b^5 - 3*a*b^3*c^2 - 2*a*b*c^4 + (4*a^2*b + b^3)*c^3 - (4*a^4*b + a^2*b^
3 - b^5)*c)*e^2*tan(e*x + d) + (4*a^6 + 7*a^4*b^2 + 4*a^2*b^4 + b^6 + (4*a^
2 - b^2)*c^4 - 4*(4*a^3 + a*b^2)*c^3 + 6*(4*a^4 + 3*a^2*b^2)*c^2 - 4*(4*a^5
 + 5*a^3*b^2 + 2*a*b^4)*c)*e^2)*sqrt(-b^2/((a^4 + 2*a^2*b^2 + b^4 - 4*a*c^3
 + c^4 + 2*(3*a^2 + b^2)*c^2 - 4*(a^3 + a*b^2)*c)*e^4)))*sqrt(c*tan(e*x + d
)^2 + b*tan(e*x + d) + a) - (2*(2*a^3*b^3 + a*b^5 + (4*a^2*b - b^3)*c^3 - 8
*(a^3*b + a*b^3)*c^2 + (4*a^4*b + 3*a^2*b^3 + 2*b^5)*c)*e*tan(e*x + d)^2 +
2*(8*a^4*b^2 + 5*a^2*b^4 + b^6 - 3*b^4*c^2 + 4*a*b^2*c^3 - 6*(2*a^3*b^2 + a
*b^4)*c)*e*tan(e*x + d) + 2*(4*a^5*b + a^3*b^3 + (4*a^3*b + a*b^3)*c^2 - (8
*a^4*b + 6*a^2*b^3 + b^5)*c)*e - ((4*a^6*b + 7*a^4*b^3 + 4*a^2*b^5 + b^7 +
8*a*b*c^5 - (12*a^2*b + 5*b^3)*c^4 - 8*(2*a^3*b - a*b^3)*c^3 + 2*(20*a^4*b
+ 11*a^2*b^3 - 2*b^5)*c^2 - 4*(6*a^5*b + 8*a^3*b^3 + 3*a*b^5)*c)*e^3*tan(e*
x + d)^2 + 2*(4*a^7 + 3*a^5*b^2 - 2*a^3*b^4 - a*b^6 - (4*a^2 - b^2)*c^5 + (
20*a^3 + 7*a*b^2)*c^4 - 2*(20*a^4 + 15*a^2*b^2 + b^4)*c^3 + 2*(20*a^5 + 19*
a^3*b^2 + 7*a*b^4)*c^2 - (20*a^6 + 19*a^4*b^2 + 10*a^2*b^4 + 3*b^6)*c)*e^3*
tan(e*x + d) - (12*a^6*b + 19*a^4*b^3 + 8*a^2*b^5 + b^7 - (4*a^2*b + b^3)*c
^4 + 6*(4*a^4*b + a^2*b^3)*c^2 - 4*(8*a^5*b + 6*a^3*b^3 + a*b^5)*c)*e^3)*sq
rt(-b^2/((a^4 + 2*a^2*b^2 + b^4 - 4*a*c^3 + c^4 + 2*(3*a^2 + b^2)*c^2 - 4*(
a^3 + a*b^2)*c)*e^4)))*sqrt(((a^2 + b^2 - 2*a*c + c^2)*e^2*sqrt(-b^2/((a^4
+ 2*a^2*b^2 + b^4 - 4*a*c^3 + c^4 + 2*(3*a^2 + b^2)*c^2 - 4*(a^3 + a*b^2)*c
)*e^4)) - a + c)/((a^2 + b^2 - 2*a*c + c^2)*e^2)))/(tan(e*x + d)^2 + 1))
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{1}{\sqrt{a+b\tan\left(d+ex\right)+c\tan^2\left(d+ex\right)}}\,dx$$

```
[In] integrate(1/(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{1}{\sqrt{c\tan\left(ex+d\right)^2+b\tan\left(ex+d\right)+a}}\,dx$$

```
[In] integrate(1/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{1}{\sqrt{c\tan\left(d+e\,x\right)^2+b\tan\left(d+e\,x\right)+a}}\,dx$$

```
[In] int(1/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)
```

```
[Out] int(1/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2), x)
```

3.16 $\int \frac{\cot(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 350

$$\begin{aligned}
\int \frac{\cot(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx
&= -\frac{\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a}e} \\
&\quad -\frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} \\
&\quad +\frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e}
\end{aligned}$$

```
[Out] -arctanh(1/2*(2*a+b*tan(e*x+d))/a^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/
2))/e/a^(1/2)-1/2*arctanh(1/2*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2)+b*tan(e*x+d))*
2^(1/2)/(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^
2)^(1/2))*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/e*2^(1/2)/(a^2-2*a*c+b^2+c^
2)^(1/2)+1/2*arctanh(1/2*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2)+b*tan(e*x+d))*2^(1/
2)/(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1
/2))*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/e*2^(1/2)/(a^2-2*a*c+b^2+c^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3781, 6857, 738, 212, 1050, 1044, 214}

$$\int \frac{\cot(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = -\frac{\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\,\text{arctanh}\left(\frac{-\sqrt{a^2-2ac+b^2+c^2}+a+b\tan(d+ex)-c}{\sqrt{2}\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}} + \frac{\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\,\text{arctanh}\left(\frac{\sqrt{a^2-2ac+b^2+c^2}+a+b\tan(d+ex)-c}{\sqrt{2}\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}} - \frac{\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a}e}$$

```
[In] Int[Cot[d + e*x]/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] -(ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x] + c*Tan
[d + e*x]^2])]/(Sqrt[a]*e)) - (Sqrt[a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]*
ArcTanh[(a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2] + b*Tan[d + e*x])/(Sqrt[2]*S
qrt[a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[
d + e*x]^2])])/(Sqrt[2]*Sqrt[a^2 + b^2 - 2*a*c + c^2]*e) + (Sqrt[a - c + Sq
rt[a^2 + b^2 - 2*a*c + c^2]]*ArcTanh[(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]
 + b*Tan[d + e*x])/(Sqrt[2]*Sqrt[a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqr
t[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*Sqrt[a^2 + b^2 - 2*a*c
 + c^2]*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x\sqrt{a+bx+cx^2}}-\frac{x}{(1+x^2)\sqrt{a+bx+cx^2}}\right)\,dx, x, \tan(d+ex)\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}-\frac{\text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}
\end{aligned}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e} + \frac{\text{Subst}\left(\int \frac{-b+\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e} - \frac{\text{Subst}\left(\int \frac{-b+\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e}$$

$$= -\frac{\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a}e} + \frac{\left(b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{-2b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)+bx^2}\,dx, x, \frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+b}{\sqrt{a+b\tan(d+ex)+c\tan}}\right)}{\sqrt{a^2+b^2-2ac+c^2}e} - \frac{\left(b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{-2b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)+bx^2}\,dx, x, \frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+b}{\sqrt{a+b\tan(d+ex)+c\tan}}\right)}{\sqrt{a^2+b^2-2ac+c^2}e}$$

$$= -\frac{\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a}e} - \frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\,\text{arctanh}\left(\frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} + \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\,\text{arctanh}\left(\frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.64

$$\int \frac{\cot(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$$

$$= \frac{-\frac{2\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a}} + \frac{\text{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a-ib-c}} + \frac{\text{arctanh}\left(\frac{2a+ib+(b+2ic}{2\sqrt{a+ib-c}\sqrt{a+b\tan(c}}\right)}{\sqrt{a+ib-c}}}{2e}$$

```
[In] Integrate[Cot[d + e*x]/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] ((-2*ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x] + c*
Tan[d + e*x]^2])])/Sqrt[a] + ArcTanh[(2*a - I*b + (b - (2*I)*c)*Tan[d + e*x
```

```
])/(2*Sqrt[a - I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])]/Sqrt[
a - I*b - c] + ArcTanh[(2*a + I*b + (b + (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a +
 I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])]/Sqrt[a + I*b - c])/
(2*e)
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int(cot(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
[Out] int(cot(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10284 vs. $2(309) = 618$.

Time = 2.29 (sec) , antiderivative size = 20605, normalized size of antiderivative = 58.87

$$\int \frac{\cot(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="fr
icas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\cot(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\cot(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$$

```
[In] integrate(cot(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2),x)
[Out] Integral(cot(d + e*x)/sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\cot(ex+d)}{\sqrt{c\tan(ex+d)^2+b\tan(ex+d)+a}}\,dx$$

[In] integrate(cot(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(cot(e*x + d)/sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a), x)

Giac [F]

$$\int \frac{\cot(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\cot(ex+d)}{\sqrt{c\tan(ex+d)^2+b\tan(ex+d)+a}}\,dx$$

[In] integrate(cot(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="giac")

[Out] integrate(cot(e*x + d)/sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\cot(d+e\,x)}{\sqrt{c\tan(d+e\,x)^2+b\tan(d+e\,x)+a}}\,dx$$

[In] int(cot(d + e*x)/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)

[Out] int(cot(d + e*x)/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2), x)

3.17 $\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 395

$$\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = -\frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} + \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} + \frac{b\,\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2a^{3/2}e} - \frac{\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{ae}$$

```
[Out] 1/2*b*arctanh(1/2*(2*a+b*tan(e*x+d))/a^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2
)^(1/2))/a^(3/2)/e-1/2*arctan(1/2*(b-(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))*tan(e*
x+d))*2^(1/2)/(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e
*x+d)^2)^(1/2))*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/e*2^(1/2)/(a^2-2*a*c+
b^2+c^2)^(1/2)+1/2*arctan(1/2*(b-(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))*tan(e*x+d)
)*2^(1/2)/(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d
)^2)^(1/2))*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/e*2^(1/2)/(a^2-2*a*c+b^2+
c^2)^(1/2)-cot(e*x+d)*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)/a/e
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3781, 6857, 744, 738, 212, 1001, 1044, 211}

$$\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \frac{b\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2a^{3/2}e} - \frac{\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\arctan\left(\frac{b-\left(-\sqrt{a^2-2ac+b^2+c^2}+a-c\right)\tan(d+ex)}{\sqrt{2}\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}} + \frac{\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\arctan\left(\frac{b-\left(\sqrt{a^2-2ac+b^2+c^2}+a-c\right)\tan(d+ex)}{\sqrt{2}\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}} - \frac{\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{ae}$$

```
[In] Int[Cot[d + e*x]^2/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] -((Sqrt[a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTan[(b - (a - c - Sqrt[a^
2 + b^2 - 2*a*c + c^2])*Tan[d + e*x])/(Sqrt[2]*Sqrt[a - c - Sqrt[a^2 + b^2
- 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*Sqr
t[a^2 + b^2 - 2*a*c + c^2]*e)) + (Sqrt[a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2
]]*ArcTan[(b - (a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2])*Tan[d + e*x])/(Sqrt[
2]*Sqrt[a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*
Tan[d + e*x]^2])])/(Sqrt[2]*Sqrt[a^2 + b^2 - 2*a*c + c^2]*e) + (b*ArcTanh[(
2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2
])])/(2*a^(3/2)*e) - (Cot[d + e*x]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]
^2])/(a*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1001

```
Int[1/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_S
ymbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist[1/(2*q), Int[(c*d
 - a*f + q + c*e*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2
*q), Int[(c*d - a*f - q + c*e*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x
]] /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x^2\sqrt{a+bx+cx^2}} + \frac{1}{(-1-x^2)\sqrt{a+bx+cx^2}}\right) dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}} dx, x, \tan(d+ex)\right)}{e} + \frac{\text{Subst}\left(\int \frac{1}{(-1-x^2)\sqrt{a+bx+cx^2}} dx, x, \tan(d+ex)\right)}{e}$$

$$= -\frac{\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{ae}$$
$$- \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x, \tan(d+ex)\right)}{2ae}$$
$$- \frac{\text{Subst}\left(\int \frac{-a+c-\sqrt{a^2+b^2-2ac+c^2}-bx}{(-1-x^2)\sqrt{a+bx+cx^2}} dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e}$$
$$+ \frac{\text{Subst}\left(\int \frac{-a+c+\sqrt{a^2+b^2-2ac+c^2}-bx}{(-1-x^2)\sqrt{a+bx+cx^2}} dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e}$$

$$= -\frac{\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{ae}$$
$$+ \frac{b\text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+b\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{ae}$$
$$+ \frac{\left(b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{-2b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)-bx^2} dx, x, \frac{b-\left(a-c-\sqrt{a^2+b^2-2ac+c}\right.}{\sqrt{a+b\tan(d+ex)+c\,\text{ta}}}\right.}{\sqrt{a^2+b^2-2ac+c^2}e}$$
$$- \frac{\left(b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{-2b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)-bx^2} dx, x, \frac{b-\left(a-c+\sqrt{a^2+b^2-2ac+c}\right.}{\sqrt{a+b\tan(d+ex)+c\,\text{ta}}}\right.}{\sqrt{a^2+b^2-2ac+c^2}e}$$

$$=$$
$$-\frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e}$$
$$+\frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b-\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e}$$
$$+\frac{b\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2a^{3/2}e}$$
$$-\frac{\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{ae}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.67

$$\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \frac{\frac{b\,\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{a^{3/2}}+\frac{i\,\text{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a-ib-c}}-\frac{i\,\text{arctanh}\left(\frac{2a+ib+(b+2\cdots}{2\sqrt{a+ib-c}\sqrt{a+b\tan(\cdots}}\right)}{\sqrt{a+ib-c}}\cdots}{2e}$$

```
[In] Integrate[Cot[d + e*x]^2/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] ((b*ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x] + c*T
an[d + e*x]^2])])/a^(3/2) + (I*ArcTanh[(2*a - I*b + (b - (2*I)*c)*Tan[d + e
*x])/(2*Sqrt[a - I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/Sq
rt[a - I*b - c] - (I*ArcTanh[(2*a + I*b + (b + (2*I)*c)*Tan[d + e*x])/(2*Sq
rt[a + I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/Sqrt[a + I*b
 - c] - (2*Cot[d + e*x]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/a)/(2*
e)
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int(cot(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
```

```
[Out] int(cot(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10076 vs. 2(352) = 704.

Time = 2.10 (sec) , antiderivative size = 20189, normalized size of antiderivative = 51.11

$$\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$$

```
[In] integrate(cot(e*x+d)**2/(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2),x)
[Out] Integral(cot(d + e*x)**2/sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\cot(ex+d)^2}{\sqrt{c\tan(ex+d)^2+b\tan(ex+d)+a}}\,dx$$

```
[In] integrate(cot(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
maxima")
[Out] integrate(cot(e*x + d)^2/sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a), x)
```

Giac [F]

$$\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\cot(ex+d)^2}{\sqrt{c\tan(ex+d)^2+b\tan(ex+d)+a}}\,dx$$

```
[In] integrate(cot(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
giac")
[Out] integrate(cot(e*x + d)^2/sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\cot(d+e\,x)^2}{\sqrt{c\tan(d+e\,x)^2+b\tan(d+e\,x)+a}}\,dx$$

```
[In] int(cot(d + e*x)^2/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)
[Out] int(cot(d + e*x)^2/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2), x)
```

3.18 $\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 500

$$\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a}e} - \frac{(3b^2-4ac)\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8a^{5/2}e} + \frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} - \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} + \frac{3b\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4a^2e} - \frac{\cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{2ae}$$

```
[Out] -1/8*(-4*a*c+3*b^2)*arctanh(1/2*(2*a+b*tan(e*x+d))/a^(1/2)/(a+b*tan(e*x+d)+
c*tan(e*x+d)^2)^(1/2))/a^(5/2)/e+arctanh(1/2*(2*a+b*tan(e*x+d))/a^(1/2)/(a+
b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))/e/a^(1/2)+1/2*arctanh(1/2*(a-c-(a^2-2*a
*c+b^2+c^2)^(1/2)+b*tan(e*x+d))*2^(1/2)/(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/
2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))^(
1/2)/e*2^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/2)-1/2*arctanh(1/2*(a-c+(a^2-2*a*c+b^
2+c^2)^(1/2)+b*tan(e*x+d))*2^(1/2)/(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a
+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/
```

```
e*2^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/2)+3/4*b*cot(e*x+d)*(a+b*tan(e*x+d)+c*tan(
e*x+d)^2)^(1/2)/a^2/e-1/2*cot(e*x+d)^2*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2
)/a/e
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3781, 6857, 758, 820, 738, 212, 1050, 1044, 214}

$$\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx$$
$$= -\frac{(3b^2-4ac)\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8a^{5/2}e} + \frac{\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\operatorname{arctanh}\left(\frac{-\sqrt{a^2-2ac+b^2+c^2}+a+b\tan(d+ex)-c}{\sqrt{2}\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}} - \frac{\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\operatorname{arctanh}\left(\frac{\sqrt{a^2-2ac+b^2+c^2}+a+b\tan(d+ex)-c}{\sqrt{2}\sqrt{\sqrt{a^2-2ac+b^2+c^2}+a-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}e\sqrt{a^2-2ac+b^2+c^2}} + \frac{3b\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4a^2e} + \frac{\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a}e} - \frac{\cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{2ae}$$

```
[In] Int[Cot[d + e*x]^3/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d
 + e*x]^2])]/(Sqrt[a]*e) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*Tan[d + e*x])/
(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(8*a^(5/2)*e) + (
Sqrt[a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTanh[(a - c - Sqrt[a^2 + b^2
 - 2*a*c + c^2] + b*Tan[d + e*x])/(Sqrt[2]*Sqrt[a - c - Sqrt[a^2 + b^2 - 2*
a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*Sqrt[a^
2 + b^2 - 2*a*c + c^2]*e) - (Sqrt[a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Ar
cTanh[(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2] + b*Tan[d + e*x])/(Sqrt[2]*Sqr
t[a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d
+ e*x]^2])])/(Sqrt[2]*Sqrt[a^2 + b^2 - 2*a*c + c^2]*e) + (3*b*Cot[d + e*x]*
Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/(4*a^2*e) - (Cot[d + e*x]^2*Sq
rt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/(2*a*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3\sqrt{a+bx+cx^2}} - \frac{1}{x\sqrt{a+bx+cx^2}} + \frac{x}{(1+x^2)\sqrt{a+bx+cx^2}}\right)\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e} + \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= -\frac{\cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{2ae} + \frac{2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{e} - \frac{\text{Subst}\left(\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2ae} - \frac{\text{Subst}\left(\int \frac{-b+\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e} + \frac{\text{Subst}\left(\int \frac{-b+\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2\sqrt{a^2+b^2-2ac+c^2}e}$$

$$= \frac{\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a}e} + \frac{3b\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4a^2e} - \frac{\cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{2ae} + \frac{(3b^2-4ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{8a^2e} - \frac{\left(b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{-2b\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)+bx^2}\,dx, x, \frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+}{\sqrt{a+b\tan(d+ex)+c\,\mathrm{ta}}}\right.}{\sqrt{a^2+b^2-2ac+c^2}e} + \frac{\left(b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{-2b\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)+bx^2}\,dx, x, \frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+}{\sqrt{a+b\tan(d+ex)+c\,\mathrm{ta}}}\right.}{\sqrt{a^2+b^2-2ac+c^2}e}$$

$$= \frac{\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a}e} + \frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} - \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} + \frac{3b\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4a^2e} - \frac{\cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{2ae} - \frac{(3b^2-4ac)\operatorname{Subst}\left(\int\frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{4a^2e}$$

$$= \frac{\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a}e} - \frac{(3b^2-4ac)\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8a^{5/2}e} + \frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} - \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+b\tan(d+ex)}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} + \frac{3b\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4a^2e} - \frac{\cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{2ae}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.01 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.63

$$\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \frac{\frac{(8a^2-3b^2+4ac)\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2a^{5/2}} - \frac{2\operatorname{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{\sqrt{a-ib-c}} - \frac{2\operatorname{arctanh}\left(\frac{}{2\sqrt{a+}}\right.}{}}{4e}$$

```
[In] Integrate[Cot[d + e*x]^3/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2],x]
```

```
[Out] (((8*a^2 - 3*b^2 + 4*a*c)*ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqrt[a
+ b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(2*a^(5/2)) - (2*ArcTanh[(2*a - I*b
 + (b - (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a - I*b - c]*Sqrt[a + b*Tan[d + e*x]
 + c*Tan[d + e*x]^2])])/Sqrt[a - I*b - c] - (2*ArcTanh[(2*a + I*b + (b + (2
*I)*c)*Tan[d + e*x])/(2*Sqrt[a + I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d
 + e*x]^2])])/Sqrt[a + I*b - c] + (3*b*Cot[d + e*x]*Sqrt[a + b*Tan[d + e*x]
 + c*Tan[d + e*x]^2])/a^2 - (2*Cot[d + e*x]^2*Sqrt[a + b*Tan[d + e*x] + c*T
an[d + e*x]^2])/a)/(4*e)
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int(cot(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
```

```
[Out] int(cot(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10435 vs. $2(441) = 882$.

Time = 2.40 (sec) , antiderivative size = 20910, normalized size of antiderivative = 41.82

$$\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\cot^3{(d+ex)}}{\sqrt{a+b\tan{(d+ex)}+c\tan^2{(d+ex)}}}\,dx$$

```
[In] integrate(cot(e*x+d)**3/(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(1/2),x)
```

```
[Out] Integral(cot(d + e*x)**3/sqrt(a + b*tan(d + e*x) + c*tan(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\cot\left(ex+d\right)^3}{\sqrt{c\tan\left(ex+d\right)^2+b\tan\left(ex+d\right)+a}}\,dx$$

```
[In] integrate(cot(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(cot(e*x + d)^3/sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a), x)
```

Giac [F]

$$\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\cot\left(ex+d\right)^3}{\sqrt{c\tan\left(ex+d\right)^2+b\tan\left(ex+d\right)+a}}\,dx$$

```
[In] integrate(cot(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(cot(e*x + d)^3/sqrt(c*tan(e*x + d)^2 + b*tan(e*x + d) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\,dx = \int \frac{\cot(d+e\,x)^3}{\sqrt{c\tan\left(d+e\,x\right)^2+b\tan\left(d+e\,x\right)+a}}\,dx$$

```
[In] int(cot(d + e*x)^3/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2),x)
```

```
[Out] int(cot(d + e*x)^3/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(1/2), x)
```

3.19 $\int \frac{\tan^7(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 1190

$$\int \frac{\tan^7(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \frac{3b\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2c^{5/2}e}$$
$$-\frac{5b(7b^2-12ac)\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{16c^{9/2}e}$$
$$-\frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{b^2-}{\sqrt{2}\sqrt{2a-2c-\sqrt{a}}}\right.}{\sqrt{2}\left(a^2+b^2-2ac+c^2\right)^{3/2}e}$$
$$+\frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{b^2-}{\sqrt{2}\sqrt{2a-2c+\sqrt{a^2}}}\right.}{\sqrt{2}\left(a^2+b^2-2ac+c^2\right)^{3/2}e}$$
$$+\frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$-\frac{2\tan^2(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+\frac{2\tan^4(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$-\frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+\frac{(7b^2-16ac)\tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{3c^2\,(b^2-4ac)\,e}$$
$$-\frac{2b\tan^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{c\,(b^2-4ac)\,e}$$
$$-\frac{(3b^2-8ac-2bc\tan(d+ex))\,\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{c^2\,(b^2-4ac)\,e}$$
$$+\frac{(105b^4-460ab^2c+256a^2c^2-2bc(35b^2-116ac)\tan(d+ex))\,\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{24c^4\,(b^2-4ac)\,e}$$

```
[Out] 3/2*b*arctanh(1/2*(b+2*c*tan(e*x+d))/c^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2
)^(1/2))/c^(5/2)/e-5/16*b*(-12*a*c+7*b^2)*arctanh(1/2*(b+2*c*tan(e*x+d))/c^
(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))/c^(9/2)/e+1/2*arctanh(1/2*(b^2
-(a-c)*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))-b*(2*a-2*c+(a^2-2*a*c+b^2+c^2)^(1/2)
)*tan(e*x+d))*2^(1/2)/(2*a-2*c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-b^2-2*
a*c+c^2-(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)
^2)^(1/2))*(2*a-2*c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)*(a^2-b^2-2*a*c+c^2-(a-
c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-2*a*c+b^2+c^2)^(3/2)/e*2^(1/2)-1/2
*arctanh(1/2*(b^2-(a-c)*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))-b*(2*a-2*c-(a^2-2*a
```

```
*c+b^2+c^2)^(1/2))*tan(e*x+d))*2^(1/2)/(2*a-2*c-(a^2-2*a*c+b^2+c^2)^(1/2))^
(1/2)/(a^2-b^2-2*a*c+c^2+(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*
x+d)+c*tan(e*x+d)^2)^(1/2))*(2*a-2*c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)*(a^2-
b^2-2*a*c+c^2+(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-2*a*c+b^2+c^2)^(3
/2)/e*2^(1/2)+1/3*(-16*a*c+7*b^2)*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan
(e*x+d)^2/c^2/(-4*a*c+b^2)/e-2*b*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*tan(
e*x+d)^3/c/(-4*a*c+b^2)/e+2*(2*a+b*tan(e*x+d))/(-4*a*c+b^2)/e/(a+b*tan(e*x+
d)+c*tan(e*x+d)^2)^(1/2)-2*tan(e*x+d)^2*(2*a+b*tan(e*x+d))/(-4*a*c+b^2)/e/(
a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)+2*tan(e*x+d)^4*(2*a+b*tan(e*x+d))/(-4*
a*c+b^2)/e/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)-(a+b*tan(e*x+d)+c*tan(e*x+
d)^2)^(1/2)*(3*b^2-8*a*c-2*b*c*tan(e*x+d))/c^2/(-4*a*c+b^2)/e-2*(a*(b^2-2*(
a-c)*c)+b*c*(a+c)*tan(e*x+d))/(b^2+(a-c)^2)/(-4*a*c+b^2)/e/(a+b*tan(e*x+d)+
c*tan(e*x+d)^2)^(1/2)+1/24*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)*(105*b^4-4
60*a*b^2*c+256*a^2*c^2-2*b*c*(-116*a*c+35*b^2)*tan(e*x+d))/c^4/(-4*a*c+b^2)
/e
```

Rubi [A] (verified)

Time = 7.24 (sec) , antiderivative size = 1190, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules

used = {3781, 6857, 650, 752, 793, 635, 212, 846, 1032, 1050, 1044, 214}

$$\int \frac{\tan^7(d+ex)}{\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}\,dx = \frac{2(2a+b\tan(d+ex))\tan^4(d+ex)}{(b^2-4ac)\,e\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}$$
$$-\frac{2b\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}\tan^3(d+ex)}{c\,(b^2-4ac)\,e}$$
$$+\frac{(7b^2-16ac)\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}\tan^2(d+ex)}{3c^2\,(b^2-4ac)\,e}$$
$$-\frac{2(2a+b\tan(d+ex))\tan^2(d+ex)}{(b^2-4ac)\,e\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}$$
$$-\frac{5b(7b^2-12ac)\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}\right)}{16c^{9/2}e}$$
$$+\frac{3b\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}\right)}{2c^{5/2}e}$$
$$-\frac{\sqrt{2a-2c-\sqrt{a^2-2ca+b^2+c^2}}\sqrt{a^2-2ca-b^2+c^2+(a-c)\sqrt{a^2-2ca+b^2+c^2}}\operatorname{arctanh}\left(\frac{b^2-\cdots}{\sqrt{2}\sqrt{2a-2c-\sqrt{a\cdots}}}\cdots\right)}{\sqrt{2}\,(a^2-2ca+b^2+c^2)^{3/2}\,e}$$
$$+\frac{\sqrt{2a-2c+\sqrt{a^2-2ca+b^2+c^2}}\sqrt{a^2-2ca-b^2+c^2-(a-c)\sqrt{a^2-2ca+b^2+c^2}}\operatorname{arctanh}\left(\frac{b^2-\cdots}{\sqrt{2}\sqrt{2a-2c+\sqrt{a^2\cdots}}}\cdots\right)}{\sqrt{2}\,(a^2-2ca+b^2+c^2)^{3/2}\,e}$$
$$-\frac{(3b^2-2c\tan(d+ex)b-8ac)\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}{c^2\,(b^2-4ac)\,e}$$
$$+\frac{(105b^4-460acb^2-2c(35b^2-116ac)\tan(d+ex)b+256a^2c^2)\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}{24c^4\,(b^2-4ac)\,e}$$
$$+\frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}$$
$$-\frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}$$

```
[In] Int[Tan[d + e*x]^7/(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2),x]
```

```
[Out] (3*b*ArcTanh[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*
Tan[d + e*x]^2])])/(2*c^(5/2)*e) - (5*b*(7*b^2 - 12*a*c)*ArcTanh[(b + 2*c*T
an[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(16*
c^(9/2)*e) - (Sqrt[2*a - 2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a^2 - b^
2 - 2*a*c + c^2 + (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTanh[(b^2 - (a
- c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - b*(2*a - 2*c - Sqrt[a^2 + b^
2 - 2*a*c + c^2])*Tan[d + e*x])/(Sqrt[2]*Sqrt[2*a - 2*c - Sqrt[a^2 + b^2 -
2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c + c^2 + (a - c)*Sqrt[a^2 + b^2 - 2*a*c
 + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2
```

```
 - 2*a*c + c^2)^(3/2)*e) + (Sqrt[2*a - 2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]
*Sqrt[a^2 - b^2 - 2*a*c + c^2 - (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcT
anh[(b^2 - (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - b*(2*a - 2*c +
 Sqrt[a^2 + b^2 - 2*a*c + c^2])*Tan[d + e*x])/(Sqrt[2]*Sqrt[2*a - 2*c + Sqr
t[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c + c^2 - (a - c)*Sqrt[a^2
 + b^2 - 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt
[2]*(a^2 + b^2 - 2*a*c + c^2)^(3/2)*e) + (2*(2*a + b*Tan[d + e*x]))/((b^2 -
 4*a*c)*e*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2]) - (2*Tan[d + e*x]^2*
(2*a + b*Tan[d + e*x]))/((b^2 - 4*a*c)*e*Sqrt[a + b*Tan[d + e*x] + c*Tan[d
+ e*x]^2]) + (2*Tan[d + e*x]^4*(2*a + b*Tan[d + e*x]))/((b^2 - 4*a*c)*e*Sqr
t[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2]) - (2*(a*(b^2 - 2*(a - c)*c) + b*c
*(a + c)*Tan[d + e*x]))/((b^2 + (a - c)^2)*(b^2 - 4*a*c)*e*Sqrt[a + b*Tan[d
 + e*x] + c*Tan[d + e*x]^2]) + ((7*b^2 - 16*a*c)*Tan[d + e*x]^2*Sqrt[a + b*
Tan[d + e*x] + c*Tan[d + e*x]^2])/(3*c^2*(b^2 - 4*a*c)*e) - (2*b*Tan[d + e*
x]^3*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/(c*(b^2 - 4*a*c)*e) - ((3
*b^2 - 8*a*c - 2*b*c*Tan[d + e*x])*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]
^2])/(c^2*(b^2 - 4*a*c)*e) + ((105*b^4 - 460*a*b^2*c + 256*a^2*c^2 - 2*b*c*
(35*b^2 - 116*a*c)*Tan[d + e*x])*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2
])/(24*c^4*(b^2 - 4*a*c)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
```

```
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^7}{(1+x^2)(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}\\ &= \frac{\text{Subst}\left(\int \left(\frac{x}{(a+bx+cx^2)^{3/2}} - \frac{x^3}{(a+bx+cx^2)^{3/2}} + \frac{x^5}{(a+bx+cx^2)^{3/2}} - \frac{x}{(1+x^2)(a+bx+cx^2)^{3/2}}\right)\,dx, x, \tan(d+ex)\right)}{e}\end{aligned}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e} - \frac{\text{Subst}\left(\int \frac{x^3}{(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e} + \frac{\text{Subst}\left(\int \frac{x^5}{(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e} - \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} - \frac{2\tan^2(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} + \frac{2\tan^4(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} - \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} + \frac{2\text{Subst}\left(\int \frac{x(4a+2bx)}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{(b^2-4ac)\,e} - \frac{2\text{Subst}\left(\int \frac{x^3(8a+4bx)}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{(b^2-4ac)\,e} + \frac{2\text{Subst}\left(\int \frac{-\frac{1}{2}b(b^2-4ac)-\frac{1}{2}(a-c)(b^2-4ac)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{(b^2+(a-c)^2)\,(b^2-4ac)\,e}$$

$$
\begin{aligned}
&= \frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} \\
&\quad - \frac{2\tan^2(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} \\
&\quad + \frac{2\tan^4(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} \\
&\quad - \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} \\
&\quad - \frac{2b\tan^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{c\,(b^2-4ac)\,e} \\
&\quad - \frac{(3b^2-8ac-2bc\tan(d+ex))\,\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{c^2\,(b^2-4ac)\,e} \\
&\quad + \frac{(3b)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2c^2e} \\
&\quad - \frac{\mathrm{Subst}\left(\int \frac{x^2(-12ab-2(7b^2-16ac)x)}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2c\,(b^2-4ac)\,e} \\
&\quad - \frac{\mathrm{Subst}\left(\int \frac{\frac{1}{2}b(b^2-4ac)\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)+\frac{1}{2}(b^2-4ac)\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+e\right.}{(b^2-4ac)\,(a^2+b^2-2ac+c^2)^{3/2}\,e} \\
&\quad + \frac{\mathrm{Subst}\left(\int \frac{\frac{1}{2}b(b^2-4ac)\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)+\frac{1}{2}(b^2-4ac)\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+e\right.}{(b^2-4ac)\,(a^2+b^2-2ac+c^2)^{3/2}\,e}
\end{aligned}
$$

$$= \frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$- \frac{2\tan^2(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{2\tan^4(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$- \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{(7b^2-16ac)\tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{3c^2\,(b^2-4ac)\,e}$$
$$- \frac{2b\tan^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{c\,(b^2-4ac)\,e}$$
$$- \frac{(3b^2-8ac-2bc\tan(d+ex))\,\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{c^2\,(b^2-4ac)\,e}$$
$$+ \frac{(3b)\mathrm{Subst}\left(\int\frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{c^2e}$$
$$- \frac{\mathrm{Subst}\left(\int\frac{x(4a(7b^2-16ac)+b(35b^2-116ac)x)}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{6c^2\,(b^2-4ac)\,e}$$
$$+ \frac{\left(b(b^2-4ac)\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\mathrm{Sub}}{}$$
$$- \frac{\left(b(b^2-4ac)\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\mathrm{Sub}}{}$$

$$= \frac{3b\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2c^{5/2}e}$$

$$-\frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\text{arctanh}\Big(}{\sqrt{2}\left(a^2+b^2-2ac+c\right.}$$

$$+\frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-\right.}}}{\sqrt{2}\left(a^2+b\right.}$$

$$+\frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$-\frac{2\tan^2(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$+\frac{2\tan^4(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$-\frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$+\frac{(7b^2-16ac)\tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{3c^2\,(b^2-4ac)\,e}$$

$$-\frac{2b\tan^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{c\,(b^2-4ac)\,e}$$

$$-\frac{(3b^2-8ac-2bc\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{c^2\,(b^2-4ac)\,e}$$

$$+\frac{(105b^4-460ab^2c+256a^2c^2-2bc(35b^2-116ac)\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+}}{24c^4\,(b^2-4ac)\,e}$$

$$-\frac{(5b(7b^2-12ac))\,\text{Subst}\left(\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{16c^4e}$$

$$= \frac{3b\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2c^{5/2}e}$$

$$- \frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{}{\sqrt{2}}\right.}{\sqrt{2}\left(a^2+b^2-2ac+c^2\right)}$$

$$+ \frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-\right.}}}{\sqrt{2}\left(a^2+b^2\right.}$$

$$+ \frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$- \frac{2\tan^2(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$+ \frac{2\tan^4(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$- \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$+ \frac{(7b^2-16ac)\tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{3c^2\,(b^2-4ac)\,e}$$

$$- \frac{2b\tan^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{c\,(b^2-4ac)\,e}$$

$$- \frac{(3b^2-8ac-2bc\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{c^2\,(b^2-4ac)\,e}$$

$$+ \frac{(105b^4-460ab^2c+256a^2c^2-2bc(35b^2-116ac)\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+e}}{24c^4\,(b^2-4ac)\,e}$$

$$- \frac{(5b(7b^2-12ac))\operatorname{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8c^4e}$$

$$= \frac{3b\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2c^{5/2}e}$$
$$- \frac{5b(7b^2-12ac)\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{16c^{9/2}e}$$
$$- \frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\text{arctanh}\Big(}{\sqrt{2}\,(a^2+b^2-2ac+c}$$
$$+ \frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-\right.}}}{\sqrt{2}\,(a^2+b^2}$$
$$+ \frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$- \frac{2\tan^2(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{2\tan^4(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$- \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{(7b^2-16ac)\tan^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{3c^2\,(b^2-4ac)\,e}$$
$$- \frac{2b\tan^3(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{c\,(b^2-4ac)\,e}$$
$$- \frac{(3b^2-8ac-2bc\tan(d+ex))\,\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{c^2\,(b^2-4ac)\,e}$$
$$+ \frac{(105b^4-460ab^2c+256a^2c^2-2bc(35b^2-116ac)\tan(d+ex))\,\sqrt{a+b\tan(d+ex)+c\tan^2(d+}}{24c^4\,(b^2-4ac)\,e}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 11.26 (sec) , antiderivative size = 2476, normalized size of antiderivative = 2.08

$$\int \frac{\tan^7(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Tan[d + e*x]^7/(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2),x]
```

```
[Out] (((-35*a^2*b^3 - 35*b^5 + 60*a^3*b*c + 130*a*b^3*c - 96*a^2*b*c^2 - 11*b^3*
c^2 + 12*a*b*c^3 + 16*b*c^4)*ArcTanh[(2*Sqrt[c]*Tan[(d + e*x)/2])/(Sqrt[a]*
```

(-1 + Tan[(d + e*x)/2]^2) - Sqrt[a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*(b + 2*c*Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2)])]*(1 + Cos[d + e*x])*Sqrt[(1 + Cos[2*(d + e*x)])/(1 + Cos[d + e*x])^2]*Sqrt[(a + c + (a - c)*Cos[2*(d + e*x)] + b*Sin[2*(d + e*x)])/(1 + Cos[2*(d + e*x)])]*(-1 + Tan[(d + e*x)/2]^2)*(1 + Tan[(d + e*x)/2]^2)*Sqrt[(a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*(b + 2*c*Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2))/(1 + Tan[(d + e*x)/2]^2)^2])/(Sqrt[c]*Sqrt[a + c + (a - c)*Cos[2*(d + e*x)] + b*Sin[2*(d + e*x)]]*Sqrt[(-1 + Tan[(d + e*x)/2]^2)^2]*Sqrt[a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*(b + 2*c*Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2)]) + ((-8*a*c^4 + 8*c^5)*(1 + Cos[d + e*x])*Sqrt[(1 + Cos[2*(d + e*x)])/(1 + Cos[d + e*x])^2]*RootSum[a^2 + b^2 + 4*b*Sqrt[c]*#1 - 2*a*#1^2 + 4*c*#1^2 + #1^4 & , (-(a*Log[-1 + Tan[(d + e*x)/2]^2]) + a*Log[#1 - 2*Sqrt[c]*Tan[(d + e*x)/2] - #1*Tan[(d + e*x)/2]^2 + Sqrt[a + 2*b*Tan[(d + e*x)/2] + (-2*a + 4*c)*Tan[(d + e*x)/2]^2 - 2*b*Tan[(d + e*x)/2]^3 + a*Tan[(d + e*x)/2]^4]] + Log[-1 + Tan[(d + e*x)/2]^2]*#1^2 - Log[#1 - 2*Sqrt[c]*Tan[(d + e*x)/2] - #1*Tan[(d + e*x)/2]^2 + Sqrt[a + 2*b*Tan[(d + e*x)/2] + (-2*a + 4*c)*Tan[(d + e*x)/2]^2 - 2*b*Tan[(d + e*x)/2]^3 + a*Tan[(d + e*x)/2]^4]]*#1^2)/(-(b*Sqrt[c]) + a*#1 - 2*c*#1 - #1^3) &]*Sqrt[(a + c + (a - c)*Cos[2*(d + e*x)] + b*Sin[2*(d + e*x)])/(1 + Cos[2*(d + e*x)])]*(-1 + Tan[(d + e*x)/2]^2)*(1 + Tan[(d + e*x)/2]^2)*Sqrt[(a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*(b + 2*c*Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2))/(1 + Tan[(d + e*x)/2]^2)^2])/(2*Sqrt[a + c + (a - c)*Cos[2*(d + e*x)] + b*Sin[2*(d + e*x)]]*Sqrt[(-1 + Tan[(d + e*x)/2]^2)^2]*Sqrt[a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*(b + 2*c*Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2)]) - (4*b*c^(7/2)*(1 + Cos[d + e*x])*Sqrt[(1 + Cos[2*(d + e*x)])/(1 + Cos[d + e*x])^2]*(Log[-1 + Tan[(d + e*x)/2]^2] - Log[-4*c*Tan[(d + e*x)/2] + b*(-1 + Tan[(d + e*x)/2]^2) + 2*Sqrt[c]*Sqrt[a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*(b + 2*c*Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2)]] + Sqrt[c]*RootSum[a^2 + b^2 + 4*b*Sqrt[c]*#1 - 2*a*#1^2 + 4*c*#1^2 + #1^4 & , (-(b*Log[-1 + Tan[(d + e*x)/2]^2]) + b*Log[#1 - 2*Sqrt[c]*Tan[(d + e*x)/2] - #1*Tan[(d + e*x)/2]^2 + Sqrt[a + 2*b*Tan[(d + e*x)/2] + (-2*a + 4*c)*Tan[(d + e*x)/2]^2 - 2*b*Tan[(d + e*x)/2]^3 + a*Tan[(d + e*x)/2]^4]] - 2*Sqrt[c]*Log[-1 + Tan[(d + e*x)/2]^2]*#1 + 2*Sqrt[c]*Log[#1 - 2*Sqrt[c]*Tan[(d + e*x)/2] - #1*Tan[(d + e*x)/2]^2 + Sqrt[a + 2*b*Tan[(d + e*x)/2] + (-2*a + 4*c)*Tan[(d + e*x)/2]^2 - 2*b*Tan[(d + e*x)/2]^3 + a*Tan[(d + e*x)/2]^4]]*#1)/(b*Sqrt[c] - a*#1 + 2*c*#1 + #1^3) &])*Sqrt[(a + c + (a - c)*Cos[2*(d + e*x)] + b*Sin[2*(d + e*x)])/(1 + Cos[2*(d + e*x)])]*(-1 + Tan[(d + e*x)/2]^2)*(1 + Tan[(d + e*x)/2]^2)*Sqrt[(a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*(b + 2*c*Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2))/(1 + Tan[(d + e*x)/2]^2)^2])/(Sqrt[a + c + (a - c)*Cos[2*(d + e*x)] + b*Sin[2*(d + e*x)]]*Sqrt[(-1 + Tan[(d + e*x)/2]^2)^2]*Sqrt[a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*(b + 2*c*Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2)]))/(8*(I*a + b - I*c)*((-I)*a + b + I*c)*c^4*e) + (Sqrt[(a + c + a*Cos[2*(d + e*x)] - c*Cos[2*(d + e*x)] + b*Sin[2*(d + e*x)])/(1 + Cos[2*(d + e*x)])]*(-1/24*(105*a^3*b^4 + 105*a*b^6 - 460*a^4*b^2*c - 727*a^2*b^4*c - 57*b^6*c + 256*a^5*

```
c^2 + 1364*a^3*b^2*c^2 + 407*a*b^4*c^2 - 448*a^4*c^3 - 740*a^2*b^2*c^3 - 25
*b^4*c^3 + 96*a^3*c^4 + 44*a*b^2*c^4 + 224*a^2*c^5 + 32*b^2*c^5 - 128*a*c^6
)/((a - c)*(a - I*b - c)*(a + I*b - c)*c^4*(-b^2 + 4*a*c)) + Sec[d + e*x]^2
/(3*c^2) + (2*(2*a^3*b^4 + 2*a*b^6 - 8*a^4*b^2*c - 12*a^2*b^4*c + 4*a^5*c^2
 + 18*a^3*b^2*c^2 - 4*a^4*c^3 + a^4*b^3*Sin[2*(d + e*x)] + 2*a^2*b^5*Sin[2*
(d + e*x)] + b^7*Sin[2*(d + e*x)] - 3*a^5*b*c*Sin[2*(d + e*x)] - 10*a^3*b^3
*c*Sin[2*(d + e*x)] - 7*a*b^5*c*Sin[2*(d + e*x)] + 10*a^4*b*c^2*Sin[2*(d +
e*x)] + 14*a^2*b^3*c^2*Sin[2*(d + e*x)] - 7*a^3*b*c^3*Sin[2*(d + e*x)]))/((
a - c)*(a - I*b - c)*(a + I*b - c)*c^3*(-b^2 + 4*a*c)*(a + c + a*Cos[2*(d +
 e*x)] - c*Cos[2*(d + e*x)] + b*Sin[2*(d + e*x)])) - (11*b*Tan[d + e*x])/(1
2*c^3)))/e
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.11 (sec) , antiderivative size = 13068421, normalized size of antiderivative = 10981.87

output too large to display

```
[In] int(tan(e*x+d)^7/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20284 vs. 2(1096) = 2192.

Time = 9.01 (sec) , antiderivative size = 40569, normalized size of antiderivative = 34.09

$$\int \frac{\tan^7(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(e*x+d)^7/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tan^7(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \int \frac{\tan^7(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(e*x+d)**7/(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(3/2),x)
[Out] Integral(tan(d + e*x)**7/(a + b*tan(d + e*x) + c*tan(d + e*x)**2)**(3/2), x
)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^7(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^7/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
maxima")
[Out] Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^7(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^7/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^7(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Hanged}$$

```
[In] int(tan(d + e*x)^7/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(3/2),x)
[Out] \text{Hanged}
```

3.20 $\int \frac{\tan^5(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 864

$$\int \frac{\tan^5(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = -\frac{3b\,\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2c^{5/2}e}$$

$$+\frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\,\text{arctanh}\left(\frac{b^2 \cdots}{\sqrt{2}\sqrt{2a-2c-\sqrt{\cdots}}\cdots}\right)}{\sqrt{2}\,(a^2+b^2-2ac+c^2)^{3/2}\,e}$$

$$-\frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}}\,\text{arctanh}\left(\frac{b \cdots}{\sqrt{2}\sqrt{2a-2c+\sqrt{\cdots}}\cdots}\right)}{\sqrt{2}\,(a^2+b^2-2ac+c^2)^{3/2}\,e}$$

$$-\frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$+\frac{2\tan^2(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$+\frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$+\frac{(3b^2-8ac-2bc\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{c^2\,(b^2-4ac)\,e}$$

```
[Out] -3/2*b*arctanh(1/2*(b+2*c*tan(e*x+d))/c^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^
2)^(1/2))/c^(5/2)/e-1/2*arctanh(1/2*(b^2-(a-c)*(a-c-(a^2-2*a*c+b^2+c^2)^(1/
2))-b*(2*a-2*c+(a^2-2*a*c+b^2+c^2)^(1/2))*tan(e*x+d))*2^(1/2)/(2*a-2*c+(a^2
-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-b^2-2*a*c+c^2-(a-c)*(a^2-2*a*c+b^2+c^2)^(
1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(2*a-2*c+(a^2-2*a*c+b^2+
c^2)^(1/2))^(1/2)*(a^2-b^2-2*a*c+c^2-(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)
```

```
/(a^2-2*a*c+b^2+c^2)^(3/2)/e*2^(1/2)+1/2*arctanh(1/2*(b^2-(a-c)*(a-c+(a^2-2
*a*c+b^2+c^2)^(1/2))-b*(2*a-2*c-(a^2-2*a*c+b^2+c^2)^(1/2))*tan(e*x+d))*2^(1
/2)/(2*a-2*c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-b^2-2*a*c+c^2+(a-c)*(a^2
-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(2*a-2*
c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)*(a^2-b^2-2*a*c+c^2+(a-c)*(a^2-2*a*c+b^2+
c^2)^(1/2))^(1/2)/(a^2-2*a*c+b^2+c^2)^(3/2)/e*2^(1/2)-2*(2*a+b*tan(e*x+d))/
(-4*a*c+b^2)/e/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)+2*tan(e*x+d)^2*(2*a+b*
tan(e*x+d))/(-4*a*c+b^2)/e/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)+(a+b*tan(e
*x+d)+c*tan(e*x+d)^2)^(1/2)*(3*b^2-8*a*c-2*b*c*tan(e*x+d))/c^2/(-4*a*c+b^2)
/e+2*(a*(b^2-2*(a-c)*c)+b*c*(a+c)*tan(e*x+d))/(b^2+(a-c)^2)/(-4*a*c+b^2)/e/
(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)
```

Rubi [A] (verified)

Time = 5.38 (sec) , antiderivative size = 864, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3781, 6857, 650, 752, 793, 635, 212, 1032, 1050, 1044, 214}

$$\int \frac{\tan^5(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx=\frac{2(2a+b\tan(d+ex))\tan^2(d+ex)}{(b^2-4ac)\,e\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}$$
$$-\frac{3b\text{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}\right)}{2c^{5/2}e}$$
$$+\frac{\sqrt{2a-2c-\sqrt{a^2-2ca+b^2+c^2}}\sqrt{a^2-2ca-b^2+c^2+(a-c)\sqrt{a^2-2ca+b^2+c^2}}\text{arctanh}\left(\frac{b^2-\cdots}{\sqrt{2}\sqrt{2a-2c-\sqrt{a^2\cdots}}}\right.}{\sqrt{2}\,(a^2-2ca+b^2+c^2)^{3/2}\,e}$$
$$-\frac{\sqrt{2a-2c+\sqrt{a^2-2ca+b^2+c^2}}\sqrt{a^2-2ca-b^2+c^2-(a-c)\sqrt{a^2-2ca+b^2+c^2}}\text{arctanh}\left(\frac{b^2\cdots}{\sqrt{2}\sqrt{2a-2c+\sqrt{a\cdots}}}\right.}{\sqrt{2}\,(a^2-2ca+b^2+c^2)^{3/2}\,e}$$
$$+\frac{(3b^2-2c\tan(d+ex)b-8ac)\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}{c^2\,(b^2-4ac)\,e}$$
$$-\frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}$$
$$+\frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}$$

```
[In] Int[Tan[d + e*x]^5/(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2),x]
```

```
[Out] (-3*b*ArcTanh[(b + 2*c*Tan[d + e*x])/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x] + c
*Tan[d + e*x]^2])])/(2*c^(5/2)*e) + (Sqrt[2*a - 2*c - Sqrt[a^2 + b^2 - 2*a*
c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c + c^2 + (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c
^2]]*ArcTanh[(b^2 - (a - c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - b*(2*
a - 2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])*Tan[d + e*x])/(Sqrt[2]*Sqrt[2*a -
```

```
2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c + c^2 + (a - c)
*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2]
)])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(3/2)*e) - (Sqrt[2*a - 2*c + Sqrt[a^
2 + b^2 - 2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c + c^2 - (a - c)*Sqrt[a^2 + b
^2 - 2*a*c + c^2]]*ArcTanh[(b^2 - (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a*c +
 c^2]) - b*(2*a - 2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])*Tan[d + e*x])/(Sqrt[
2]*Sqrt[2*a - 2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c +
 c^2 - (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*T
an[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(3/2)*e) - (2*(2*a + b
*Tan[d + e*x]))/((b^2 - 4*a*c)*e*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2
]) + (2*Tan[d + e*x]^2*(2*a + b*Tan[d + e*x]))/((b^2 - 4*a*c)*e*Sqrt[a + b*
Tan[d + e*x] + c*Tan[d + e*x]^2]) + (2*(a*(b^2 - 2*(a - c)*c) + b*c*(a + c)
*Tan[d + e*x]))/((b^2 + (a - c)^2)*(b^2 - 4*a*c)*e*Sqrt[a + b*Tan[d + e*x]
+ c*Tan[d + e*x]^2]) + ((3*b^2 - 8*a*c - 2*b*c*Tan[d + e*x])*Sqrt[a + b*Tan
[d + e*x] + c*Tan[d + e*x]^2])/(c^2*(b^2 - 4*a*c)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
```

```
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
```

```
NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{x}{(a+bx+cx^2)^{3/2}}+\frac{x^3}{(a+bx+cx^2)^{3/2}}+\frac{x}{(1+x^2)(a+bx+cx^2)^{3/2}}\right)\,dx, x, \tan(d+ex)\right)}{e}$$

$$= -\frac{\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e} + \frac{\text{Subst}\left(\int \frac{x^3}{(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e} + \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= -\frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} + \frac{2\tan^2(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} + \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} - \frac{2\text{Subst}\left(\int \frac{x(4a+2bx)}{\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{(b^2-4ac)\,e} - \frac{2\text{Subst}\left(\int \frac{-\frac{1}{2}b(b^2-4ac)-\frac{1}{2}(a-c)(b^2-4ac)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{(b^2+(a-c)^2)\,(b^2-4ac)\,e}$$

$$= -\frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{2\tan^2(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{(3b^2-8ac-2bc\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{c^2\,(b^2-4ac)\,e}$$
$$- \frac{(3b)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{2c^2e}$$
$$+ \frac{\text{Subst}\left(\int\frac{\frac{1}{2}b(b^2-4ac)\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)+\frac{1}{2}(b^2-4ac)\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex\right.}{(b^2-4ac)\left(a^2+b^2-2ac+c^2\right)^{3/2}e}$$
$$- \frac{\text{Subst}\left(\int\frac{\frac{1}{2}b(b^2-4ac)\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)+\frac{1}{2}(b^2-4ac)\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex\right.}{(b^2-4ac)\left(a^2+b^2-2ac+c^2\right)^{3/2}e}$$

$$= -\frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{2\tan^2(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{(3b^2-8ac-2bc\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{c^2\,(b^2-4ac)\,e}$$
$$- \frac{(3b)\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2c\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{c^2e}$$
$$- \frac{\left(b(b^2-4ac)\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Sub}}{}$$
$$+ \frac{\left(b(b^2-4ac)\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Sub}}{}$$

$$= -\frac{3b\operatorname{arctanh}\left(\frac{b+2c\tan(d+ex)}{2\sqrt{c}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2c^{5/2}e}$$
$$+ \frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\Big(-}{\sqrt{2}\left(a^2+b^2-2ac+c\right.}$$
$$- \frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-\right.}}}{\sqrt{2}\left(a^2+b^2\right.}$$
$$- \frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{2\tan^2(d+ex)(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{(3b^2-8ac-2bc\tan(d+ex))\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{c^2\,(b^2-4ac)\,e}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 7.70 (sec) , antiderivative size = 2272, normalized size of antiderivative = 2.63

$$\int \frac{\tan^5(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Tan[d + e*x]^5/(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[(a + c + a*Cos[2*(d + e*x)] - c*Cos[2*(d + e*x)] + b*Sin[2*(d + e*x)]
)/(1 + Cos[2*(d + e*x)])]*((-3*a^3*b^2 - 3*a*b^4 + 8*a^4*c + 15*a^2*b^2*c +
 b^4*c - 16*a^3*c^2 - 7*a*b^2*c^2 + 12*a^2*c^3 + b^2*c^3 - 4*a*c^4)/((a - c
)*(a - I*b - c)*(a + I*b - c)*c^2*(-b^2 + 4*a*c)) - (2*(-2*a^3*b^2 - 2*a*b^
4 + 4*a^4*c + 8*a^2*b^2*c - 4*a^3*c^2 - a^4*b*Sin[2*(d + e*x)] - 2*a^2*b^3*
Sin[2*(d + e*x)] - b^5*Sin[2*(d + e*x)] + 6*a^3*b*c*Sin[2*(d + e*x)] + 5*a*
b^3*c*Sin[2*(d + e*x)] - 5*a^2*b*c^2*Sin[2*(d + e*x)]))/((a - c)*(a - I*b -
 c)*(a + I*b - c)*c*(-b^2 + 4*a*c)*(a + c + a*Cos[2*(d + e*x)] - c*Cos[2*(d
 + e*x)] + b*Sin[2*(d + e*x)]))))/e - (((3*a^2*b + 3*b^3 - 6*a*b*c + 2*b*c^
2)*ArcTanh[(2*Sqrt[c]*Tan[(d + e*x)/2])/(Sqrt[a]*(-1 + Tan[(d + e*x)/2]^2)
- Sqrt[a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*(b + 2*c*Tan[(d +
 e*x)/2] - b*Tan[(d + e*x)/2]^2)])]*(1 + Cos[d + e*x])*Sqrt[(1 + Cos[2*(d +
 e*x)])/(1 + Cos[d + e*x])^2]*Sqrt[(a + c + (a - c)*Cos[2*(d + e*x)] + b*Si
n[2*(d + e*x)])/(1 + Cos[2*(d + e*x)])]*(-1 + Tan[(d + e*x)/2]^2)*(1 + Tan[
```

```
(d + e*x)/2]^2)*Sqrt[(a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*(b
 + 2*c*Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2))/(1 + Tan[(d + e*x)/2]^2)^2
])/(Sqrt[c]*Sqrt[a + c + (a - c)*Cos[2*(d + e*x)] + b*Sin[2*(d + e*x)]]*Sqr
t[(-1 + Tan[(d + e*x)/2]^2)^2]*Sqrt[a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(
d + e*x)/2]*(b + 2*c*Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2)]) + ((-(a*c^2
) + c^3)*(1 + Cos[d + e*x])*Sqrt[(1 + Cos[2*(d + e*x)])/(1 + Cos[d + e*x])^
2]*RootSum[a^2 + b^2 + 4*b*Sqrt[c]*#1 - 2*a*#1^2 + 4*c*#1^2 + #1^4 & , (-(a
*Log[-1 + Tan[(d + e*x)/2]^2]) + a*Log[#1 - 2*Sqrt[c]*Tan[(d + e*x)/2] - #1
*Tan[(d + e*x)/2]^2 + Sqrt[a + 2*b*Tan[(d + e*x)/2] + (-2*a + 4*c)*Tan[(d +
 e*x)/2]^2 - 2*b*Tan[(d + e*x)/2]^3 + a*Tan[(d + e*x)/2]^4]] + Log[-1 + Tan
[(d + e*x)/2]^2]*#1^2 - Log[#1 - 2*Sqrt[c]*Tan[(d + e*x)/2] - #1*Tan[(d + e
*x)/2]^2 + Sqrt[a + 2*b*Tan[(d + e*x)/2] + (-2*a + 4*c)*Tan[(d + e*x)/2]^2
- 2*b*Tan[(d + e*x)/2]^3 + a*Tan[(d + e*x)/2]^4]]*#1^2)/(-(b*Sqrt[c]) + a*#
1 - 2*c*#1 - #1^3) & ]*Sqrt[(a + c + (a - c)*Cos[2*(d + e*x)] + b*Sin[2*(d
+ e*x)])/(1 + Cos[2*(d + e*x)])]*(-1 + Tan[(d + e*x)/2]^2)*(1 + Tan[(d + e*
x)/2]^2)*Sqrt[(a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*(b + 2*c*
Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2))/(1 + Tan[(d + e*x)/2]^2)^2])/(2*S
qrt[a + c + (a - c)*Cos[2*(d + e*x)] + b*Sin[2*(d + e*x)]]*Sqrt[(-1 + Tan[(
d + e*x)/2]^2)^2]*Sqrt[a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*(
b + 2*c*Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2)]) - (b*c^(3/2)*(1 + Cos[d
+ e*x])*Sqrt[(1 + Cos[2*(d + e*x)])/(1 + Cos[d + e*x])^2]*(Log[-1 + Tan[(d
+ e*x)/2]^2] - Log[-4*c*Tan[(d + e*x)/2] + b*(-1 + Tan[(d + e*x)/2]^2) + 2*
Sqrt[c]*Sqrt[a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*(b + 2*c*Ta
n[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2)]] + Sqrt[c]*RootSum[a^2 + b^2 + 4*b*
Sqrt[c]*#1 - 2*a*#1^2 + 4*c*#1^2 + #1^4 & , (-(b*Log[-1 + Tan[(d + e*x)/2]^
2]) + b*Log[#1 - 2*Sqrt[c]*Tan[(d + e*x)/2] - #1*Tan[(d + e*x)/2]^2 + Sqrt[
a + 2*b*Tan[(d + e*x)/2] + (-2*a + 4*c)*Tan[(d + e*x)/2]^2 - 2*b*Tan[(d + e
*x)/2]^3 + a*Tan[(d + e*x)/2]^4]] - 2*Sqrt[c]*Log[-1 + Tan[(d + e*x)/2]^2]*
#1 + 2*Sqrt[c]*Log[#1 - 2*Sqrt[c]*Tan[(d + e*x)/2] - #1*Tan[(d + e*x)/2]^2
+ Sqrt[a + 2*b*Tan[(d + e*x)/2] + (-2*a + 4*c)*Tan[(d + e*x)/2]^2 - 2*b*Tan
[(d + e*x)/2]^3 + a*Tan[(d + e*x)/2]^4]]*#1)/(b*Sqrt[c] - a*#1 + 2*c*#1 + #
1^3) & ])*Sqrt[(a + c + (a - c)*Cos[2*(d + e*x)] + b*Sin[2*(d + e*x)])/(1 +
 Cos[2*(d + e*x)])]*(-1 + Tan[(d + e*x)/2]^2)*(1 + Tan[(d + e*x)/2]^2)*Sqrt
[(a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*(b + 2*c*Tan[(d + e*x)
/2] - b*Tan[(d + e*x)/2]^2))/(1 + Tan[(d + e*x)/2]^2)^2])/(2*Sqrt[a + c + (
a - c)*Cos[2*(d + e*x)] + b*Sin[2*(d + e*x)]]*Sqrt[(-1 + Tan[(d + e*x)/2]^2
)^2]*Sqrt[a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*(b + 2*c*Tan[(
d + e*x)/2] - b*Tan[(d + e*x)/2]^2)]))/((I*a + b - I*c)*((-I)*a + b + I*c)*
c^2*e)
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.37 (sec) , antiderivative size = 13066870, normalized size of antiderivative = 15123.69

output too large to display

```
[In] int(tan(e*x+d)^5/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x)
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19865 vs. 2(794) = 1588.

Time = 4.70 (sec) , antiderivative size = 39731, normalized size of antiderivative = 45.98

$$\int \frac{\tan^5(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(e*x+d)^5/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tan^5(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \int \frac{\tan^5(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(e*x+d)**5/(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(3/2),x)
[Out] Integral(tan(d + e*x)**5/(a + b*tan(d + e*x) + c*tan(d + e*x)**2)**(3/2), x
)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^5(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^5/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
maxima")
[Out] Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^5(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^5/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \int \frac{\tan(d+e\,x)^5}{\left(c\tan\left(d+e\,x\right)^2+b\tan\left(d+e\,x\right)+a\right)^{3/2}}\,dx$$

```
[In] int(tan(d + e*x)^5/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(3/2),x)
```

```
[Out] int(tan(d + e*x)^5/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(3/2), x)
```

3.21 $\int \frac{\tan^3(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}} \, dx$

Optimal result

Integrand size = 33, antiderivative size = 686

$$\int \frac{\tan^3(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}} \, dx =$$

$$-\frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\,\text{arctanh}\left(\frac{b}{\sqrt{2}\sqrt{2a-2c-\sqrt{}}}\right)}{\sqrt{2}\,(a^2+b^2-2ac+c^2)^{3/2}\,e}$$

$$+\frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}}\,\text{arctanh}\left(\frac{b^2}{\sqrt{2}\sqrt{2a-2c+\sqrt{}}}\right)}{\sqrt{2}\,(a^2+b^2-2ac+c^2)^{3/2}\,e}$$

$$+\frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$-\frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

```
[Out] 1/2*arctanh(1/2*(b^2-(a-c)*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))-b*(2*a-2*c+(a^2-
2*a*c+b^2+c^2)^(1/2))*tan(e*x+d))*2^(1/2)/(2*a-2*c+(a^2-2*a*c+b^2+c^2)^(1/2
))^(1/2)/(a^2-b^2-2*a*c+c^2-(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan
(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(2*a-2*c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)*(a
^2-b^2-2*a*c+c^2-(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-2*a*c+b^2+c^2)
^(3/2)/e*2^(1/2)-1/2*arctanh(1/2*(b^2-(a-c)*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))
-b*(2*a-2*c-(a^2-2*a*c+b^2+c^2)^(1/2))*tan(e*x+d))*2^(1/2)/(2*a-2*c-(a^2-2*
a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-b^2-2*a*c+c^2+(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2
))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(2*a-2*c-(a^2-2*a*c+b^2+c^2
)^(1/2))^(1/2)*(a^2-b^2-2*a*c+c^2+(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a
^2-2*a*c+b^2+c^2)^(3/2)/e*2^(1/2)+2*(2*a+b*tan(e*x+d))/(-4*a*c+b^2)/e/(a+b*
```

```
tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)-2*(a*(b^2-2*(a-c)*c)+b*c*(a+c)*tan(e*x+d))
/(b^2+(a-c)^2)/(-4*a*c+b^2)/e/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)
```

Rubi [A] (verified)

Time = 5.06 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3781, 6857, 650, 1032, 1050, 1044, 214}

$$\int \frac{\tan^3(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx =$$
$$-\frac{\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+2a-2c}\sqrt{(a-c)\sqrt{a^2-2ac+b^2+c^2}+a^2-2ac-b^2+c^2}\,\text{arctanh}\Bigg(\frac{-}{\sqrt{2}\sqrt{-\sqrt{a^2-2}}}}{\sqrt{2}e\,(a^2-2ac+b^2+c^2)^{3/2}}$$
$$+\frac{\sqrt{\sqrt{a^2-2ac+b^2+c^2}+2a-2c}\sqrt{-(a-c)\sqrt{a^2-2ac+b^2+c^2}+a^2-2ac-b^2+c^2}\,\text{arctanh}\Bigg(\frac{-}{\sqrt{2}\sqrt{\sqrt{a^2-2ac}}}}{\sqrt{2}e\,(a^2-2ac+b^2+c^2)^{3/2}}$$
$$+\frac{2(2a+b\tan(d+ex))}{e\,(b^2-4ac)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$-\frac{2(a(b^2-2c(a-c))+bc(a+c)\tan(d+ex))}{e\,((a-c)^2+b^2)\,(b^2-4ac)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

```
[In] Int[Tan[d + e*x]^3/(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2),x]
```

```
[Out] -((Sqrt[2*a - 2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c +
 c^2 + (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTanh[(b^2 - (a - c)*(a - c
 + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - b*(2*a - 2*c - Sqrt[a^2 + b^2 - 2*a*c +
 c^2])*Tan[d + e*x])/(Sqrt[2]*Sqrt[2*a - 2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2
]]*Sqrt[a^2 - b^2 - 2*a*c + c^2 + (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sq
rt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c +
c^2)^(3/2)*e)) + (Sqrt[2*a - 2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a^2
- b^2 - 2*a*c + c^2 - (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTanh[(b^2 -
 (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - b*(2*a - 2*c + Sqrt[a^2
+ b^2 - 2*a*c + c^2])*Tan[d + e*x])/(Sqrt[2]*Sqrt[2*a - 2*c + Sqrt[a^2 + b^
2 - 2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c + c^2 - (a - c)*Sqrt[a^2 + b^2 - 2
*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 +
 b^2 - 2*a*c + c^2)^(3/2)*e) + (2*(2*a + b*Tan[d + e*x]))/((b^2 - 4*a*c)*e*
Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2]) - (2*(a*(b^2 - 2*(a - c)*c) +
b*c*(a + c)*Tan[d + e*x]))/((b^2 + (a - c)^2)*(b^2 - 4*a*c)*e*Sqrt[a + b*Ta
n[d + e*x] + c*Tan[d + e*x]^2])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{x}{(a+bx+cx^2)^{3/2}} - \frac{x}{(1+x^2)(a+bx+cx^2)^{3/2}}\right)dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e} - \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} - \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} + \frac{2\text{Subst}\left(\int \frac{-\frac{1}{2}b(b^2-4ac)-\frac{1}{2}(a-c)(b^2-4ac)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{(b^2+(a-c)^2)(b^2-4ac)\,e}$$

$$= \frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} - \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}b(b^2-4ac)\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)+\frac{1}{2}(b^2-4ac)\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex\right.}{(b^2-4ac)(a^2+b^2-2ac+c^2)^{3/2}\,e} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}b(b^2-4ac)\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)+\frac{1}{2}(b^2-4ac)\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex\right.}{(b^2-4ac)(a^2+b^2-2ac+c^2)^{3/2}\,e}$$

$$= \frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$- \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{\left(b(b^2-4ac)\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\,\mathrm{Su}}{}$$
$$- \frac{\left(b(b^2-4ac)\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\,\mathrm{Su}}{}$$

$$= -\frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\,\mathrm{arctanh}\Big(}{\sqrt{2}\,(a^2+b^2-2ac+c}$$
$$+ \frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-}\right.}}{\sqrt{2}\,(a^2+b}$$
$$+ \frac{2(2a+b\tan(d+ex))}{(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$- \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 7.33 (sec) , antiderivative size = 2339, normalized size of antiderivative = 3.41

$$\int \frac{\tan^3(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Tan[d + e*x]^3/(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[(a + c + a*Cos[2*(d + e*x)] - c*Cos[2*(d + e*x)] + b*Sin[2*(d + e*x)]
)/(1 + Cos[2*(d + e*x)])]*((-2*a*(2*a^2 + b^2 - 2*a*c))/((a - c)*(a - I*b -
 c)*(-(a*b^2) - I*b^3 + 4*a^2*c + (4*I)*a*b*c + b^2*c - 4*a*c^2)) + ((Cos[2
*(d + e*x)] - I*Sin[2*(d + e*x)])*(I*a^3*b + (2*I)*a^2*b*c + I*b^3*c - (3*I
)*a*b*c^2 + 8*a^3*c*Cos[2*(d + e*x)] + 4*a*b^2*c*Cos[2*(d + e*x)] - 8*a^2*c
^2*Cos[2*(d + e*x)] - I*a^3*b*Cos[4*(d + e*x)] - (2*I)*a^2*b*c*Cos[4*(d + e
*x)] - I*b^3*c*Cos[4*(d + e*x)] + (3*I)*a*b*c^2*Cos[4*(d + e*x)] + (8*I)*a^
3*c*Sin[2*(d + e*x)] + (4*I)*a*b^2*c*Sin[2*(d + e*x)] - (8*I)*a^2*c^2*Sin[2
*(d + e*x)] + a^3*b*Sin[4*(d + e*x)] + 2*a^2*b*c*Sin[4*(d + e*x)] + b^3*c*S
```

```
in[4*(d + e*x)] - 3*a*b*c^2*Sin[4*(d + e*x)]))/((a - c)*(a - I*b - c)*(a +
I*b - c)*(-b^2 + 4*a*c)*(a + c + a*Cos[2*(d + e*x)] - c*Cos[2*(d + e*x)] +
b*Sin[2*(d + e*x)]))))/e - ((b*ArcTanh[(2*Sqrt[c]*Tan[(d + e*x)/2])/(Sqrt[a
]*(-1 + Tan[(d + e*x)/2]^2) - Sqrt[a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d
 + e*x)/2]*(b + 2*c*Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2)])]*(1 + Cos[d
+ e*x])*Sqrt[(1 + Cos[2*(d + e*x)])/(1 + Cos[d + e*x])^2]*Sqrt[(a + c + (a
- c)*Cos[2*(d + e*x)] + b*Sin[2*(d + e*x)])/(1 + Cos[2*(d + e*x)])]*(-1 + T
an[(d + e*x)/2]^2)*(1 + Tan[(d + e*x)/2]^2)*Sqrt[(a*(-1 + Tan[(d + e*x)/2]^
2)^2 + 2*Tan[(d + e*x)/2]*(b + 2*c*Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2)
)/(1 + Tan[(d + e*x)/2]^2)^2])/(Sqrt[c]*Sqrt[a + c + (a - c)*Cos[2*(d + e*x
)] + b*Sin[2*(d + e*x)]]*Sqrt[(-1 + Tan[(d + e*x)/2]^2)^2]*Sqrt[a*(-1 + Tan
[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*(b + 2*c*Tan[(d + e*x)/2] - b*Tan[(
d + e*x)/2]^2)]) + ((a - c)*(1 + Cos[d + e*x])*Sqrt[(1 + Cos[2*(d + e*x)])/
(1 + Cos[d + e*x])^2]*RootSum[a^2 + b^2 + 4*b*Sqrt[c]*#1 - 2*a*#1^2 + 4*c*#
1^2 + #1^4 & , (-(a*Log[-1 + Tan[(d + e*x)/2]^2]) + a*Log[#1 - 2*Sqrt[c]*Ta
n[(d + e*x)/2] - #1*Tan[(d + e*x)/2]^2 + Sqrt[a + 2*b*Tan[(d + e*x)/2] + (-
2*a + 4*c)*Tan[(d + e*x)/2]^2 - 2*b*Tan[(d + e*x)/2]^3 + a*Tan[(d + e*x)/2]
^4]] + Log[-1 + Tan[(d + e*x)/2]^2]*#1^2 - Log[#1 - 2*Sqrt[c]*Tan[(d + e*x)
/2] - #1*Tan[(d + e*x)/2]^2 + Sqrt[a + 2*b*Tan[(d + e*x)/2] + (-2*a + 4*c)*
Tan[(d + e*x)/2]^2 - 2*b*Tan[(d + e*x)/2]^3 + a*Tan[(d + e*x)/2]^4]]*#1^2)/
(-(b*Sqrt[c]) + a*#1 - 2*c*#1 - #1^3) & ]*Sqrt[(a + c + (a - c)*Cos[2*(d +
e*x)] + b*Sin[2*(d + e*x)])/(1 + Cos[2*(d + e*x)])]*(-1 + Tan[(d + e*x)/2]^
2)*(1 + Tan[(d + e*x)/2]^2)*Sqrt[(a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d
+ e*x)/2]*(b + 2*c*Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2))/(1 + Tan[(d +
e*x)/2]^2)^2])/(2*Sqrt[a + c + (a - c)*Cos[2*(d + e*x)] + b*Sin[2*(d + e*x)
]]*Sqrt[(-1 + Tan[(d + e*x)/2]^2)^2]*Sqrt[a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2
*Tan[(d + e*x)/2]*(b + 2*c*Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2)]) + (b*
(1 + Cos[d + e*x])*Sqrt[(1 + Cos[2*(d + e*x)])/(1 + Cos[d + e*x])^2]*(Log[-
1 + Tan[(d + e*x)/2]^2] - Log[-4*c*Tan[(d + e*x)/2] + b*(-1 + Tan[(d + e*x)
/2]^2) + 2*Sqrt[c]*Sqrt[a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*
(b + 2*c*Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2)]] + Sqrt[c]*RootSum[a^2 +
 b^2 + 4*b*Sqrt[c]*#1 - 2*a*#1^2 + 4*c*#1^2 + #1^4 & , (-(b*Log[-1 + Tan[(d
 + e*x)/2]^2]) + b*Log[#1 - 2*Sqrt[c]*Tan[(d + e*x)/2] - #1*Tan[(d + e*x)/2
]^2 + Sqrt[a + 2*b*Tan[(d + e*x)/2] + (-2*a + 4*c)*Tan[(d + e*x)/2]^2 - 2*b
*Tan[(d + e*x)/2]^3 + a*Tan[(d + e*x)/2]^4]] - 2*Sqrt[c]*Log[-1 + Tan[(d +
e*x)/2]^2]*#1 + 2*Sqrt[c]*Log[#1 - 2*Sqrt[c]*Tan[(d + e*x)/2] - #1*Tan[(d +
 e*x)/2]^2 + Sqrt[a + 2*b*Tan[(d + e*x)/2] + (-2*a + 4*c)*Tan[(d + e*x)/2]^
2 - 2*b*Tan[(d + e*x)/2]^3 + a*Tan[(d + e*x)/2]^4]]*#1)/(b*Sqrt[c] - a*#1 +
 2*c*#1 + #1^3) & ])*Sqrt[(a + c + (a - c)*Cos[2*(d + e*x)] + b*Sin[2*(d +
e*x)])/(1 + Cos[2*(d + e*x)])]*(-1 + Tan[(d + e*x)/2]^2)*(1 + Tan[(d + e*x)
/2]^2)*Sqrt[(a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x)/2]*(b + 2*c*Ta
n[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2))/(1 + Tan[(d + e*x)/2]^2)^2])/(2*Sqr
t[c]*Sqrt[a + c + (a - c)*Cos[2*(d + e*x)] + b*Sin[2*(d + e*x)]]*Sqrt[(-1 +
 Tan[(d + e*x)/2]^2)^2]*Sqrt[a*(-1 + Tan[(d + e*x)/2]^2)^2 + 2*Tan[(d + e*x
)/2]*(b + 2*c*Tan[(d + e*x)/2] - b*Tan[(d + e*x)/2]^2)]))/((I*a + b - I*c)*
```

```
((-I)*a + b + I*c)*e)
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.25 (sec) , antiderivative size = 13067312, normalized size of antiderivative = 19048.56

output too large to display

```
[In] int(tan(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x)
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19371 vs. 2(629) = 1258.

Time = 2.87 (sec) , antiderivative size = 19371, normalized size of antiderivative = 28.24

$$\int \frac{\tan^3(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tan^3(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \int \frac{\tan^3\left(d+ex\right)}{\left(a+b\tan\left(d+ex\right)+c\tan^2\left(d+ex\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(e*x+d)**3/(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(3/2),x)
[Out] Integral(tan(d + e*x)**3/(a + b*tan(d + e*x) + c*tan(d + e*x)**2)**(3/2), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^3(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(tan(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^3(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \int \frac{\tan(d+e\,x)^3}{\left(c\tan\left(d+e\,x\right)^2+b\tan\left(d+e\,x\right)+a\right)^{3/2}}\,dx$$

```
[In] int(tan(d + e*x)^3/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(3/2),x)
```

```
[Out] int(tan(d + e*x)^3/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(3/2), x)
```

3.22 $\int \frac{\tan^2(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 638

$$\int \frac{\tan^2(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx =$$

$$-\frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b(2\cdots}{\sqrt{2}\sqrt{2a-2c+\sqrt{\cdots}}}\cdots\right.}{\sqrt{2}\,(a^2+b^2-2ac+c^2)^{3/2}\,e\cdots}$$

$$+\frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b(2\cdots}{\sqrt{2}\sqrt{2a-2c-\sqrt{\cdots}}}\cdots\right.}{\sqrt{2}\,(a^2+b^2-2ac+c^2)^{3/2}\,e\cdots}$$

$$-\frac{2(ab(a+c)+c(2a^2+b^2-2ac)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

```
[Out] -1/2*arctan(1/2*(b*(2*a-2*c+(a^2-2*a*c+b^2+c^2)^(1/2))+(b^2-(a-c)*(a-c-(a^2
-2*a*c+b^2+c^2)^(1/2)))*tan(e*x+d))*2^(1/2)/(2*a-2*c+(a^2-2*a*c+b^2+c^2)^(1
/2))^(1/2)/(a^2-b^2-2*a*c+c^2-(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*t
an(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(2*a-2*c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)*
(a^2-b^2-2*a*c+c^2-(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-2*a*c+b^2+c^
2)^(3/2)/e*2^(1/2)+1/2*arctan(1/2*(b*(2*a-2*c-(a^2-2*a*c+b^2+c^2)^(1/2))+(b
^2-(a-c)*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2)))*tan(e*x+d))*2^(1/2)/(2*a-2*c-(a^2
-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-b^2-2*a*c+c^2+(a-c)*(a^2-2*a*c+b^2+c^2)^(
1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(2*a-2*c-(a^2-2*a*c+b^2+
c^2)^(1/2))^(1/2)*(a^2-b^2-2*a*c+c^2+(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)
/(a^2-2*a*c+b^2+c^2)^(3/2)/e*2^(1/2)-2*(a*b*(a+c)+c*(2*a^2-2*a*c+b^2)*tan(e
*x+d))/(b^2+(a-c)^2)/(-4*a*c+b^2)/e/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)
```

Rubi [A] (verified)

Time = 4.30 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3781, 1079, 1050, 1044, 211}

$$\int \frac{\tan^2(d+ex)}{\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}\,dx =$$

$$-\frac{\sqrt{\sqrt{a^2-2ac+b^2+c^2}+2a-2c}\sqrt{-(a-c)\sqrt{a^2-2ac+b^2+c^2}+a^2-2ac-b^2+c^2}\arctan\left(\frac{(b^2}{\sqrt{2}\sqrt{\sqrt{a^2-2ac}}}\right.}{\sqrt{2}e\left(a^2-2ac+b^2+c^2\right)^{3/2}}$$

$$+\frac{\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+2a-2c}\sqrt{(a-c)\sqrt{a^2-2ac+b^2+c^2}+a^2-2ac-b^2+c^2}\arctan\left(\frac{(b^2}{\sqrt{2}\sqrt{-\sqrt{a^2-2a}}}\right.}{\sqrt{2}e\left(a^2-2ac+b^2+c^2\right)^{3/2}}$$

$$-\frac{2(c(2a^2-2ac+b^2)\tan(d+ex)+ab(a+c))}{e\left((a-c)^2+b^2\right)\left(b^2-4ac\right)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

```
[In] Int[Tan[d + e*x]^2/(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2),x]
```

```
[Out] -((Sqrt[2*a - 2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c +
 c^2 - (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTan[(b*(2*a - 2*c + Sqrt[a
^2 + b^2 - 2*a*c + c^2]) + (b^2 - (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a*c +
 c^2]))*Tan[d + e*x])/(Sqrt[2]*Sqrt[2*a - 2*c + Sqrt[a^2 + b^2 - 2*a*c + c^
2]]*Sqrt[a^2 - b^2 - 2*a*c + c^2 - (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*S
qrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c +
 c^2)^(3/2)*e)) + (Sqrt[2*a - 2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a^2
 - b^2 - 2*a*c + c^2 + (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTan[(b*(2*
a - 2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) + (b^2 - (a - c)*(a - c + Sqrt[a^2
 + b^2 - 2*a*c + c^2]))*Tan[d + e*x])/(Sqrt[2]*Sqrt[2*a - 2*c - Sqrt[a^2 +
b^2 - 2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c + c^2 + (a - c)*Sqrt[a^2 + b^2 -
 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2
 + b^2 - 2*a*c + c^2)^(3/2)*e) - (2*(a*b*(a + c) + c*(2*a^2 + b^2 - 2*a*c)*
Tan[d + e*x]))/((b^2 + (a - c)^2)*(b^2 - 4*a*c)*e*Sqrt[a + b*Tan[d + e*x] +
 c*Tan[d + e*x]^2])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
```

```
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 1079

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2
)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((A*c - a*C)*(
(-b)*(c*d + a*f)) + (A*b)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(2*a*f)) + C*(b^2*d - 2*a*(c*d - a*f)))*x), x] + Dist[1/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) +
(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) -
c*d*(p + 2)) - (2*f*((A*c - a*C)*((-b)*(c*d + a*f)) + (A*b)*(2*c^2*d + b^2*
f - c*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*
C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f)
 - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d
, f, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*
d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= -\frac{2(ab(a+c)+c(2a^2+b^2-2ac)\tan(d+ex))}{(b^2+(a-c)^2)(b^2-4ac)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} - \frac{2\text{Subst}\left(\int \frac{\frac{1}{2}(a-c)(b^2-4ac)-\frac{1}{2}b(b^2-4ac)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{(b^2+(a-c)^2)(b^2-4ac)e}$$

$$= -\frac{2(ab(a+c)+c(2a^2+b^2-2ac)\tan(d+ex))}{(b^2+(a-c)^2)(b^2-4ac)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(b^2-4ac)\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)-\frac{1}{2}b(b^2-4ac)\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex\right.}{(b^2-4ac)(a^2+b^2-2ac+c^2)^{3/2}e} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(b^2-4ac)\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)-\frac{1}{2}b(b^2-4ac)\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex\right.}{(b^2-4ac)(a^2+b^2-2ac+c^2)^{3/2}e}$$

$$= -\frac{2(ab(a+c)+c(2a^2+b^2-2ac)\tan(d+ex))}{(b^2+(a-c)^2)(b^2-4ac)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} - \frac{\left(b(b^2-4ac)\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Sub}}{} + \frac{\left(b(b^2-4ac)\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Sub}}{}$$

$$= -\frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-\right.}}}{\sqrt{2}\,(a^2+b^2}$$

$$+\frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{}{\sqrt{2}}\right.}{\sqrt{2}\,(a^2+b^2-2ac+c^2)}$$

$$-\frac{2(ab(a+c)+c(2a^2+b^2-2ac)\tan(d+ex))}{(b^2+(a-c)^2)(b^2-4ac)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.51

$$\int \frac{\tan^2(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \frac{\frac{\left(-b^2(b+ic)-4ia^2c+a\left(ib^2+4bc+4ic^2\right)\right)\operatorname{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex \cdots}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan \cdots}}\right.}{\sqrt{a-ib-c}} \cdots}{\cdots}$$

```
[In] Integrate[Tan[d + e*x]^2/(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2),x]
```

```
[Out] (((-(b^2*(b + I*c)) - (4*I)*a^2*c + a*(I*b^2 + 4*b*c + (4*I)*c^2))*ArcTanh[
(2*a - I*b + (b - (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a - I*b - c]*Sqrt[a + b*Ta
n[d + e*x] + c*Tan[d + e*x]^2])])/Sqrt[a - I*b - c] + (I*(4*a^2*c + b^2*(I*
b + c) - a*(b^2 + (4*I)*b*c + 4*c^2))*ArcTanh[(2*a + I*b + (b + (2*I)*c)*Ta
n[d + e*x])/(2*Sqrt[a + I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2
])])/Sqrt[a + I*b - c] - (4*(a*b*(a + c) + c*(2*a^2 + b^2 - 2*a*c)*Tan[d +
e*x]))/Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/(2*(b^2 + (a - c)^2)*(b
^2 - 4*a*c)*e)
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.13 (sec) , antiderivative size = 11848772, normalized size of antiderivative = 18571.74

output too large to display

```
[In] int(tan(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19326 vs. 2(587) = 1174.

Time = 3.09 (sec) , antiderivative size = 19326, normalized size of antiderivative = 30.29

$$\int \frac{\tan^2(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tan^2(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \int \frac{\tan^2{(d+ex)}}{(a+b\tan{(d+ex)}+c\tan^2{(d+ex)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(e*x+d)**2/(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(3/2),x)
[Out] Integral(tan(d + e*x)**2/(a + b*tan(d + e*x) + c*tan(d + e*x)**2)**(3/2), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^2(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(tan(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^2(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx=\int \frac{\tan(d+e\,x)^2}{\left(c\tan\left(d+e\,x\right)^2+b\tan\left(d+e\,x\right)+a\right)^{3/2}}\,dx$$

```
[In] int(tan(d + e*x)^2/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(3/2),x)
[Out] int(tan(d + e*x)^2/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(3/2), x)
```

3.23 $\int \frac{\tan(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 635

$$\int \frac{\tan(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-}{}$$

$$-\frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{b^2-}{\sqrt{2}\sqrt{2a-2c+\sqrt{a}}}\right.}{\sqrt{2}\,(a^2+b^2-2ac+c^2)^{3/2}\,e}$$

$$+\frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

```
[Out] -1/2*arctanh(1/2*(b^2-(a-c)*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))-b*(2*a-2*c+(a^2
-2*a*c+b^2+c^2)^(1/2))*tan(e*x+d))*2^(1/2)/(2*a-2*c+(a^2-2*a*c+b^2+c^2)^(1/
2))^(1/2)/(a^2-b^2-2*a*c+c^2-(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*ta
n(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(2*a-2*c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)*(
a^2-b^2-2*a*c+c^2-(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-2*a*c+b^2+c^2
)^(3/2)/e*2^(1/2)+1/2*arctanh(1/2*(b^2-(a-c)*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2)
)-b*(2*a-2*c-(a^2-2*a*c+b^2+c^2)^(1/2))*tan(e*x+d))*2^(1/2)/(2*a-2*c-(a^2-2
*a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-b^2-2*a*c+c^2+(a-c)*(a^2-2*a*c+b^2+c^2)^(1/
2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(2*a-2*c-(a^2-2*a*c+b^2+c^
2)^(1/2))^(1/2)*(a^2-b^2-2*a*c+c^2+(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(
a^2-2*a*c+b^2+c^2)^(3/2)/e*2^(1/2)+2*(a*(b^2-2*(a-c)*c)+b*c*(a+c)*tan(e*x+d
))/(b^2+(a-c)^2)/(-4*a*c+b^2)/e/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)
```

Rubi [A] (verified)

Time = 4.30 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3781, 1032, 1050, 1044, 214}

$$\int \frac{\tan(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \frac{\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+2a-2c}\sqrt{(a-c)\sqrt{a^2-2ac+b^2}}}{}$$

$$-\frac{\sqrt{\sqrt{a^2-2ac+b^2+c^2}+2a-2c}\sqrt{-(a-c)\sqrt{a^2-2ac+b^2+c^2}+a^2-2ac-b^2+c^2}\,\mathrm{arctanh}\left(\frac{}{\sqrt{2}\sqrt{\sqrt{a^2-2}}}\right.}{\sqrt{2}e\,(a^2-2ac+b^2+c^2)^{3/2}}$$

$$+\frac{2(a(b^2-2c(a-c))+bc(a+c)\tan(d+ex))}{e\,((a-c)^2+b^2)\,(b^2-4ac)\,\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

```
[In] Int[Tan[d + e*x]/(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2*a - 2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c + c
^2 + (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTanh[(b^2 - (a - c)*(a - c +
 Sqrt[a^2 + b^2 - 2*a*c + c^2]) - b*(2*a - 2*c - Sqrt[a^2 + b^2 - 2*a*c + c
^2])*Tan[d + e*x])/(Sqrt[2]*Sqrt[2*a - 2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]
*Sqrt[a^2 - b^2 - 2*a*c + c^2 + (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt
[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^
2)^(3/2)*e) - (Sqrt[2*a - 2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a^2 - b
^2 - 2*a*c + c^2 - (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTanh[(b^2 - (a
 - c)*(a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - b*(2*a - 2*c + Sqrt[a^2 + b
^2 - 2*a*c + c^2])*Tan[d + e*x])/(Sqrt[2]*Sqrt[2*a - 2*c + Sqrt[a^2 + b^2 -
 2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c + c^2 - (a - c)*Sqrt[a^2 + b^2 - 2*a*
c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^
2 - 2*a*c + c^2)^(3/2)*e) + (2*(a*(b^2 - 2*(a - c)*c) + b*c*(a + c)*Tan[d +
 e*x]))/((b^2 + (a - c)^2)*(b^2 - 4*a*c)*e*Sqrt[a + b*Tan[d + e*x] + c*Tan[
d + e*x]^2])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
```

```
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} - \frac{2\text{Subst}\left(\int \frac{-\frac{1}{2}b(b^2-4ac)-\frac{1}{2}(a-c)(b^2-4ac)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{(b^2+(a-c)^2)\,(b^2-4ac)\,e}$$

$$= \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)(b^2-4ac)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}b(b^2-4ac)\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)+\frac{1}{2}(b^2-4ac)\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}dx, x, \tan(d+e}{(b^2-4ac)(a^2+b^2-2ac+c^2)^{3/2}e} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}b(b^2-4ac)\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)+\frac{1}{2}(b^2-4ac)\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}dx, x, \tan(d+e}{(b^2-4ac)(a^2+b^2-2ac+c^2)^{3/2}e}$$

$$= \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)(b^2-4ac)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} - \frac{\left(b(b^2-4ac)\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Su}}{} + \frac{\left(b(b^2-4ac)\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Su}}{}$$

$$= \frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\,\text{arctanh}\left(\frac{}{\sqrt{2}\sqrt{}}\right.}{\sqrt{2}(a^2+b^2-2ac+c^2)} - \frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-}\right.}}{\sqrt{2}(a^2+b^2} + \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)(b^2-4ac)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.80 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.50

$$\int \frac{\tan(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}dx = \frac{\frac{(4a^2c+b^2(-ib+c)-a(b^2-4ibc+4c^2))\,\text{arctanh}\left(\frac{2a-ib+(b-2ic)\tan(d+ex)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+ex)+c\tan^2}}\right.}{\sqrt{a-ib-c}}}{}$$

```
[In] Integrate[Tan[d + e*x]/(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2),x]
```

```
[Out] (((4*a^2*c + b^2*((-I)*b + c) - a*(b^2 - (4*I)*b*c + 4*c^2))*ArcTanh[(2*a -
 I*b + (b - (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a - I*b - c]*Sqrt[a + b*Tan[d +
e*x] + c*Tan[d + e*x]^2])])/Sqrt[a - I*b - c] + ((4*a^2*c + b^2*(I*b + c) -
```

```
a*(b^2 + (4*I)*b*c + 4*c^2))*ArcTanh[(2*a + I*b + (b + (2*I)*c)*Tan[d + e*
x])/(2*Sqrt[a + I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/Sqr
t[a + I*b - c] + (4*(a*(b^2 + 2*c*(-a + c)) + b*c*(a + c)*Tan[d + e*x]))/Sq
rt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/(2*(b^2 + (a - c)^2)*(b^2 - 4*a*
c)*e)
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 0.88 (sec) , antiderivative size = 13066372, normalized size of antiderivative = 20576.96

output too large to display

```
[In] int(tan(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x)
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19368 vs. 2(580) = 1160.

Time = 2.91 (sec) , antiderivative size = 19368, normalized size of antiderivative = 30.50

$$\int \frac{\tan(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="fr
icas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tan(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \int \frac{\tan(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(3/2),x)
[Out] Integral(tan(d + e*x)/(a + b*tan(d + e*x) + c*tan(d + e*x)**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(tan(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \int \frac{\tan(d+e\,x)}{\left(c\tan\left(d+e\,x\right)^2+b\tan\left(d+e\,x\right)+a\right)^{3/2}}\,dx$$

```
[In] int(tan(d + e*x)/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(3/2),x)
```

```
[Out] int(tan(d + e*x)/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(3/2), x)
```

3.24 $\int \frac{\cot(d+ex)}{\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 750

$$\int \frac{\cot(d+ex)}{\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{a^{3/2}e}$$

$$-\frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{b^2-\cdots}{\sqrt{2}\sqrt{2a-2c-\sqrt{a\cdots}}}\right.}{\sqrt{2}\left(a^2+b^2-2ac+c^2\right)^{3/2}e}$$

$$+\frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{b^2-\cdots}{\sqrt{2}\sqrt{2a-2c+\sqrt{a^2\cdots}}}\right.}{\sqrt{2}\left(a^2+b^2-2ac+c^2\right)^{3/2}e}$$

$$+\frac{2(b^2-2ac+bc\tan(d+ex))}{a\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$-\frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{\left(b^2+(a-c)^2\right)\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

```
[Out] -arctanh(1/2*(2*a+b*tan(e*x+d))/a^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/
2))/a^(3/2)/e+1/2*arctanh(1/2*(b^2-(a-c)*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))-b*
(2*a-2*c+(a^2-2*a*c+b^2+c^2)^(1/2))*tan(e*x+d))*2^(1/2)/(2*a-2*c+(a^2-2*a*c
+b^2+c^2)^(1/2))^(1/2)/(a^2-b^2-2*a*c+c^2-(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^
(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(2*a-2*c+(a^2-2*a*c+b^2+c^2)^(
1/2))^(1/2)*(a^2-b^2-2*a*c+c^2-(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-
2*a*c+b^2+c^2)^(3/2)/e*2^(1/2)-1/2*arctanh(1/2*(b^2-(a-c)*(a-c+(a^2-2*a*c+b
^2+c^2)^(1/2))-b*(2*a-2*c-(a^2-2*a*c+b^2+c^2)^(1/2))*tan(e*x+d))*2^(1/2)/(2
*a-2*c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-b^2-2*a*c+c^2+(a-c)*(a^2-2*a*c
+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(2*a-2*c-(a^2
-2*a*c+b^2+c^2)^(1/2))^(1/2)*(a^2-b^2-2*a*c+c^2+(a-c)*(a^2-2*a*c+b^2+c^2)^(
```

```
1/2))^(1/2)/(a^2-2*a*c+b^2+c^2)^(3/2)/e*2^(1/2)+2*(b^2-2*a*c+b*c*tan(e*x+d)
)/a/(-4*a*c+b^2)/e/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)-2*(a*(b^2-2*(a-c)*
c)+b*c*(a+c)*tan(e*x+d))/(b^2+(a-c)^2)/(-4*a*c+b^2)/e/(a+b*tan(e*x+d)+c*tan
(e*x+d)^2)^(1/2)
```

Rubi [A] (verified)

Time = 5.15 (sec) , antiderivative size = 750, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3781, 6857, 754, 12, 738, 212, 1032, 1050, 1044, 214}

$$\int \frac{\cot(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{a^{3/2}e} - \frac{\sqrt{-\sqrt{a^2-2ac+b^2+c^2}+2a-2c}\sqrt{(a-c)\sqrt{a^2-2ac+b^2+c^2}+a^2-2ac-b^2+c^2}\operatorname{arctanh}\left(\frac{\ldots}{\sqrt{2}\sqrt{-\sqrt{a^2-\ldots}}}\right)}{\sqrt{2}e\left(a^2-2ac+b^2+c^2\right)^{3/2}} + \frac{\sqrt{\sqrt{a^2-2ac+b^2+c^2}+2a-2c}\sqrt{-(a-c)\sqrt{a^2-2ac+b^2+c^2}+a^2-2ac-b^2+c^2}\operatorname{arctanh}\left(\frac{\ldots}{\sqrt{2}\sqrt{\sqrt{a^2-2a\ldots}}}\right)}{\sqrt{2}e\left(a^2-2ac+b^2+c^2\right)^{3/2}} + \frac{2(-2ac+b^2+bc\tan(d+ex))}{ae(b^2-4ac)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} - \frac{2(a(b^2-2c(a-c))+bc(a+c)\tan(d+ex))}{e((a-c)^2+b^2)(b^2-4ac)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

```
[In] Int[Cot[d + e*x]/(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x] + c*Tan
[d + e*x]^2])]/(a^(3/2)*e)) - (Sqrt[2*a - 2*c - Sqrt[a^2 + b^2 - 2*a*c + c^
2]]*Sqrt[a^2 - b^2 - 2*a*c + c^2 + (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*A
rcTanh[(b^2 - (a - c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - b*(2*a - 2*
c - Sqrt[a^2 + b^2 - 2*a*c + c^2])*Tan[d + e*x])/(Sqrt[2]*Sqrt[2*a - 2*c -
Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c + c^2 + (a - c)*Sqrt[
a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(S
qrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(3/2)*e) + (Sqrt[2*a - 2*c + Sqrt[a^2 + b^
2 - 2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c + c^2 - (a - c)*Sqrt[a^2 + b^2 - 2
*a*c + c^2]]*ArcTanh[(b^2 - (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2])
 - b*(2*a - 2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])*Tan[d + e*x])/(Sqrt[2]*Sqr
t[2*a - 2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c + c^2 -
 (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d +
 e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(3/2)*e) + (2*(b^2 - 2*a*c +
 b*c*Tan[d + e*x]))/(a*(b^2 - 4*a*c)*e*Sqrt[a + b*Tan[d + e*x] + c*Tan[d +
e*x]^2]) - (2*(a*(b^2 - 2*(a - c)*c) + b*c*(a + c)*Tan[d + e*x]))/((b^2 + (
a - c)^2)*(b^2 - 4*a*c)*e*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
```

```
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x(a+bx+cx^2)^{3/2}} - \frac{x}{(1+x^2)(a+bx+cx^2)^{3/2}}\right)dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e} - \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{2(b^2-2ac+bc\tan(d+ex))}{a\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$- \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$- \frac{2\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{a\,(b^2-4ac)\,e}$$
$$+ \frac{2\text{Subst}\left(\int \frac{-\frac{1}{2}b(b^2-4ac)-\frac{1}{2}(a-c)(b^2-4ac)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{(b^2+(a-c)^2)\,(b^2-4ac)\,e}$$

$$= \frac{2(b^2-2ac+bc\tan(d+ex))}{a\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$- \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{ae}$$
$$- \frac{\text{Subst}\left(\int \frac{\frac{1}{2}b(b^2-4ac)\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)+\frac{1}{2}(b^2-4ac)\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex\right.}{(b^2-4ac)\,(a^2+b^2-2ac+c^2)^{3/2}\,e}$$
$$+ \frac{\text{Subst}\left(\int \frac{\frac{1}{2}b(b^2-4ac)\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)+\frac{1}{2}(b^2-4ac)\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex\right.}{(b^2-4ac)\,(a^2+b^2-2ac+c^2)^{3/2}\,e}$$

$$= \frac{2(b^2-2ac+bc\tan(d+ex))}{a\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$- \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$- \frac{2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{ae}$$
$$+ \frac{\left(b(b^2-4ac)\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Sub}}{}$$
$$- \frac{\left(b(b^2-4ac)\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Sub}}{}$$

$$= -\frac{\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{a^{3/2}e}$$

$$-\frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\Big(-}{\sqrt{2}\,(a^2+b^2-2ac+c}$$

$$+\frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-\right.}}}{\sqrt{2}\,(a^2+b^2}$$

$$+\frac{2(b^2-2ac+bc\tan(d+ex))}{a\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$-\frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.90 (sec) , antiderivative size = 450, normalized size of antiderivative = 0.60

$$\int \frac{\cot(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \frac{2\left(\frac{\left(-\frac{b^2}{2}+2ac\right)\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{a^{3/2}}+\frac{-\frac{(4a^2c+b^2(}{}}{}\right.}{}$$

```
[In] Integrate[Cot[d + e*x]/(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2),x]
```

```
[Out] (2*(((-1/2*b^2 + 2*a*c)*ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqrt[a +
b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/a^(3/2) + (-1/4*((4*a^2*c + b^2*((-I)
*b + c) - a*(b^2 - (4*I)*b*c + 4*c^2))*ArcTanh[(2*a - I*b + (b - (2*I)*c)*T
an[d + e*x])/(2*Sqrt[a - I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^
2])])/Sqrt[a - I*b - c] - ((4*a^2*c + b^2*(I*b + c) - a*(b^2 + (4*I)*b*c +
4*c^2))*ArcTanh[(2*a + I*b + (b + (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a + I*b -
c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(4*Sqrt[a + I*b - c]))/(b
^2 + (a - c)^2) + (b^2 - 2*a*c + b*c*Tan[d + e*x])/(a*Sqrt[a + b*Tan[d + e*
x] + c*Tan[d + e*x]^2]) + (-(a*b^2) + 2*a*(a - c)*c - b*c*(a + c)*Tan[d + e
*x])/((a^2 + b^2 - 2*a*c + c^2)*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2]
)))/((b^2 - 4*a*c)*e)
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int(cot(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x)
[Out] int(cot(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39197 vs. 2(685) = 1370.

Time = 13.50 (sec) , antiderivative size = 78431, normalized size of antiderivative = 104.57

$$\int \frac{\cot(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="fr
icas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\cot(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \int \frac{\cot{\left(d+ex \right)}}{\left(a+b\tan{\left(d+ex \right)}+c\tan^{2}{\left(d+ex \right)}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(3/2),x)
[Out] Integral(cot(d + e*x)/(a + b*tan(d + e*x) + c*tan(d + e*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="ma
xima")
[Out] Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(e*x+d)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \int \frac{\cot(d+e\,x)}{\left(c\tan\left(d+e\,x\right)^2+b\tan\left(d+e\,x\right)+a\right)^{3/2}}\,dx$$

```
[In] int(cot(d + e*x)/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(3/2),x)
```

```
[Out] int(cot(d + e*x)/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(3/2), x)
```

3.25 $\int \frac{\cot^2(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 829

$$\int \frac{\cot^2(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx =$$

$$-\frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b(2a\cdots}{\sqrt{2}\sqrt{2a-2c+\sqrt{a^2\cdots}}}\right.}{\sqrt{2}\,(a^2+b^2-2ac+c^2)^{3/2}\,e}$$

$$+\frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(\frac{b(2a\cdots}{\sqrt{2}\sqrt{2a-2c-\sqrt{a^2\cdots}}}\right.}{\sqrt{2}\,(a^2+b^2-2ac+c^2)^{3/2}\,e}$$

$$+\frac{3b\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2a^{5/2}e}$$

$$+\frac{2\cot(d+ex)\,(b^2-2ac+bc\tan(d+ex))}{a\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$+\frac{2(b(b^2-(3a-c)c)+c(b^2-2(a-c)c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$-\frac{(3b^2-8ac)\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{a^2\,(b^2-4ac)\,e}$$

```
[Out] 3/2*b*arctanh(1/2*(2*a+b*tan(e*x+d))/a^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2
)^(1/2))/a^(5/2)/e-1/2*arctan(1/2*(b*(2*a-2*c+(a^2-2*a*c+b^2+c^2)^(1/2))+(b
^2-(a-c)*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2)))*tan(e*x+d))*2^(1/2)/(2*a-2*c+(a^2
-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-b^2-2*a*c+c^2-(a-c)*(a^2-2*a*c+b^2+c^2)^(
1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(2*a-2*c+(a^2-2*a*c+b^2+
c^2)^(1/2))^(1/2)*(a^2-b^2-2*a*c+c^2-(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)
```

```
/(a^2-2*a*c+b^2+c^2)^(3/2)/e*2^(1/2)+1/2*arctan(1/2*(b*(2*a-2*c-(a^2-2*a*c+
b^2+c^2)^(1/2))+(b^2-(a-c)*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2)))*tan(e*x+d))*2^(
1/2)/(2*a-2*c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-b^2-2*a*c+c^2+(a-c)*(a^
2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))*(2*a-2
*c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)*(a^2-b^2-2*a*c+c^2+(a-c)*(a^2-2*a*c+b^2
+c^2)^(1/2))^(1/2)/(a^2-2*a*c+b^2+c^2)^(3/2)/e*2^(1/2)-(-8*a*c+3*b^2)*cot(e
*x+d)*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)/a^2/(-4*a*c+b^2)/e+2*cot(e*x+d)
*(b^2-2*a*c+b*c*tan(e*x+d))/a/(-4*a*c+b^2)/e/(a+b*tan(e*x+d)+c*tan(e*x+d)^2
)^(1/2)+2*(b*(b^2-(3*a-c)*c)+c*(b^2-2*(a-c)*c)*tan(e*x+d))/(b^2+(a-c)^2)/(-
4*a*c+b^2)/e/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)
```

Rubi [A] (verified)

Time = 5.49 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3781, 6857, 754, 820, 738, 212, 989, 1050, 1044, 211}

$$\int \frac{\cot^2(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx =$$

$$-\frac{\sqrt{2a-2c+\sqrt{a^2-2ca+b^2+c^2}}\sqrt{a^2-2ca-b^2+c^2-(a-c)\sqrt{a^2-2ca+b^2+c^2}}\arctan\left(\frac{b(2\ldots}{\sqrt{2}\sqrt{2a-2c+\sqrt{\ldots}}\ldots}\right.}{\sqrt{2}\left(a^2-2ca+b^2+c^2\right)^{3/2}e}$$

$$+\frac{\sqrt{2a-2c-\sqrt{a^2-2ca+b^2+c^2}}\sqrt{a^2-2ca-b^2+c^2+(a-c)\sqrt{a^2-2ca+b^2+c^2}}\arctan\left(\frac{b(2\ldots}{\sqrt{2}\sqrt{2a-2c-\sqrt{a\ldots}}\ldots}\right.}{\sqrt{2}\left(a^2-2ca+b^2+c^2\right)^{3/2}e}$$

$$+\frac{3b\,\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}\right)}{2a^{5/2}e}$$

$$-\frac{(3b^2-8ac)\cot(d+ex)\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}{a^2\left(b^2-4ac\right)e}$$

$$+\frac{2\cot(d+ex)\left(b^2+c\tan(d+ex)b-2ac\right)}{a\left(b^2-4ac\right)e\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}$$

$$+\frac{2(b(b^2-(3a-c)c)+c(b^2-2(a-c)c)\tan(d+ex))}{\left(b^2+(a-c)^2\right)\left(b^2-4ac\right)e\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}$$

```
[In] Int[Cot[d + e*x]^2/(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2),x]

[Out] -((Sqrt[2*a - 2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c +
 c^2 - (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTan[(b*(2*a - 2*c + Sqrt[a
^2 + b^2 - 2*a*c + c^2]) + (b^2 - (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a*c +
 c^2]))*Tan[d + e*x])/(Sqrt[2]*Sqrt[2*a - 2*c + Sqrt[a^2 + b^2 - 2*a*c + c^
2]]*Sqrt[a^2 - b^2 - 2*a*c + c^2 - (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*S
qrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c +
```

```
c^2)^(3/2)*e)) + (Sqrt[2*a - 2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a^2
 - b^2 - 2*a*c + c^2 + (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTan[(b*(2*
a - 2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) + (b^2 - (a - c)*(a - c + Sqrt[a^2
 + b^2 - 2*a*c + c^2]))*Tan[d + e*x])/(Sqrt[2]*Sqrt[2*a - 2*c - Sqrt[a^2 +
b^2 - 2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c + c^2 + (a - c)*Sqrt[a^2 + b^2 -
 2*a*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2
 + b^2 - 2*a*c + c^2)^(3/2)*e) + (3*b*ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqr
t[a]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(2*a^(5/2)*e) + (2*Cot[
d + e*x]*(b^2 - 2*a*c + b*c*Tan[d + e*x]))/(a*(b^2 - 4*a*c)*e*Sqrt[a + b*Ta
n[d + e*x] + c*Tan[d + e*x]^2]) + (2*(b*(b^2 - (3*a - c)*c) + c*(b^2 - 2*(a
 - c)*c)*Tan[d + e*x]))/((b^2 + (a - c)^2)*(b^2 - 4*a*c)*e*Sqrt[a + b*Tan[d
 + e*x] + c*Tan[d + e*x]^2]) - ((3*b^2 - 8*a*c)*Cot[d + e*x]*Sqrt[a + b*Tan
[d + e*x] + c*Tan[d + e*x]^2])/(a^2*(b^2 - 4*a*c)*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
```

```
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 989

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*f
))*x)*(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x^2(a+bx+cx^2)^{3/2}}+\frac{1}{(-1-x^2)(a+bx+cx^2)^{3/2}}\right)dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}+\frac{\text{Subst}\left(\int \frac{1}{(-1-x^2)(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{2\cot(d+ex)\left(b^2-2ac+bc\tan(d+ex)\right)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} \\
&\quad+\frac{2(b(b^2-(3a-c)c)+c(b^2-2(a-c)c)\tan(d+ex))}{\left(b^2+(a-c)^2\right)\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} \\
&\quad-\frac{2\text{Subst}\left(\int \frac{\frac{1}{2}(-3b^2+8ac)-bcx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{a\left(b^2-4ac\right)e} \\
&\quad+\frac{2\text{Subst}\left(\int \frac{\frac{1}{2}(a-c)(b^2-4ac)-\frac{1}{2}b(b^2-4ac)x}{(-1-x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{\left(b^2+(a-c)^2\right)\left(b^2-4ac\right)e} \\
&= \frac{2\cot(d+ex)\left(b^2-2ac+bc\tan(d+ex)\right)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} \\
&\quad+\frac{2(b(b^2-(3a-c)c)+c(b^2-2(a-c)c)\tan(d+ex))}{\left(b^2+(a-c)^2\right)\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}} \\
&\quad-\frac{\left(3b^2-8ac\right)\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{a^2\left(b^2-4ac\right)e} \\
&\quad-\frac{(3b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+ex)\right)}{2a^2e} \\
&\quad+\frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(b^2-4ac)\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)+\frac{1}{2}b(b^2-4ac)\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)x}{(-1-x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+e\right.}{\left(b^2-4ac\right)\left(a^2+b^2-2ac+c^2\right)^{3/2}e} \\
&\quad-\frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(b^2-4ac)\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)+\frac{1}{2}b(b^2-4ac)\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)x}{(-1-x^2)\sqrt{a+bx+cx^2}}\,dx, x, \tan(d+e\right.}{\left(b^2-4ac\right)\left(a^2+b^2-2ac+c^2\right)^{3/2}e}
\end{aligned}$$

$$= \frac{2\cot(d+ex)\left(b^2-2ac+bc\tan(d+ex)\right)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$+ \frac{2(b(b^2-(3a-c)c)+c(b^2-2(a-c)c)\tan(d+ex))}{\left(b^2+(a-c)^2\right)\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$- \frac{(3b^2-8ac)\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{a^2\left(b^2-4ac\right)e}$$

$$+ \frac{(3b)\text{Subst}\left(\int\frac{1}{4a-x^2}\,dx,x,\frac{2a+b\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{a^2e}$$

$$+ \frac{\left(b(b^2-4ac)\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Su}}{}$$

$$- \frac{\left(b(b^2-4ac)\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Su}}{}$$

$$=$$

$$- \frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-\right.}}}{\sqrt{2}\left(a^2+b^2\right.}$$

$$+ \frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\arctan\left(-\right.}{\sqrt{2}\left(a^2+b^2-2ac+c\right.}$$

$$+ \frac{3b\,\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{2a^{5/2}e}$$

$$+ \frac{2\cot(d+ex)\left(b^2-2ac+bc\tan(d+ex)\right)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$+ \frac{2(b(b^2-(3a-c)c)+c(b^2-2(a-c)c)\tan(d+ex))}{\left(b^2+(a-c)^2\right)\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$- \frac{(3b^2-8ac)\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{a^2\left(b^2-4ac\right)e}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.24 (sec) , antiderivative size = 583, normalized size of antiderivative = 0.70

$$\int \frac{\cot^2(d+ex)}{\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}\,dx = \frac{2\left(-\frac{4\sqrt{a-ib-c}\left(-\frac{1}{4}b\left(b^2-4ac\right)+\frac{1}{4}i(a-c)\left(b^2-4ac\right)\right)\text{arctanh}\left(\frac{-2a+ib-(b-2ic)}{2\sqrt{a-ib-c}\sqrt{a+b\tan(d+}}\right.}{4a-4ib-4c}\right.}{}$$

[In] Integrate[Cot[d + e*x]^2/(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2),x]

[Out] ((2*((-4*Sqrt[a - I*b - c]*(-1/4*(b*(b^2 - 4*a*c)) + (I/4)*(a - c)*(b^2 - 4 *a*c))*ArcTanh[(-2*a + I*b - (b - (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a - I*b - c]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(4*a - (4*I)*b - 4*c) - (4*Sqrt[a + I*b - c]*(-1/4*(b*(b^2 - 4*a*c)) - (I/4)*(a - c)*(b^2 - 4*a*c))* ArcTanh[(-2*a - I*b - (b + (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a + I*b - c]*Sqrt [a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(4*a + (4*I)*b - 4*c)))/((b^2 + (a - c)^2)*(b^2 - 4*a*c)) - (2*Cot[d + e*x]*(-b^2 + 2*a*c - b*c*Tan[d + e*x]))/(a*(b^2 - 4*a*c)*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2]) - (2*(-b^ 3 + b*(3*a - c)*c + c*(-b^2 + 2*a*c - 2*c^2)*Tan[d + e*x]))/((b^2 + (a - c) ^2)*(b^2 - 4*a*c)*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2]) - (2*(((2*a* b*c + (b*(-3*b^2 + 8*a*c))/2)*ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqr t[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(2*a^(3/2)) + ((3*b^2 - 8*a*c)* Cot[d + e*x]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/(2*a)))/(a*(b^2 - 4*a*c)))/e

Maple [F(-1)]

Timed out.

hanged

[In] int(cot(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x)

[Out] int(cot(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39249 vs. 2(764) = 1528.

Time = 12.46 (sec) , antiderivative size = 78535, normalized size of antiderivative = 94.73

$$\int \frac{\cot^2(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\cot^2(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \int \frac{\cot^2(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(e*x+d)**2/(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(3/2),x)
```

```
[Out] Integral(cot(d + e*x)**2/(a + b*tan(d + e*x) + c*tan(d + e*x)**2)**(3/2), x
)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^2(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^2(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(e*x+d)^2/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx=\int \frac{\cot(d+e\,x)^2}{\left(c\tan\left(d+e\,x\right)^2+b\tan\left(d+e\,x\right)+a\right)^{3/2}}\,dx$$

```
[In] int(cot(d + e*x)^2/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(3/2),x)
[Out] int(cot(d + e*x)^2/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(3/2), x)
```

3.26 $\int \frac{\cot^3(d+ex)}{\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 1007

$$\int \frac{\cot^3(d+ex)}{\left(a+b\tan(d+ex)+c\tan^2(d+ex)\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{a^{3/2}e}$$

$$- \frac{3(5b^2-4ac)\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8a^{7/2}e}$$

$$+ \frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{b^2 \cdots}{\sqrt{2}\sqrt{2a-2c-\sqrt{\cdots}}}\right.}{\sqrt{2}\left(a^2+b^2-2ac+c^2\right)^{3/2}e}$$

$$- \frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{b \cdots}{\sqrt{2}\sqrt{2a-2c+\sqrt{\cdots}}}\right.}{\sqrt{2}\left(a^2+b^2-2ac+c^2\right)^{3/2}e}$$

$$- \frac{2(b^2-2ac+bc\tan(d+ex))}{a(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$+ \frac{2\cot^2(d+ex)\,(b^2-2ac+bc\tan(d+ex))}{a(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$+ \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)\,(b^2-4ac)\,e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$+ \frac{b(15b^2-52ac)\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4a^3(b^2-4ac)\,e}$$

$$- \frac{(5b^2-12ac)\cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{2a^2(b^2-4ac)\,e}$$

```
[Out] arctanh(1/2*(2*a+b*tan(e*x+d))/a^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2
))/a^(3/2)/e-3/8*(-4*a*c+5*b^2)*arctanh(1/2*(2*a+b*tan(e*x+d))/a^(1/2)/(a+b
```

```
*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))/a^(7/2)/e-1/2*arctanh(1/2*(b^2-(a-c)*(a-
c-(a^2-2*a*c+b^2+c^2)^(1/2))-b*(2*a-2*c+(a^2-2*a*c+b^2+c^2)^(1/2))*tan(e*x+
d))*2^(1/2)/(2*a-2*c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-b^2-2*a*c+c^2-(a
-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2))
*(2*a-2*c+(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)*(a^2-b^2-2*a*c+c^2-(a-c)*(a^2-2*
a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-2*a*c+b^2+c^2)^(3/2)/e*2^(1/2)+1/2*arctanh(1
/2*(b^2-(a-c)*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))-b*(2*a-2*c-(a^2-2*a*c+b^2+c^2
)^(1/2))*tan(e*x+d))*2^(1/2)/(2*a-2*c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a^2
-b^2-2*a*c+c^2+(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)+c*tan
(e*x+d)^2)^(1/2))*(2*a-2*c-(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)*(a^2-b^2-2*a*c+
c^2+(a-c)*(a^2-2*a*c+b^2+c^2)^(1/2))^(1/2)/(a^2-2*a*c+b^2+c^2)^(3/2)/e*2^(1
/2)+1/4*b*(-52*a*c+15*b^2)*cot(e*x+d)*(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)
/a^3/(-4*a*c+b^2)/e-1/2*(-12*a*c+5*b^2)*cot(e*x+d)^2*(a+b*tan(e*x+d)+c*tan(
e*x+d)^2)^(1/2)/a^2/(-4*a*c+b^2)/e-2*(b^2-2*a*c+b*c*tan(e*x+d))/a/(-4*a*c+b
^2)/e/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)+2*cot(e*x+d)^2*(b^2-2*a*c+b*c*t
an(e*x+d))/a/(-4*a*c+b^2)/e/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(1/2)+2*(a*(b^2
-2*(a-c)*c)+b*c*(a+c)*tan(e*x+d))/(b^2+(a-c)^2)/(-4*a*c+b^2)/e/(a+b*tan(e*x
+d)+c*tan(e*x+d)^2)^(1/2)
```

Rubi [A] (verified)

Time = 5.63 (sec) , antiderivative size = 1007, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules

used = {3781, 6857, 754, 848, 820, 738, 212, 12, 1032, 1050, 1044, 214}

$$\int \frac{\cot^3(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx =$$

$$-\frac{(5b^2-12ac)\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}\cot^2(d+ex)}{2a^2(b^2-4ac)e}$$

$$+\frac{2(b^2+c\tan(d+ex)b-2ac)\cot^2(d+ex)}{a(b^2-4ac)e\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}$$

$$+\frac{b(15b^2-52ac)\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}\cot(d+ex)}{4a^3(b^2-4ac)e}$$

$$-\frac{3(5b^2-4ac)\,\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}\right)}{8a^{7/2}e}$$

$$+\frac{\text{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}\right)}{a^{3/2}e}$$

$$+\frac{\sqrt{2a-2c-\sqrt{a^2-2ca+b^2+c^2}}\sqrt{a^2-2ca-b^2+c^2+(a-c)\sqrt{a^2-2ca+b^2+c^2}}\text{arctanh}\left(\frac{b^2\cdots}{\sqrt{2}\sqrt{2a-2c-\cdots}}\right.}{\sqrt{2}(a^2-2ca+b^2+c^2)^{3/2}e}$$

$$-\frac{\sqrt{2a-2c+\sqrt{a^2-2ca+b^2+c^2}}\sqrt{a^2-2ca-b^2+c^2-(a-c)\sqrt{a^2-2ca+b^2+c^2}}\text{arctanh}\left(\frac{b\cdots}{\sqrt{2}\sqrt{2a-2c+\cdots}}\right.}{\sqrt{2}(a^2-2ca+b^2+c^2)^{3/2}e}$$

$$-\frac{2(b^2+c\tan(d+ex)b-2ac)}{a(b^2-4ac)e\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}$$

$$+\frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)(b^2-4ac)e\sqrt{c\tan^2(d+ex)+b\tan(d+ex)+a}}$$

```
[In] Int[Cot[d + e*x]^3/(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d
 + e*x]^2])]/(a^(3/2)*e) - (3*(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*Tan[d + e*x]
)/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(8*a^(7/2)*e) +
 (Sqrt[2*a - 2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c +
c^2 + (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTanh[(b^2 - (a - c)*(a - c
+ Sqrt[a^2 + b^2 - 2*a*c + c^2]) - b*(2*a - 2*c - Sqrt[a^2 + b^2 - 2*a*c +
c^2])*Tan[d + e*x])/(Sqrt[2]*Sqrt[2*a - 2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2]
]*Sqrt[a^2 - b^2 - 2*a*c + c^2 + (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqr
t[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c
^2)^(3/2)*e) - (Sqrt[2*a - 2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2]]*Sqrt[a^2 -
b^2 - 2*a*c + c^2 - (a - c)*Sqrt[a^2 + b^2 - 2*a*c + c^2]]*ArcTanh[(b^2 - (
a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - b*(2*a - 2*c + Sqrt[a^2 +
b^2 - 2*a*c + c^2])*Tan[d + e*x])/(Sqrt[2]*Sqrt[2*a - 2*c + Sqrt[a^2 + b^2
- 2*a*c + c^2]]*Sqrt[a^2 - b^2 - 2*a*c + c^2 - (a - c)*Sqrt[a^2 + b^2 - 2*a
```

```
*c + c^2]]*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])])/(Sqrt[2]*(a^2 + b
^2 - 2*a*c + c^2)^(3/2)*e) - (2*(b^2 - 2*a*c + b*c*Tan[d + e*x]))/(a*(b^2 -
 4*a*c)*e*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2]) + (2*Cot[d + e*x]^2*
(b^2 - 2*a*c + b*c*Tan[d + e*x]))/(a*(b^2 - 4*a*c)*e*Sqrt[a + b*Tan[d + e*x
] + c*Tan[d + e*x]^2]) + (2*(a*(b^2 - 2*(a - c)*c) + b*c*(a + c)*Tan[d + e*
x]))/((b^2 + (a - c)^2)*(b^2 - 4*a*c)*e*Sqrt[a + b*Tan[d + e*x] + c*Tan[d +
 e*x]^2]) + (b*(15*b^2 - 52*a*c)*Cot[d + e*x]*Sqrt[a + b*Tan[d + e*x] + c*T
an[d + e*x]^2])/(4*a^3*(b^2 - 4*a*c)*e) - ((5*b^2 - 12*a*c)*Cot[d + e*x]^2*
Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2])/(2*a^2*(b^2 - 4*a*c)*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f))))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
```

```
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3(a+bx+cx^2)^{3/2}} - \frac{1}{x(a+bx+cx^2)^{3/2}} + \frac{x}{(1+x^2)(a+bx+cx^2)^{3/2}}\right)\,dx, x, \tan(d+ex)\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}\\
&\quad + \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)(a+bx+cx^2)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}
\end{aligned}$$

$$= -\frac{2(b^2-2ac+bc\tan(d+ex))}{a\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+\frac{2\cot^2(d+ex)\left(b^2-2ac+bc\tan(d+ex)\right)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+\frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{\left(b^2+(a-c)^2\right)\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+\frac{2\text{Subst}\left(\int\frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{a\left(b^2-4ac\right)e}$$
$$-\frac{2\text{Subst}\left(\int\frac{\frac{1}{2}\left(-5b^2+12ac\right)-2bcx}{x^3\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{a\left(b^2-4ac\right)e}$$
$$-\frac{2\text{Subst}\left(\int\frac{-\frac{1}{2}b\left(b^2-4ac\right)-\frac{1}{2}(a-c)\left(b^2-4ac\right)x}{\left(1+x^2\right)\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{\left(b^2+(a-c)^2\right)\left(b^2-4ac\right)e}$$

$$= -\frac{2(b^2-2ac+bc\tan(d+ex))}{a\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+\frac{2\cot^2(d+ex)\left(b^2-2ac+bc\tan(d+ex)\right)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+\frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{\left(b^2+(a-c)^2\right)\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$-\frac{\left(5b^2-12ac\right)\cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{2a^2\left(b^2-4ac\right)e}$$
$$-\frac{\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{ae}$$
$$+\frac{\text{Subst}\left(\int\frac{-\frac{1}{4}b\left(15b^2-52ac\right)-\frac{1}{2}c\left(5b^2-12ac\right)x}{x^2\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{a^2\left(b^2-4ac\right)e}$$
$$+\frac{\text{Subst}\left(\int\frac{\frac{1}{2}b\left(b^2-4ac\right)\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)+\frac{1}{2}\left(b^2-4ac\right)\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{\left(1+x^2\right)\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+e\right.}{\left(b^2-4ac\right)\left(a^2+b^2-2ac+c^2\right)^{3/2}e}$$
$$-\frac{\text{Subst}\left(\int\frac{\frac{1}{2}b\left(b^2-4ac\right)\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)+\frac{1}{2}\left(b^2-4ac\right)\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)x}{\left(1+x^2\right)\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+e\right.}{\left(b^2-4ac\right)\left(a^2+b^2-2ac+c^2\right)^{3/2}e}$$

$$= -\frac{2(b^2-2ac+bc\tan(d+ex))}{a\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+\frac{2\cot^2(d+ex)\left(b^2-2ac+bc\tan(d+ex)\right)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+\frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{\left(b^2+(a-c)^2\right)\left(b^2-4ac\right)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+\frac{b(15b^2-52ac)\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4a^3\left(b^2-4ac\right)e}$$
$$-\frac{(5b^2-12ac)\cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{2a^2\left(b^2-4ac\right)e}$$
$$+\frac{2\text{Subst}\left(\int\frac{1}{4a-x^2}\,dx,x,\frac{2a+b\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{ae}$$
$$+\frac{(3(5b^2-4ac))\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,\tan(d+ex)\right)}{8a^3e}$$
$$-\frac{\left(b(b^2-4ac)\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Sub}}{}$$
$$+\frac{\left(b(b^2-4ac)\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Sub}}{}$$

$$= \frac{\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{a^{3/2}e}$$

$$+ \frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\Big(}{\sqrt{2}\left(a^2+b^2-2ac+c\right.}$$

$$- \frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-\right.}}}{\sqrt{2}\left(a^2+b^2\right.}$$

$$- \frac{2(b^2-2ac+bc\tan(d+ex))}{a(b^2-4ac)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$+ \frac{2\cot^2(d+ex)(b^2-2ac+bc\tan(d+ex))}{a(b^2-4ac)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$+ \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)(b^2-4ac)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$

$$+ \frac{b(15b^2-52ac)\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4a^3(b^2-4ac)e}$$

$$- \frac{(5b^2-12ac)\cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{2a^2(b^2-4ac)e}$$

$$- \frac{(3(5b^2-4ac))\operatorname{Subst}\left(\int\frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tan(d+ex)}{\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{4a^3e}$$

$$= \frac{\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{a^{3/2}e}$$
$$- \frac{3(5b^2-4ac)\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{8a^{7/2}e}$$
$$+ \frac{\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\operatorname{arctanh}\left(\frac{}{\sqrt{2}}\right.}{\sqrt{2}\left(a^2+b^2-2ac+c^2\right)}$$
$$- \frac{\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-\right.}}}{\sqrt{2}\left(a^2+b^2\right.}$$
$$- \frac{2(b^2-2ac+bc\tan(d+ex))}{a(b^2-4ac)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{2\cot^2(d+ex)(b^2-2ac+bc\tan(d+ex))}{a(b^2-4ac)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{2(a(b^2-2(a-c)c)+bc(a+c)\tan(d+ex))}{(b^2+(a-c)^2)(b^2-4ac)e\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}$$
$$+ \frac{b(15b^2-52ac)\cot(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{4a^3(b^2-4ac)e}$$
$$- \frac{(5b^2-12ac)\cot^2(d+ex)\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}{2a^2(b^2-4ac)e}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.21 (sec) , antiderivative size = 786, normalized size of antiderivative = 0.78

$$\int \frac{\cot^3(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \frac{-\frac{2\left(-\frac{b^2}{2}+2ac\right)\operatorname{arctanh}\left(\frac{2a+b\tan(d+ex)}{2\sqrt{a}\sqrt{a+b\tan(d+ex)+c\tan^2(d+ex)}}\right)}{a^{3/2}(b^2-4ac)} - \frac{2\left(-\frac{4\sqrt{a+ib-c}}{}\right.}{}}{}$$

```
[In] Integrate[Cot[d + e*x]^3/(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^(3/2),x]
```

```
[Out] ((-2*(-1/2*b^2 + 2*a*c)*ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqrt[a +
b*Tan[d + e*x] + c*Tan[d + e*x]^2)])])/(a^(3/2)*(b^2 - 4*a*c)) - (2*((-4*Sqr
t[a + I*b - c]*((I/4)*b*(b^2 - 4*a*c) - ((a - c)*(b^2 - 4*a*c))/4)*ArcTanh[
(2*a + I*b - (-b - (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a + I*b - c]*Sqrt[a + b*T
an[d + e*x] + c*Tan[d + e*x]^2)])])/(4*a + (4*I)*b - 4*c) - (4*Sqrt[a - I*b
- c]*((-1/4*I)*b*(b^2 - 4*a*c) - ((a - c)*(b^2 - 4*a*c))/4)*ArcTanh[(2*a -
```

```
I*b - (-b + (2*I)*c)*Tan[d + e*x])/(2*Sqrt[a - I*b - c]*Sqrt[a + b*Tan[d +
e*x] + c*Tan[d + e*x]^2])])/(4*a - (4*I)*b - 4*c)))/((b^2 - 4*a*c)*(b^2 + (
-a + c)^2)) + (2*(-b^2 + 2*a*c - b*c*Tan[d + e*x]))/(a*(b^2 - 4*a*c)*Sqrt[a
 + b*Tan[d + e*x] + c*Tan[d + e*x]^2]) - (2*Cot[d + e*x]^2*(-b^2 + 2*a*c -
b*c*Tan[d + e*x]))/(a*(b^2 - 4*a*c)*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x
]^2]) - (2*(-(a*(b^2 - 2*a*c + 2*c^2)) + c*(-(a*b) - b*c)*Tan[d + e*x]))/((
b^2 - 4*a*c)*(b^2 + (-a + c)^2)*Sqrt[a + b*Tan[d + e*x] + c*Tan[d + e*x]^2]
) - (2*(-1/4*((-5*b^2 + 12*a*c)*Cot[d + e*x]^2*Sqrt[a + b*Tan[d + e*x] + c*
Tan[d + e*x]^2])/a - (((-1/4*(b^2*(15*b^2 - 52*a*c)) + a*c*(5*b^2 - 12*a*c)
)*ArcTanh[(2*a + b*Tan[d + e*x])/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x] + c*Tan
[d + e*x]^2])])/(2*a^(3/2)) + (b*(15*b^2 - 52*a*c)*Cot[d + e*x]*Sqrt[a + b*
Tan[d + e*x] + c*Tan[d + e*x]^2])/(4*a))/(2*a)))/(a*(b^2 - 4*a*c)))/e
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int(cot(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x)
```

```
[Out] int(cot(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39674 vs. $2(922) = 1844$.

Time = 13.37 (sec) , antiderivative size = 79389, normalized size of antiderivative = 78.84

$$\int \frac{\cot^3(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\cot^3(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \int \frac{\cot^3(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(e*x+d)**3/(a+b*tan(e*x+d)+c*tan(e*x+d)**2)**(3/2),x)
```

```
[Out] Integral(cot(d + e*x)**3/(a + b*tan(d + e*x) + c*tan(d + e*x)**2)**(3/2), x
)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^3(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^3(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(e*x+d)^3/(a+b*tan(e*x+d)+c*tan(e*x+d)^2)^(3/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(d+ex)}{(a+b\tan(d+ex)+c\tan^2(d+ex))^{3/2}}\,dx = \text{Hanged}$$

```
[In] int(cot(d + e*x)^3/(a + b*tan(d + e*x) + c*tan(d + e*x)^2)^(3/2),x)
```

```
[Out] \text{Hanged}
```

3.27 $\int \tan^5(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 270

$$
\begin{aligned}
&\int \tan^5(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx \\
&= -\frac{\sqrt{a-b+c}\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} \\
&\quad+\frac{(b^3+2b^2c-4b(a-2c)c-8c^2(a+2c))\operatorname{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{32c^{5/2}e} \\
&\quad-\frac{((b-2c)(b+4c)+2c(b+2c)\tan^2(d+ex))\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{16c^2e} \\
&\quad+\frac{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}{6ce}
\end{aligned}
$$

```
[Out] 1/32*(b^3+2*b^2*c-4*b*(a-2*c)*c-8*c^2*(a+2*c))*arctanh(1/2*(b+2*c*tan(e*x+d
)^2)/c^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))/c^(5/2)/e-1/2*arctanh
(1/2*(2*a-b+(b-2*c)*tan(e*x+d)^2)/(a-b+c)^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x
+d)^4)^(1/2))*(a-b+c)^(1/2)/e-1/16*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*
((b-2*c)*(b+4*c)+2*c*(b+2*c)*tan(e*x+d)^2)/c^2/e+1/6*(a+b*tan(e*x+d)^2+c*ta
n(e*x+d)^4)^(3/2)/c/e
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3781, 1265, 1667, 828, 857, 635, 212, 738}

$$\int \tan^5(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$= \frac{(-4bc(a-2c)-8c^2(a+2c)+b^3+2b^2c)\operatorname{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{32c^{5/2}e}$$
$$-\frac{\sqrt{a-b+c}\operatorname{arctanh}\left(\frac{2a+(b-2c)\tan^2(d+ex)-b}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e}$$
$$-\frac{(2c(b+2c)\tan^2(d+ex)+(b-2c)(b+4c))\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{16c^2e}$$
$$+\frac{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}{6ce}$$

```
[In] Int[Tan[d + e*x]^5*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] -1/2*(Sqrt[a - b + c]*ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[
a - b + c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])])/e + ((b^3 + 2*b
^2*c - 4*b*(a - 2*c)*c - 8*c^2*(a + 2*c))*ArcTanh[(b + 2*c*Tan[d + e*x]^2)/
(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])])/(32*c^(5/2)*e)
- (((b - 2*c)*(b + 4*c) + 2*c*(b + 2*c)*Tan[d + e*x]^2)*Sqrt[a + b*Tan[d +
e*x]^2 + c*Tan[d + e*x]^4])/(16*c^2*e) + (a + b*Tan[d + e*x]^2 + c*Tan[d +
e*x]^4)^(3/2)/(6*c*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
```

```
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^5\sqrt{a+bx^2+cx^4}}{1+x^2}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx+cx^2}}{1+x}\,dx, x, \tan^2(d+ex)\right)}{2e}$$

$$= \frac{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}{6ce} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{3b}{2}-\frac{3}{2}(b+2c)x\right)\sqrt{a+bx+cx^2}}{1+x}\,dx, x, \tan^2(d+ex)\right)}{6ce}$$

$$= -\frac{\left((b-2c)(b+4c)+2c(b+2c)\tan^2(d+ex)\right)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{16c^2e} + \frac{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}{6ce} - \frac{\text{Subst}\left(\int \frac{-\frac{3}{4}(b-2c)\left(b^2-4ac+4bc\right)-\frac{3}{4}\left(b^3+2b^2c-4b(a-2c)c-8c^2(a+2c)\right)x}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{24c^2e}$$

$$= -\frac{\left((b-2c)(b+4c)+2c(b+2c)\tan^2(d+ex)\right)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{16c^2e} + \frac{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}{6ce} + \frac{(a-b+c)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e} + \frac{\left(b^3+2b^2c-4b(a-2c)c-8c^2(a+2c)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{32c^2e}$$

$$= -\frac{\left((b-2c)(b+4c)+2c(b+2c)\tan^2(d+ex)\right)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{16c^2e} + \frac{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}{6ce} - \frac{(a-b+c)\text{Subst}\left(\int \frac{1}{4a-4b+4c-x^2}\,dx, x, \frac{2a-b-(-b+2c)\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{e} + \frac{\left(b^3+2b^2c-4b(a-2c)c-8c^2(a+2c)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{16c^2e}$$

$$= -\frac{\sqrt{a-b+c}\,\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e}$$
$$+ \frac{(b^3+2b^2c-4b(a-2c)c-8c^2(a+2c))\,\text{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{32c^{5/2}e}$$
$$- \frac{((b-2c)(b+4c)+2c(b+2c)\tan^2(d+ex))\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{16c^2e}$$
$$+ \frac{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}{6ce}$$

Mathematica [A] (verified)

Time = 6.06 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.07

$$\int \tan^5(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$= \frac{-48c^{5/2}\sqrt{a-b+c}\,\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)+3(b^3+2b^2c-4b(a-2c)c-8c^2(a+2c)}{}$$

```
[In] Integrate[Tan[d + e*x]^5*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] (-48*c^(5/2)*Sqrt[a - b + c]*ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(
2*Sqrt[a - b + c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])] + 3*(b^3
+ 2*b^2*c - 4*b*(a - 2*c)*c - 8*c^2*(a + 2*c))*ArcTanh[(b + 2*c*Tan[d + e*x
]^2)/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])] + (Sqrt[c]*
(-9*b^2 + 24*a*c - 16*b*c + 84*c^2 - 4*(3*b^2 + 6*b*c - 8*c*(a + 2*c))*Cos[
2*(d + e*x)] + (-3*b^2 + 8*a*c - 8*b*c + 44*c^2)*Cos[4*(d + e*x)])*Sec[d +
e*x]^4*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])/4)/(96*c^(5/2)*e)
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.69

method	result
derivativedivides	$\frac{\frac{(a+b\tan(ex+d)^2+c\tan(ex+d)^4)^{\frac{3}{2}}}{6c}-\frac{b\left(\frac{(b+2c\tan(ex+d)^2)\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+c\tan(ex+d)^2}{\sqrt{c}}+\sqrt{}\right.}{8c^{\frac{3}{2}}}\right.}{4c}}{}$
default	$\frac{\frac{(a+b\tan(ex+d)^2+c\tan(ex+d)^4)^{\frac{3}{2}}}{6c}-\frac{b\left(\frac{(b+2c\tan(ex+d)^2)\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+c\tan(ex+d)^2}{\sqrt{c}}+\sqrt{}\right.}{8c^{\frac{3}{2}}}\right.}{4c}}{}$

```
[In] int((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)^5,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/e*(1/6*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2)/c-1/4*b/c*(1/4*(b+2*c*tan(
e*x+d)^2)/c*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)
*ln((1/2*b+c*tan(e*x+d)^2)/c^(1/2)+(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))
)-1/8*(b+2*c*tan(e*x+d)^2)/c*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)-1/16*(
4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*tan(e*x+d)^2)/c^(1/2)+(a+b*tan(e*x+d)^2+c*ta
n(e*x+d)^4)^(1/2))+1/2*(c*(1+tan(e*x+d)^2)^2+(b-2*c)*(1+tan(e*x+d)^2)+a-b+c
)^(1/2)+1/4*(b-2*c)*ln((1/2*b-c+(1+tan(e*x+d)^2)*c)/c^(1/2)+(c*(1+tan(e*x+d
)^2)^2+(b-2*c)*(1+tan(e*x+d)^2)+a-b+c)^(1/2))/c^(1/2)-1/2*(a-b+c)^(1/2)*ln(
(2*a-2*b+2*c+(b-2*c)*(1+tan(e*x+d)^2)+2*(a-b+c)^(1/2)*(c*(1+tan(e*x+d)^2)^2
+(b-2*c)*(1+tan(e*x+d)^2)+a-b+c)^(1/2))/(1+tan(e*x+d)^2)))
```

Fricas [A] (verification not implemented)

none

Time = 2.89 (sec) , antiderivative size = 1405, normalized size of antiderivative = 5.20

$$\int \tan^5(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)^5,x, algorithm
="fricas")
```

```
[Out] [1/192*(48*sqrt(a - b + c)*c^3*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x +
 d)^4 + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 - 4*sqrt(c*tan(e*x +
 d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a -
 b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2 +
 1)) - 3*(b^3 - 8*(a - b)*c^2 - 16*c^3 - 2*(2*a*b - b^2)*c)*sqrt(c)*log(8*c
^2*tan(e*x + d)^4 + 8*b*c*tan(e*x + d)^2 + b^2 - 4*sqrt(c*tan(e*x + d)^4 +
b*tan(e*x + d)^2 + a)*(2*c*tan(e*x + d)^2 + b)*sqrt(c) + 4*a*c) + 4*(8*c^3*
tan(e*x + d)^4 - 3*b^2*c + 2*(4*a - 3*b)*c^2 + 24*c^3 + 2*(b*c^2 - 6*c^3)*t
an(e*x + d)^2)*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a))/(c^3*e), 1/96
*(24*sqrt(a - b + c)*c^3*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x + d)^4
+ 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 - 4*sqrt(c*tan(e*x + d)^4
+ b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a - b + c
) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2 + 1)) -
 3*(b^3 - 8*(a - b)*c^2 - 16*c^3 - 2*(2*a*b - b^2)*c)*sqrt(-c)*arctan(1/2*s
qrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*tan(e*x + d)^2 + b)*sqrt(
-c)/(c^2*tan(e*x + d)^4 + b*c*tan(e*x + d)^2 + a*c)) + 2*(8*c^3*tan(e*x + d
)^4 - 3*b^2*c + 2*(4*a - 3*b)*c^2 + 24*c^3 + 2*(b*c^2 - 6*c^3)*tan(e*x + d)
^2)*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a))/(c^3*e), -1/192*(96*sqrt
(-a + b - c)*c^3*arctan(-1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*
((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(-a + b - c)/(((a - b)*c + c^2)*ta
```

```
n(e*x + d)^4 + (a*b - b^2 + b*c)*tan(e*x + d)^2 + a^2 - a*b + a*c)) + 3*(b^
3 - 8*(a - b)*c^2 - 16*c^3 - 2*(2*a*b - b^2)*c)*sqrt(c)*log(8*c^2*tan(e*x +
 d)^4 + 8*b*c*tan(e*x + d)^2 + b^2 - 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x +
d)^2 + a)*(2*c*tan(e*x + d)^2 + b)*sqrt(c) + 4*a*c) - 4*(8*c^3*tan(e*x + d)
^4 - 3*b^2*c + 2*(4*a - 3*b)*c^2 + 24*c^3 + 2*(b*c^2 - 6*c^3)*tan(e*x + d)^
2)*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a))/(c^3*e), -1/96*(48*sqrt(-
a + b - c)*c^3*arctan(-1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((
b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(-a + b - c)/(((a - b)*c + c^2)*tan(
e*x + d)^4 + (a*b - b^2 + b*c)*tan(e*x + d)^2 + a^2 - a*b + a*c)) + 3*(b^3
- 8*(a - b)*c^2 - 16*c^3 - 2*(2*a*b - b^2)*c)*sqrt(-c)*arctan(1/2*sqrt(c*ta
n(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*tan(e*x + d)^2 + b)*sqrt(-c)/(c^2
*tan(e*x + d)^4 + b*c*tan(e*x + d)^2 + a*c)) - 2*(8*c^3*tan(e*x + d)^4 - 3*
b^2*c + 2*(4*a - 3*b)*c^2 + 24*c^3 + 2*(b*c^2 - 6*c^3)*tan(e*x + d)^2)*sqrt
(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a))/(c^3*e)]
```

Sympy [F]

$$\int \tan^5(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \int \sqrt{a+b\tan^2\left(d+ex\right)+c\tan^4\left(d+ex\right)}\tan^5\left(d+ex\right)dx$$

```
[In] integrate((a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2)*tan(e*x+d)**5,x)
[Out] Integral(sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4)*tan(d + e*x)**5, x
)
```

Maxima [F]

$$\int \tan^5(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \int \sqrt{c\tan\left(ex+d\right)^4+b\tan\left(ex+d\right)^2+a}\tan\left(ex+d\right)^5\,dx$$

```
[In] integrate((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)^5,x, algorithm
="maxima")
[Out] integrate(sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*tan(e*x + d)^5, x)
```

Giac [F(-1)]

Timed out.

$$\int \tan^5(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)^5,x, algorithm
="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\begin{aligned}&\int \tan^5(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx\\&=\int \tan(d+e\,x)^5\,\sqrt{c\tan\left(d+e\,x\right)^4+b\tan\left(d+e\,x\right)^2+a}\,dx\end{aligned}$$

```
[In] int(tan(d + e*x)^5*(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)
[Out] int(tan(d + e*x)^5*(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)
```

3.28 $\int \tan^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 209

$$\int \tan^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \frac{\sqrt{a-b+c}\,\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} - \frac{(b^2+4bc-4c(a+2c))\,\text{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{16c^{3/2}e} + \frac{(b-4c+2c\tan^2(d+ex))\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{8ce}$$

```
[Out] -1/16*(b^2+4*b*c-4*c*(a+2*c))*arctanh(1/2*(b+2*c*tan(e*x+d)^2)/c^(1/2)/(a+b
*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))/c^(3/2)/e+1/2*arctanh(1/2*(2*a-b+(b-2*
c)*tan(e*x+d)^2)/(a-b+c)^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))*(a-
b+c)^(1/2)/e+1/8*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*(b-4*c+2*c*tan(e*x
+d)^2)/c/e
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

= {3781, 1265, 828, 857, 635, 212, 738}

$$\int \tan^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$= -\frac{(-4c(a+2c)+b^2+4bc)\operatorname{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{16c^{3/2}e}$$
$$+ \frac{\sqrt{a-b+c}\operatorname{arctanh}\left(\frac{2a+(b-2c)\tan^2(d+ex)-b}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e}$$
$$+ \frac{(b+2c\tan^2(d+ex)-4c)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{8ce}$$

```
[In] Int[Tan[d + e*x]^3*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] (Sqrt[a - b + c]*ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b
 + c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])])/(2*e) - ((b^2 + 4*b*
c - 4*c*(a + 2*c))*ArcTanh[(b + 2*c*Tan[d + e*x]^2)/(2*Sqrt[c]*Sqrt[a + b*T
an[d + e*x]^2 + c*Tan[d + e*x]^4])])/(16*c^(3/2)*e) + ((b - 4*c + 2*c*Tan[d
 + e*x]^2)*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])/(8*c*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
```

```
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^3\sqrt{a+bx^2+cx^4}}{1+x^2}\,dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx+cx^2}}{1+x}\,dx, x, \tan^2(d+ex)\right)}{2e} \\
&= \frac{(b-4c+2c\tan^2(d+ex))\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{8ce} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(b^2+4ac-4bc)+\frac{1}{2}(b^2+4bc-4c(a+2c))x}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{8ce}
\end{aligned}$$

$$= \frac{(b-4c+2c\tan^2(d+ex))\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{8ce} - \frac{(a-b+c)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e} - \frac{(b^2+4bc-4c(a+2c))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{16ce}$$

$$= \frac{(b-4c+2c\tan^2(d+ex))\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{8ce} + \frac{(a-b+c)\text{Subst}\left(\int \frac{1}{4a-4b+4c-x^2}\,dx, x, \frac{2a-b-(-b+2c)\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{e} - \frac{(b^2+4bc-4c(a+2c))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{8ce}$$

$$= \frac{\sqrt{a-b+c}\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} - \frac{(b^2+4bc-4c(a+2c))\,\text{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{16c^{3/2}e} + \frac{(b-4c+2c\tan^2(d+ex))\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{8ce}$$

Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00

$$\int \tan^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$

$$= \frac{8c^{3/2}\sqrt{a-b+c}\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right) - (b^2+4bc-4c(a+2c))\,\text{arctanh}\left(\frac{b+2c}{2\sqrt{c}\sqrt{a+b\tan^2}}\right.}{16c^{3/2}e}$$

```
[In] Integrate[Tan[d + e*x]^3*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] (8*c^(3/2)*Sqrt[a - b + c]*ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*
Sqrt[a - b + c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])] - (b^2 + 4*
b*c - 4*c*(a + 2*c))*ArcTanh[(b + 2*c*Tan[d + e*x]^2)/(2*Sqrt[c]*Sqrt[a + b
*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])] + 2*Sqrt[c]*(b - 4*c + 2*c*Tan[d + e*
x]^2)*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])/(16*c^(3/2)*e)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.52

method	result
derivativedivides	$\frac{\frac{\left(b+2c\tan(ex+d)^2\right)\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}{8c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+c\tan(ex+d)^2}{\sqrt{c}}+\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}\right)}{16c^{\frac{3}{2}}}-\sqrt{c\left(1-\right.}}{}$
default	$\frac{\frac{\left(b+2c\tan(ex+d)^2\right)\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}{8c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+c\tan(ex+d)^2}{\sqrt{c}}+\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}\right)}{16c^{\frac{3}{2}}}-\sqrt{c\left(1-\right.}}{}$

```
[In] int((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/e*(1/8*(b+2*c*tan(e*x+d)^2)/c*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)+1/1
6*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*tan(e*x+d)^2)/c^(1/2)+(a+b*tan(e*x+d)^2+c
*tan(e*x+d)^4)^(1/2))-1/2*(c*(1+tan(e*x+d)^2)^2+(b-2*c)*(1+tan(e*x+d)^2)+a-
b+c)^(1/2)-1/4*(b-2*c)*ln((1/2*b-c+(1+tan(e*x+d)^2)*c)/c^(1/2)+(c*(1+tan(e*
x+d)^2)^2+(b-2*c)*(1+tan(e*x+d)^2)+a-b+c)^(1/2))/c^(1/2)+1/2*(a-b+c)^(1/2)*
ln((2*a-2*b+2*c+(b-2*c)*(1+tan(e*x+d)^2)+2*(a-b+c)^(1/2)*(c*(1+tan(e*x+d)^2
)^2+(b-2*c)*(1+tan(e*x+d)^2)+a-b+c)^(1/2))/(1+tan(e*x+d)^2)))
```

Fricas [A] (verification not implemented)

none

Time = 2.25 (sec) , antiderivative size = 1199, normalized size of antiderivative = 5.74

$$\int \tan^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)^3,x, algorithm
="fricas")
```

```
[Out] [1/32*(8*sqrt(a - b + c)*c^2*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x + d
)^4 + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 + 4*sqrt(c*tan(e*x + d
)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a - b
 + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2 + 1
)) - (b^2 - 4*(a - b)*c - 8*c^2)*sqrt(c)*log(8*c^2*tan(e*x + d)^4 + 8*b*c*t
an(e*x + d)^2 + b^2 + 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*
tan(e*x + d)^2 + b)*sqrt(c) + 4*a*c) + 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x
+ d)^2 + a)*(2*c^2*tan(e*x + d)^2 + b*c - 4*c^2))/(c^2*e), 1/16*(4*sqrt(a -
 b + c)*c^2*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x + d)^4 + 2*(4*a*b -
3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 + 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x +
```

```
d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8
*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2 + 1)) + (b^2 - 4*(a
- b)*c - 8*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^
2 + a)*(2*c*tan(e*x + d)^2 + b)*sqrt(-c)/(c^2*tan(e*x + d)^4 + b*c*tan(e*x
+ d)^2 + a*c)) + 2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c^2*tan
(e*x + d)^2 + b*c - 4*c^2))/(c^2*e), 1/32*(16*sqrt(-a + b - c)*c^2*arctan(-
1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2
 + 2*a - b)*sqrt(-a + b - c)/(((a - b)*c + c^2)*tan(e*x + d)^4 + (a*b - b^2
 + b*c)*tan(e*x + d)^2 + a^2 - a*b + a*c)) - (b^2 - 4*(a - b)*c - 8*c^2)*sq
rt(c)*log(8*c^2*tan(e*x + d)^4 + 8*b*c*tan(e*x + d)^2 + b^2 + 4*sqrt(c*tan(
e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*tan(e*x + d)^2 + b)*sqrt(c) + 4*a*c
) + 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c^2*tan(e*x + d)^2 +
 b*c - 4*c^2))/(c^2*e), 1/16*(8*sqrt(-a + b - c)*c^2*arctan(-1/2*sqrt(c*tan
(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sq
rt(-a + b - c)/(((a - b)*c + c^2)*tan(e*x + d)^4 + (a*b - b^2 + b*c)*tan(e*
x + d)^2 + a^2 - a*b + a*c)) + (b^2 - 4*(a - b)*c - 8*c^2)*sqrt(-c)*arctan(
1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*tan(e*x + d)^2 + b)*
sqrt(-c)/(c^2*tan(e*x + d)^4 + b*c*tan(e*x + d)^2 + a*c)) + 2*sqrt(c*tan(e*
x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c^2*tan(e*x + d)^2 + b*c - 4*c^2))/(c^2
*e)]
```

Sympy [F]

$$\int \tan^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int\sqrt{a+b\tan^2{(d+ex)}+c\tan^4{(d+ex)}}\tan^3{(d+ex)}\,dx$$

```
[In] integrate((a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2)*tan(e*x+d)**3,x)
[Out] Integral(sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4)*tan(d + e*x)**3, x
)
```

Maxima [F]

$$\int \tan^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int\sqrt{c\tan(ex+d)^4+b\tan(ex+d)^2+a}\tan(ex+d)^3\,dx$$

```
[In] integrate((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)^3,x, algorithm
="maxima")
[Out] integrate(sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*tan(e*x + d)^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \tan^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)^3,x, algorithm
="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \tan^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int \tan(d+e\,x)^3\,\sqrt{c\tan\left(d+e\,x\right)^4+b\tan\left(d+e\,x\right)^2+a}\,dx$$

```
[In] int(tan(d + e*x)^3*(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)
[Out] int(tan(d + e*x)^3*(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)
```

3.29 $\int \tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 179

$$\int \tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = -\frac{\sqrt{a-b+c}\,\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} + \frac{(b-2c)\text{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4\sqrt{c}e} + \frac{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2e}$$

```
[Out] 1/4*(b-2*c)*arctanh(1/2*(b+2*c*tan(e*x+d)^2)/c^(1/2)/(a+b*tan(e*x+d)^2+c*ta
n(e*x+d)^4)^(1/2))/e/c^(1/2)-1/2*arctanh(1/2*(2*a-b+(b-2*c)*tan(e*x+d)^2)/(
a-b+c)^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))*(a-b+c)^(1/2)/e+1/2*(
a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used

= {3781, 1261, 748, 857, 635, 212, 738}

$$\int \tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$= -\frac{\sqrt{a-b+c}\,\text{arctanh}\left(\frac{2a+(b-2c)\tan^2(d+ex)-b}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} + \frac{(b-2c)\text{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4\sqrt{c}e} + \frac{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2e}$$

```
[In] Int[Tan[d + e*x]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] -1/2*(Sqrt[a - b + c]*ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[
a - b + c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])])/e + ((b - 2*c)*
ArcTanh[(b + 2*c*Tan[d + e*x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x]^2 + c*T
an[d + e*x]^4])])/(4*Sqrt[c]*e) + Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x
]^4]/(2*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
```

```
& !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx^2+cx^4}}{1+x^2}\,dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{1+x}\,dx, x, \tan^2(d+ex)\right)}{2e} \\
&= \frac{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2e} - \frac{\text{Subst}\left(\int \frac{-2a+b-(b-2c)x}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{4e} \\
&= \frac{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2e} \\
&\quad + \frac{(b-2c)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{4e} \\
&\quad + \frac{(a-b+c)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e}
\end{aligned}$$

$$= \frac{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2e} + \frac{(b-2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} - \frac{(a-b+c)\text{Subst}\left(\int \frac{1}{4a-4b+4c-x^2}\,dx, x, \frac{2a-b-(-b+2c)\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{e}$$

$$= -\frac{\sqrt{a-b+c}\,\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} + \frac{(b-2c)\text{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4\sqrt{c}e} + \frac{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2e}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01

$$\int \tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \frac{-2\sqrt{c}\sqrt{a-b+c}\,\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right) + (b-2c)\text{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d-}\right.}{4\sqrt{c}e}$$

```
[In] Integrate[Tan[d + e*x]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[a - b + c]*ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2
*Sqrt[a - b + c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])] + (b - 2*c
)*ArcTanh[(b + 2*c*Tan[d + e*x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x]^2 + c
*Tan[d + e*x]^4])] + 2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4
])/(4*Sqrt[c]*e)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{\sqrt{c\left(1+\tan(ex+d)^2\right)^2+(b-2c)\left(1+\tan(ex+d)^2\right)+a-b+c}}{2}+\frac{(b-2c)\ln\left(\frac{\frac{b}{2}-c+\left(1+\tan(ex+d)^2\right)c}{\sqrt{c}}+\sqrt{c\left(1+\tan(ex+d)^2\right)^2+(b-2c)\left(1+\tan(\right.}\right.}{4\sqrt{c}}$
default	$\frac{\sqrt{c\left(1+\tan(ex+d)^2\right)^2+(b-2c)\left(1+\tan(ex+d)^2\right)+a-b+c}}{2}+\frac{(b-2c)\ln\left(\frac{\frac{b}{2}-c+\left(1+\tan(ex+d)^2\right)c}{\sqrt{c}}+\sqrt{c\left(1+\tan(ex+d)^2\right)^2+(b-2c)\left(1+\tan(\right.}\right.}{4\sqrt{c}}$

[In] int((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d),x,method=_RETURNVERB
OSE)

[Out] 1/e*(1/2*(c*(1+tan(e*x+d)^2)^2+(b-2*c)*(1+tan(e*x+d)^2)+a-b+c)^(1/2)+1/4*(b
-2*c)*ln((1/2*b-c+(1+tan(e*x+d)^2)*c)/c^(1/2)+(c*(1+tan(e*x+d)^2)^2+(b-2*c)
*(1+tan(e*x+d)^2)+a-b+c)^(1/2))/c^(1/2)-1/2*(a-b+c)^(1/2)*ln((2*a-2*b+2*c+(
b-2*c)*(1+tan(e*x+d)^2)+2*(a-b+c)^(1/2)*(c*(1+tan(e*x+d)^2)^2+(b-2*c)*(1+ta
n(e*x+d)^2)+a-b+c)^(1/2))/(1+tan(e*x+d)^2)))

Fricas [A] (verification not implemented)

none

Time = 1.70 (sec) , antiderivative size = 1057, normalized size of antiderivative = 5.91

$$\int \tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \text{Too large to display}$$

[In] integrate((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d),x, algorithm="
fricas")

[Out] [-1/8*((b - 2*c)*sqrt(c)*log(8*c^2*tan(e*x + d)^4 + 8*b*c*tan(e*x + d)^2 +
b^2 - 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*tan(e*x + d)^2 +
b)*sqrt(c) + 4*a*c) - 2*sqrt(a - b + c)*c*log(((b^2 + 4*(a - 2*b)*c + 8*c^
2)*tan(e*x + d)^4 + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 - 4*sqrt
(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a -
b)*sqrt(a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(
e*x + d)^2 + 1)) - 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*c)/(c*e)
, -1/4*((b - 2*c)*sqrt(-c)*arctan(1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d
)^2 + a)*(2*c*tan(e*x + d)^2 + b)*sqrt(-c)/(c^2*tan(e*x + d)^4 + b*c*tan(e*
x + d)^2 + a*c)) - sqrt(a - b + c)*c*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan
(e*x + d)^4 + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 - 4*sqrt(c*tan
(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sq
rt(a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x +
d)^2 + 1)) - 2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*c)/(c*e), -1/8
*(4*sqrt(-a + b - c)*c*arctan(-1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2
+ a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(-a + b - c)/(((a - b)*c + c

```
^2)*tan(e*x + d)^4 + (a*b - b^2 + b*c)*tan(e*x + d)^2 + a^2 - a*b + a*c)) +
 (b - 2*c)*sqrt(c)*log(8*c^2*tan(e*x + d)^4 + 8*b*c*tan(e*x + d)^2 + b^2 -
4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*tan(e*x + d)^2 + b)*sq
rt(c) + 4*a*c) - 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*c)/(c*e),
-1/4*(2*sqrt(-a + b - c)*c*arctan(-1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x +
d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(-a + b - c)/(((a - b)*c
 + c^2)*tan(e*x + d)^4 + (a*b - b^2 + b*c)*tan(e*x + d)^2 + a^2 - a*b + a*c
)) + (b - 2*c)*sqrt(-c)*arctan(1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2
 + a)*(2*c*tan(e*x + d)^2 + b)*sqrt(-c)/(c^2*tan(e*x + d)^4 + b*c*tan(e*x +
 d)^2 + a*c)) - 2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*c)/(c*e)]
```

Sympy [F]

$$\int \tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \int \sqrt{a+b\tan^2{(d+ex)}+c\tan^4{(d+ex)}}\tan{(d+ex)}\,dx$$

```
[In] integrate((a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2)*tan(e*x+d),x)
[Out] Integral(sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4)*tan(d + e*x), x)
```

Maxima [F]

$$\int \tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \int \sqrt{c\tan(ex+d)^4+b\tan(ex+d)^2+a}\tan(ex+d)\,dx$$

```
[In] integrate((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d),x, algorithm="
maxima")
[Out] integrate(sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*tan(e*x + d), x)
```

Giac [F(-1)]

Timed out.

$$\int \tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$= \int \tan(d+e\,x)\,\sqrt{c\tan\left(d+e\,x\right)^4+b\tan\left(d+e\,x\right)^2+a}\,dx$$

```
[In] int(tan(d + e*x)*(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)
```

```
[Out] int(tan(d + e*x)*(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)
```

3.30 $\int \cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 203

$$
\begin{aligned}
&\int \cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx \\
&= -\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} \\
&\quad+\frac{\sqrt{a-b+c}\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} \\
&\quad+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e}
\end{aligned}
$$

```
[Out] -1/2*arctanh(1/2*(2*a+b*tan(e*x+d)^2)/a^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d
)^4)^(1/2))*a^(1/2)/e+1/2*arctanh(1/2*(b+2*c*tan(e*x+d)^2)/c^(1/2)/(a+b*tan
(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))*c^(1/2)/e+1/2*arctanh(1/2*(2*a-b+(b-2*c)*t
an(e*x+d)^2)/(a-b+c)^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))*(a-b+c)
^(1/2)/e
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used

$= \{3781, 1265, 909, 738, 212, 857, 635\}$

$$\int \cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$= -\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} + \frac{\sqrt{a-b+c}\text{arctanh}\left(\frac{2a+(b-2c)\tan^2(d+ex)-b}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} + \frac{\sqrt{c}\text{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e}$$

```
[In] Int[Cot[d + e*x]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] -1/2*(Sqrt[a]*ArcTanh[(2*a + b*Tan[d + e*x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tan[d
+ e*x]^2 + c*Tan[d + e*x]^4])])/e + (Sqrt[a - b + c]*ArcTanh[(2*a - b + (b
- 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan
[d + e*x]^4])])/(2*e) + (Sqrt[c]*ArcTanh[(b + 2*c*Tan[d + e*x]^2)/(2*Sqrt[c
]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])])/(2*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 909

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p -
1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2+cx^4}}{x(1+x^2)}\,dx, x, \tan(d+ex)\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x(1+x)}\,dx, x, \tan^2(d+ex)\right)}{2e}\\
&= -\frac{\text{Subst}\left(\int \frac{a-b-cx}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e}+\frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e}\\
&= -\frac{a\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{e}\\
&\quad+\frac{c\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e}\\
&\quad-\frac{(a-b+c)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e}
\end{aligned}$$

$$= -\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} + \frac{c\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{e} + \frac{(a-b+c)\text{Subst}\left(\int \frac{1}{4a-4b+4c-x^2}\,dx, x, \frac{2a-b-(-b+2c)\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{e}$$

$$= -\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} + \frac{\sqrt{a-b+c}\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} + \frac{\sqrt{c}\operatorname{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e}$$

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.95

$$\int \cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = -\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right) + \sqrt{a-b+c}\operatorname{arctanh}\left(\frac{-2a+b-(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right) - \sqrt{}}{2e}$$

```
[In] Integrate[Cot[d + e*x]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] -1/2*(Sqrt[a]*ArcTanh[(2*a + b*Tan[d + e*x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tan[d
+ e*x]^2 + c*Tan[d + e*x]^4])] + Sqrt[a - b + c]*ArcTanh[(-2*a + b - (b - 2
*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d
+ e*x]^4])] - Sqrt[c]*ArcTanh[(b + 2*c*Tan[d + e*x]^2)/(2*Sqrt[c]*Sqrt[a +
b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])])/e
```

Maple [F]

$$\int \cot(ex+d)\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}\,dx$$

```
[In] int(cot(e*x+d)*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x)
```

```
[Out] int(cot(e*x+d)*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 2097, normalized size of antiderivative = 10.33

$$\int \cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(e*x+d)*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*(sqrt(c)*log(8*c^2*tan(e*x + d)^4 + 8*b*c*tan(e*x + d)^2 + b^2 + 4*sqr
t(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*tan(e*x + d)^2 + b)*sqrt(c)
 + 4*a*c) + sqrt(a - b + c)*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x + d)
^4 + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 + 4*sqrt(c*tan(e*x + d)
^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a - b
+ c) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2 + 1)
) + sqrt(a)*log(((b^2 + 4*a*c)*tan(e*x + d)^4 + 8*a*b*tan(e*x + d)^2 - 4*sq
rt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(b*tan(e*x + d)^2 + 2*a)*sqrt(a
) + 8*a^2)/tan(e*x + d)^4))/e, -1/4*(2*sqrt(-c)*arctan(2*sqrt(c*tan(e*x + d
)^4 + b*tan(e*x + d)^2 + a)*sqrt(-c)/(2*c*tan(e*x + d)^2 + b)) - sqrt(a - b
 + c)*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x + d)^4 + 2*(4*a*b - 3*b^2
- 4*(a - b)*c)*tan(e*x + d)^2 + 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2
+ a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b +
 b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2 + 1)) - sqrt(a)*log(((b^2
+ 4*a*c)*tan(e*x + d)^4 + 8*a*b*tan(e*x + d)^2 - 4*sqrt(c*tan(e*x + d)^4 +
b*tan(e*x + d)^2 + a)*(b*tan(e*x + d)^2 + 2*a)*sqrt(a) + 8*a^2)/tan(e*x + d
)^4))/e, 1/4*(2*sqrt(-a)*arctan(2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2
+ a)*sqrt(-a)/(b*tan(e*x + d)^2 + 2*a)) + sqrt(c)*log(8*c^2*tan(e*x + d)^4
+ 8*b*c*tan(e*x + d)^2 + b^2 + 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 +
 a)*(2*c*tan(e*x + d)^2 + b)*sqrt(c) + 4*a*c) + sqrt(a - b + c)*log(((b^2 +
 4*(a - 2*b)*c + 8*c^2)*tan(e*x + d)^4 + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*ta
n(e*x + d)^2 + 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*t
an(e*x + d)^2 + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(ta
n(e*x + d)^4 + 2*tan(e*x + d)^2 + 1)))/e, 1/4*(2*sqrt(-a)*arctan(2*sqrt(c*t
an(e*x + d)^4 + b*tan(e*x + d)^2 + a)*sqrt(-a)/(b*tan(e*x + d)^2 + 2*a)) -
2*sqrt(-c)*arctan(2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*sqrt(-c)/
(2*c*tan(e*x + d)^2 + b)) + sqrt(a - b + c)*log(((b^2 + 4*(a - 2*b)*c + 8*c
^2)*tan(e*x + d)^4 + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 + 4*sqr
t(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a
- b)*sqrt(a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan
(e*x + d)^2 + 1)))/e, 1/4*(2*sqrt(-a + b - c)*arctan(-2*sqrt(c*tan(e*x + d)
^4 + b*tan(e*x + d)^2 + a)*sqrt(-a + b - c)/((b - 2*c)*tan(e*x + d)^2 + 2*a
 - b)) + sqrt(c)*log(8*c^2*tan(e*x + d)^4 + 8*b*c*tan(e*x + d)^2 + b^2 + 4*
sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*tan(e*x + d)^2 + b)*sqrt
```

```
(c) + 4*a*c) + sqrt(a)*log(((b^2 + 4*a*c)*tan(e*x + d)^4 + 8*a*b*tan(e*x +
d)^2 - 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(b*tan(e*x + d)^2 +
2*a)*sqrt(a) + 8*a^2)/tan(e*x + d)^4))/e, 1/4*(2*sqrt(-a + b - c)*arctan(-2
*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*sqrt(-a + b - c)/((b - 2*c)*
tan(e*x + d)^2 + 2*a - b)) - 2*sqrt(-c)*arctan(2*sqrt(c*tan(e*x + d)^4 + b*
tan(e*x + d)^2 + a)*sqrt(-c)/(2*c*tan(e*x + d)^2 + b)) + sqrt(a)*log(((b^2
+ 4*a*c)*tan(e*x + d)^4 + 8*a*b*tan(e*x + d)^2 - 4*sqrt(c*tan(e*x + d)^4 +
b*tan(e*x + d)^2 + a)*(b*tan(e*x + d)^2 + 2*a)*sqrt(a) + 8*a^2)/tan(e*x + d
)^4))/e, 1/4*(2*sqrt(-a)*arctan(2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2
+ a)*sqrt(-a)/(b*tan(e*x + d)^2 + 2*a)) + 2*sqrt(-a + b - c)*arctan(-2*sqrt
(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*sqrt(-a + b - c)/((b - 2*c)*tan(e
*x + d)^2 + 2*a - b)) + sqrt(c)*log(8*c^2*tan(e*x + d)^4 + 8*b*c*tan(e*x +
d)^2 + b^2 + 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*tan(e*x +
 d)^2 + b)*sqrt(c) + 4*a*c))/e, 1/2*(sqrt(-a)*arctan(2*sqrt(c*tan(e*x + d)^
4 + b*tan(e*x + d)^2 + a)*sqrt(-a)/(b*tan(e*x + d)^2 + 2*a)) + sqrt(-a + b
- c)*arctan(-2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*sqrt(-a + b -
c)/((b - 2*c)*tan(e*x + d)^2 + 2*a - b)) - sqrt(-c)*arctan(2*sqrt(c*tan(e*x
 + d)^4 + b*tan(e*x + d)^2 + a)*sqrt(-c)/(2*c*tan(e*x + d)^2 + b)))/e]
```

Sympy [F]

$$\int \cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \int \sqrt{a+b\tan^2{(d+ex)}+c\tan^4{(d+ex)}}\cot{(d+ex)}\,dx$$

```
[In] integrate(cot(e*x+d)*(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2),x)
[Out] Integral(sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4)*cot(d + e*x), x)
```

Maxima [F]

$$\int \cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \int \sqrt{c\tan\left(ex+d\right)^4+b\tan\left(ex+d\right)^2+a}\cot\left(ex+d\right)\,dx$$

```
[In] integrate(cot(e*x+d)*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm="
maxima")
[Out] integrate(sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*cot(e*x + d), x)
```

Giac [F]

$$\int \cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int \sqrt{c\tan\left(ex+d\right)^4+b\tan\left(ex+d\right)^2+a}\cot\left(ex+d\right)\,dx$$

[In] integrate(cot(e*x+d)*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*cot(e*x + d), x)

Mupad [F(-1)]

Timed out.

$$\int \cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int \cot(d+e\,x)\,\sqrt{c\tan\left(d+e\,x\right)^4+b\tan\left(d+e\,x\right)^2+a}\,dx$$

[In] int(cot(d + e*x)*(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)

[Out] int(cot(d + e*x)*(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)

3.31 $\int \cot^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 435

$$\begin{aligned}
&\int \cot^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx\\
&=\frac{\sqrt{a}\,\text{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e}-\frac{b\,\text{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4\sqrt{a}e}\\
&-\frac{\sqrt{a-b+c}\,\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e}\\
&-\frac{b\,\text{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4\sqrt{c}e}\\
&+\frac{(b-2c)\text{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4\sqrt{c}e}\\
&+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e}\\
&-\frac{\cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2e}
\end{aligned}$$

```
[Out] -1/4*b*arctanh(1/2*(2*a+b*tan(e*x+d)^2)/a^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x
+d)^4)^(1/2))/e/a^(1/2)+1/2*arctanh(1/2*(2*a+b*tan(e*x+d)^2)/a^(1/2)/(a+b*t
an(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))*a^(1/2)/e-1/4*b*arctanh(1/2*(b+2*c*tan(e
*x+d)^2)/c^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))/e/c^(1/2)+1/4*(b-
2*c)*arctanh(1/2*(b+2*c*tan(e*x+d)^2)/c^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d
)^4)^(1/2))/e/c^(1/2)+1/2*arctanh(1/2*(b+2*c*tan(e*x+d)^2)/c^(1/2)/(a+b*tan
(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))*c^(1/2)/e-1/2*arctanh(1/2*(2*a-b+(b-2*c)*t
an(e*x+d)^2)/(a-b+c)^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))*(a-b+c)
^(1/2)/e-1/2*cot(e*x+d)^2*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {3781, 1265, 974, 746, 857, 635, 212, 738, 748}

$$\int \cot^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$= -\frac{b\,\text{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4\sqrt{a}e}+\frac{\sqrt{a}\,\text{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e}$$
$$-\frac{\sqrt{a-b+c}\,\text{arctanh}\left(\frac{2a+(b-2c)\tan^2(d+ex)-b}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e}$$
$$+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e}-\frac{b\,\text{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4\sqrt{c}e}$$
$$+\frac{(b-2c)\,\text{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4\sqrt{c}e}$$
$$-\frac{\cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2e}$$

```
[In] Int[Cot[d + e*x]^3*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] (Sqrt[a]*ArcTanh[(2*a + b*Tan[d + e*x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x
]^2 + c*Tan[d + e*x]^4])])/(2*e) - (b*ArcTanh[(2*a + b*Tan[d + e*x]^2)/(2*S
qrt[a]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])])/(4*Sqrt[a]*e) - (Sq
rt[a - b + c]*ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b +
c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])])/(2*e) - (b*ArcTanh[(b +
 2*c*Tan[d + e*x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^
4])])/(4*Sqrt[c]*e) + ((b - 2*c)*ArcTanh[(b + 2*c*Tan[d + e*x]^2)/(2*Sqrt[c
]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])])/(4*Sqrt[c]*e) + (Sqrt[c]
*ArcTanh[(b + 2*c*Tan[d + e*x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x]^2 + c*
Tan[d + e*x]^4])])/(2*e) - (Cot[d + e*x]^2*Sqrt[a + b*Tan[d + e*x]^2 + c*Ta
n[d + e*x]^4])/(2*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(1+x^2)}\,dx, x, \tan(d+ex)\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2(1+x)}\,dx, x, \tan^2(d+ex)\right)}{2e}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\sqrt{a+bx+cx^2}}{x^2}-\frac{\sqrt{a+bx+cx^2}}{x}+\frac{\sqrt{a+bx+cx^2}}{1+x}\right)\,dx, x, \tan^2(d+ex)\right)}{2e}\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx, x, \tan^2(d+ex)\right)}{2e}-\frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, \tan^2(d+ex)\right)}{2e}\\
&\quad+\frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{1+x}\,dx, x, \tan^2(d+ex)\right)}{2e}\\
&= -\frac{\cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2e}\\
&\quad+\frac{\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{4e}\\
&\quad-\frac{\text{Subst}\left(\int \frac{-2a+b-(b-2c)x}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{4e}\\
&\quad+\frac{\text{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{4e}
\end{aligned}$$

$$= -\frac{\cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2e} - \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{4e} + \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{4e} + \frac{(b-2c)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{4e} + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e} + \frac{(a-b+c)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e}$$

$$= -\frac{\cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2e} + \frac{a\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{e} - \frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} - \frac{b\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} + \frac{(b-2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} + \frac{c\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{e} - \frac{(a-b+c)\text{Subst}\left(\int \frac{1}{4a-4b+4c-x^2}\,dx, x, \frac{2a-b-(-b+2c)\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{e}$$

$$
\begin{aligned}
&= \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} \\
&- \frac{b\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4\sqrt{a}e} \\
&- \frac{\sqrt{a-b+c}\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} \\
&- \frac{b\operatorname{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4\sqrt{c}e} \\
&+ \frac{(b-2c)\operatorname{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4\sqrt{c}e} \\
&+ \frac{\sqrt{c}\operatorname{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} \\
&- \frac{\cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.43

$$
\int \cot^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx
$$

$$
= \frac{(2a-b)\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right) - 2\sqrt{a}\left(\sqrt{a-b+c}\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d\cdots}}\right.\right.}{4\sqrt{a}e}
$$

```
[In] Integrate[Cot[d + e*x]^3*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] ((2*a - b)*ArcTanh[(2*a + b*Tan[d + e*x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e
*x]^2 + c*Tan[d + e*x]^4])] - 2*Sqrt[a]*(Sqrt[a - b + c]*ArcTanh[(2*a - b +
 (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a + b*Tan[d + e*x]^2 + c
*Tan[d + e*x]^4])] + Cot[d + e*x]^2*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e
*x]^4]))/(4*Sqrt[a]*e)
```

Maple [F]

$$\int \cot(ex+d)^3 \sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}dx$$

```
[In] int(cot(e*x+d)^3*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x)
```

```
[Out] int(cot(e*x+d)^3*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 1.78 (sec) , antiderivative size = 1186, normalized size of antiderivative = 2.73

$$\int \cot^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(e*x+d)^3*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/8*(2*sqrt(a - b + c)*a*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x + d)^4
 + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 - 4*sqrt(c*tan(e*x + d)^4
 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a - b +
c) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2 + 1))*
tan(e*x + d)^2 - (2*a - b)*sqrt(a)*log(((b^2 + 4*a*c)*tan(e*x + d)^4 + 8*a*
b*tan(e*x + d)^2 - 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(b*tan(e
*x + d)^2 + 2*a)*sqrt(a) + 8*a^2)/tan(e*x + d)^4)*tan(e*x + d)^2 - 4*sqrt(c
*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*a)/(a*e*tan(e*x + d)^2), -1/8*(4*a*
sqrt(-a + b - c)*arctan(-1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*
((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(-a + b - c)/(((a - b)*c + c^2)*ta
n(e*x + d)^4 + (a*b - b^2 + b*c)*tan(e*x + d)^2 + a^2 - a*b + a*c))*tan(e*x
 + d)^2 + (2*a - b)*sqrt(a)*log(((b^2 + 4*a*c)*tan(e*x + d)^4 + 8*a*b*tan(e
*x + d)^2 - 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(b*tan(e*x + d)
^2 + 2*a)*sqrt(a) + 8*a^2)/tan(e*x + d)^4)*tan(e*x + d)^2 + 4*sqrt(c*tan(e*
x + d)^4 + b*tan(e*x + d)^2 + a)*a)/(a*e*tan(e*x + d)^2), -1/4*(sqrt(-a)*(2
*a - b)*arctan(1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(b*tan(e*x
 + d)^2 + 2*a)*sqrt(-a)/(a*c*tan(e*x + d)^4 + a*b*tan(e*x + d)^2 + a^2))*ta
n(e*x + d)^2 - sqrt(a - b + c)*a*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x
 + d)^4 + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 - 4*sqrt(c*tan(e*x
 + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a
 - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2
 + 1))*tan(e*x + d)^2 + 2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*a)/
(a*e*tan(e*x + d)^2), -1/4*(sqrt(-a)*(2*a - b)*arctan(1/2*sqrt(c*tan(e*x +
d)^4 + b*tan(e*x + d)^2 + a)*(b*tan(e*x + d)^2 + 2*a)*sqrt(-a)/(a*c*tan(e*x
 + d)^4 + a*b*tan(e*x + d)^2 + a^2))*tan(e*x + d)^2 + 2*a*sqrt(-a + b - c)*
arctan(-1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*
```

```
x + d)^2 + 2*a - b)*sqrt(-a + b - c)/(((a - b)*c + c^2)*tan(e*x + d)^4 + (a
*b - b^2 + b*c)*tan(e*x + d)^2 + a^2 - a*b + a*c))*tan(e*x + d)^2 + 2*sqrt(
c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*a)/(a*e*tan(e*x + d)^2)]
```

Sympy [F]

$$\int \cot^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int \sqrt{a+b\tan^2{(d+ex)}+c\tan^4{(d+ex)}}\cot^3{(d+ex)}\,dx$$

```
[In] integrate(cot(e*x+d)**3*(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2),x)
[Out] Integral(sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4)*cot(d + e*x)**3, x
)
```

Maxima [F]

$$\int \cot^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int \sqrt{c\tan{(ex+d)}^4+b\tan{(ex+d)}^2+a}\cot{(ex+d)}^3\,dx$$

```
[In] integrate(cot(e*x+d)^3*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="maxima")
[Out] integrate(sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*cot(e*x + d)^3, x)
```

Giac [F]

$$\int \cot^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int \sqrt{c\tan{(ex+d)}^4+b\tan{(ex+d)}^2+a}\cot{(ex+d)}^3\,dx$$

```
[In] integrate(cot(e*x+d)^3*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="giac")
[Out] integrate(sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*cot(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int \cot(d+e\,x)^3\,\sqrt{c\tan\left(d+e\,x\right)^4+b\tan\left(d+e\,x\right)^2+a}\,dx$$

```
[In] int(cot(d + e*x)^3*(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)
[Out] int(cot(d + e*x)^3*(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)
```

3.32 $\int \tan^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1254

$$\int \tan^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$= -\frac{\sqrt{a-b+c}\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e}$$
$$+\frac{\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{3e}$$
$$+\frac{b\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{3\sqrt{c}e\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)}$$
$$-\frac{\sqrt{c}\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{e\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)}$$
$$-\frac{\sqrt[4]{a}bE\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{3c^{3/4}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$+\frac{\sqrt[4]{a}\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$+\frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(}{(\sqrt{a}+}}}{6c^{3/4}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$-\frac{\left(b+\sqrt{a}\sqrt{c}-c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(}{(\sqrt{a}+}}}{2\sqrt[4]{a}\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$+\frac{\sqrt[4]{c}(a-b+c)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+e}{(\sqrt{a}+\sqrt{c}}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$-\frac{\left(\sqrt{a}+\sqrt{c}\right)(a-b+c)\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

```
[Out] -1/2*arctan((a-b+c)^(1/2)*tan(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2
))*(a-b+c)^(1/2)/e+1/3*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)/e
+1/3*b*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)/e/c^(1/2)/(a^(1/2
)+c^(1/2)*tan(e*x+d)^2)-c^(1/2)*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan
(e*x+d)/e/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)-1/3*a^(1/4)*b*(cos(2*arctan(c^(1/4
)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*E
llipticE(sin(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2)
```

```
)^(1/2))*((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^
2)^(1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^2)/c^(3/4)/e/(a+b*tan(e*x+d)^2+c*tan(e
*x+d)^4)^(1/2)+a^(1/4)*c^(1/4)*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2
)^(1/2)/cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticE(sin(2*arctan(c^
(1/4)*tan(e*x+d)/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*((a+b*tan(e*x+d
)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2
)*tan(e*x+d)^2)/e/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)+1/2*c^(1/4)*(a-b+
c)*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)
*tan(e*x+d)/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4))),1
/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)
+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^2)/a^(1/4)/e/(a
^(1/2)-c^(1/2))/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)-1/4*(a-b+c)*(cos(2*
arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*tan(e*x+d
)/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4))),-1/4*(a^(1
/2)-c^(1/2))^2/a^(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+c^
(1/2))*((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)
^(1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^2)/a^(1/4)/c^(1/4)/e/(a^(1/2)-c^(1/2))/(
a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)-1/2*(cos(2*arctan(c^(1/4)*tan(e*x+d)
/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticF(sin
(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(b-
c+a^(1/2)*c^(1/2))*((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(
e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^2)/a^(1/4)/c^(1/4)/e/(a+b*ta
n(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)+1/6*a^(1/4)*(cos(2*arctan(c^(1/4)*tan(e*x+
d)/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticF(s
in(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(
b+2*a^(1/2)*c^(1/2))*((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*ta
n(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^2)/c^(3/4)/e/(a+b*tan(e*x+
d)^2+c*tan(e*x+d)^4)^(1/2)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 1254, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used

$= \{3781, 1349, 1105, 1211, 1117, 1209, 1222, 1230, 1720\}$

$$\int \tan^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$

$$= -\frac{\sqrt{a-b+c}\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}\right)}{2e}$$
$$+\frac{\tan(d+ex)\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}{3e}$$
$$-\frac{\sqrt{c}\tan(d+ex)\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}{e\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)}$$
$$+\frac{b\tan(d+ex)\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}{3\sqrt{c}e\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)}$$
$$+\frac{\sqrt[4]{a}\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c\tan^4(d+ex)+b\tan^2(d+ex)+a}{\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)^2}}}{e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}$$
$$-\frac{\sqrt[4]{a}bE\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c\tan^4(d+ex)+b\tan^2(d+ex)+a}{\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)^2}}}{3c^{3/4}e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}$$
$$+\frac{\sqrt[4]{c}(a-b+c)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c\tan^4(d+ex)}{(\sqrt{c}\tan^2(}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}$$
$$-\frac{\left(b-c+\sqrt{a}\sqrt{c}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c\tan^4(d+}{(\sqrt{c}\tan}}}{2\sqrt[4]{a}\sqrt[4]{c}e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}$$
$$+\frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c\tan^4(d+}{(\sqrt{c}\tan}}}{6c^{3/4}e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}$$
$$-\frac{\left(\sqrt{a}+\sqrt{c}\right)(a-b+c)\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}$$

```
[In] Int[Tan[d + e*x]^2*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] -1/2*(Sqrt[a - b + c]*ArcTan[(Sqrt[a - b + c]*Tan[d + e*x])/Sqrt[a + b*Tan[
d + e*x]^2 + c*Tan[d + e*x]^4]])/e + (Tan[d + e*x]*Sqrt[a + b*Tan[d + e*x]^
2 + c*Tan[d + e*x]^4])/(3*e) + (b*Tan[d + e*x]*Sqrt[a + b*Tan[d + e*x]^2 +
c*Tan[d + e*x]^4])/(3*Sqrt[c]*e*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)) - (Sqrt
[c]*Tan[d + e*x]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])/(e*(Sqrt[a]
 + Sqrt[c]*Tan[d + e*x]^2)) - (a^(1/4)*b*EllipticE[2*ArcTan[(c^(1/4)*Tan[d
+ e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan[d + e
*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/(Sqrt[a] + Sqrt[c]*Ta
```

```
n[d + e*x]^2)^2])/(3*c^(3/4)*e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4
]) + (a^(1/4)*c^(1/4)*EllipticE[2*ArcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (
2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*
Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(
e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) + (a^(1/4)*(b + 2*Sqrt[a]*
Sqrt[c])*EllipticF[2*ArcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a
]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^
2 + c*Tan[d + e*x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(6*c^(3/4)*e*S
qrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) - ((b + Sqrt[a]*Sqrt[c] - c)*
EllipticF[2*ArcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]
))/4]*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan
[d + e*x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(2*a^(1/4)*c^(1/4)*e*Sq
rt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) + (c^(1/4)*(a - b + c)*Ellipti
cF[2*ArcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(
Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*
x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(2*a^(1/4)*(Sqrt[a] - Sqrt[c])
*e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) - ((Sqrt[a] + Sqrt[c])*(a
 - b + c)*EllipticPi[-1/4*(Sqrt[a] - Sqrt[c])^2/(Sqrt[a]*Sqrt[c]), 2*ArcTan
[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a] + S
qrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/(Sqrt
[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[c])*c^(1/4)*e*
Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2+cx^4}}{1+x^2}\,dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{\text{Subst}\left(\int \left(\sqrt{a+bx^2+cx^4}-\frac{\sqrt{a+bx^2+cx^4}}{1+x^2}\right)dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{\text{Subst}\left(\int \sqrt{a+bx^2+cx^4}\,dx, x, \tan(d+ex)\right)}{e}-\frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2+cx^4}}{1+x^2}\,dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{3e} \\
&\quad+\frac{\text{Subst}\left(\int \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{3e} \\
&\quad+\frac{\text{Subst}\left(\int \frac{-b+c-cx^2}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e} \\
&\quad-\frac{(a-b+c)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e}
\end{aligned}
$$

$$= \frac{\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{3e}$$
$$+ \frac{\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{3e}$$
$$- \frac{(b+\sqrt{a}\sqrt{c}-c)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e}$$
$$- \frac{(\sqrt{a}b)\,\mathrm{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{3\sqrt{c}e}$$
$$+ \frac{(\sqrt{a}\sqrt{c})\,\mathrm{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e}$$
$$- \frac{(\sqrt{a}(a-b+c))\,\mathrm{Subst}\left(\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\left(\sqrt{a}-\sqrt{c}\right)e}$$
$$+ \frac{(\sqrt{c}(a-b+c))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\left(\sqrt{a}-\sqrt{c}\right)e}$$

$$= -\frac{\sqrt{a-b+c}\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e}$$
$$+ \frac{\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{3e}$$
$$+ \frac{b\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{3\sqrt{c}e\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)}$$
$$- \frac{\sqrt{c}\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{e\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)}$$
$$- \frac{\sqrt[4]{a}bE\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{3c^{3/4}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$+ \frac{\sqrt[4]{a}\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$+ \frac{\sqrt[4]{a}(b+2\sqrt{a}\sqrt{c})\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\ldots}{6c^{3/4}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$- \frac{(b+\sqrt{a}\sqrt{c}-c)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\ldots}{2\sqrt[4]{a}\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$+ \frac{\sqrt[4]{c}(a-b+c)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\ldots}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$- \frac{\left(\sqrt{a}+\sqrt{c}\right)(a-b+c)\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\ldots\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+\ldots}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.58 (sec) , antiderivative size = 633, normalized size of antiderivative = 0.50

$$\int \tan^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$= \frac{\sqrt{\frac{3a+b+3c+4a\cos(2(d+ex))-4c\cos(2(d+ex))+a\cos(4(d+ex))-b\cos(4(d+ex))+c\cos(4(d+ex))}{3+4\cos(2(d+ex))+\cos(4(d+ex))}}\left(\frac{(b-3c)\sin(2(d+ex))}{6c}+\frac{1}{3}\tan(d+ex)\ldots\right.}{e}$$
$$+ \frac{\frac{i\sqrt{2}\left((b-3c)\left(-b+\sqrt{b^2-4ac}\right)E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\tan(d+ex)\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+\left(b^2-b\left(-3c+\sqrt{b^2-4ac}\right)+c\left(-4a-6c+3\sqrt{b^2-4ac}\ldots\right.\right.\right.}{}}{}$$

```
[In] Integrate[Tan[d + e*x]^2*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] (Sqrt[(3*a + b + 3*c + 4*a*Cos[2*(d + e*x)] - 4*c*Cos[2*(d + e*x)] + a*Cos[
4*(d + e*x)] - b*Cos[4*(d + e*x)] + c*Cos[4*(d + e*x)])/(3 + 4*Cos[2*(d + e
*x)] + Cos[4*(d + e*x)])]*(((b - 3*c)*Sin[2*(d + e*x)])/(6*c) + Tan[d + e*x
]/3))/e + ((I*Sqrt[2]*((b - 3*c)*(-b + Sqrt[b^2 - 4*a*c])*EllipticE[I*ArcSi
nh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (b^2 - b*(-3*c + Sqrt[b^2 - 4*a*c]) + c*(
-4*a - 6*c + 3*Sqrt[b^2 - 4*a*c]))*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b +
Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
4*a*c])] + 6*c*(a - b + c)*EllipticPi[(b + Sqrt[b^2 - 4*a*c])/(2*c), I*ArcS
inh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c])])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*Tan[d +
 e*x]^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*Tan[d + e*x]^2)/(b - Sqrt[b
^2 - 4*a*c])])/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])] - 4*(b - 3*c)*Cos[d + e*x]*S
in[d + e*x]*(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4))/(12*c*e*Sqrt[a + b*T
an[d + e*x]^2 + c*Tan[d + e*x]^4])
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 1945, normalized size of antiderivative = 1.55

method	result	size
derivativedivides	Expression too large to display	1945
default	Expression too large to display	1945

```
[In] int((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/e*(1/3*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)+1/6*a*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*tan(e*x+d)^
2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*tan(e*x+d)^2)^(1/2)/(a+b*tan(e*x+d)^
2+c*tan(e*x+d)^4)^(1/2)*EllipticF(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/6*b*a*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*tan(e*
x+d)^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*tan(e*x+d)^2)^(1/2)/(a+b*tan(e*
x+d)^2+c*tan(e*x+d)^4)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*tan(e*x+
d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1
/2))/a/c)^(1/2))-EllipticE(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))-1/4*2^(1/2)/(-1/a*
b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2-2/a*tan(e*x+d)^2*(-4*
a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2+2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(
1/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*EllipticF(1/2*tan(e*x+d
)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/
2))/a/c)^(1/2))*b+1/4*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*
```

```
b*tan(e*x+d)^2-2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+
d)^2+2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x
+d)^4)^(1/2)*EllipticF(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*c+1/2*c*a*2^(1/2)/(-1/a
*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2-2/a*tan(e*x+d)^2*(-4
*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2+2/a*tan(e*x+d)^2*(-4*a*c+b^2)^
(1/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
*EllipticF(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4
+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*c*a*2^(1/2)/(-1/a*b+1/a*(-4*a*c
+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2-2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2
))^(1/2)*(4+2/a*b*tan(e*x+d)^2+2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(
a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/2
*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*
c+b^2)^(1/2))/a/c)^(1/2))-a*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(
1+1/2/a*b*tan(e*x+d)^2-1/2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/
a*b*tan(e*x+d)^2+1/2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+b*tan(e*x+
d)^2+c*tan(e*x+d)^4)^(1/2)*EllipticPi(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a,(-1/2*(b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))+b*2^(1/2)/(-1/a*b+1/
a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*tan(e*x+d)^2-1/2/a*tan(e*x+d)^2*(-4*
a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*tan(e*x+d)^2+1/2/a*tan(e*x+d)^2*(-4*a*c+b^
2)^(1/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*EllipticPi(1/2*tan(
e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))
*a,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2))-c*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*tan(e*x
+d)^2-1/2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*tan(e*x+d)^2+
1/2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)
^4)^(1/2)*EllipticPi(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2),-2/(-b+(-4*a*c+b^2)^(1/2))*a,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \tan^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)^2,x, algorithm
="fricas")

[Out] Timed out
```

Sympy [F]

$$\int \tan^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int \sqrt{a+b\tan^2{(d+ex)}+c\tan^4{(d+ex)}}\tan^2{(d+ex)}\,dx$$

```
[In] integrate((a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2)*tan(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4)*tan(d + e*x)**2, x
)
```

Maxima [F]

$$\int \tan^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int \sqrt{c\tan{(ex+d)}^4+b\tan{(ex+d)}^2+a}\tan{(ex+d)}^2\,dx$$

```
[In] integrate((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)^2,x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*tan(e*x + d)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \tan^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)^2,x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \tan^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int \tan(d+e\,x)^2\,\sqrt{c\tan\left(d+e\,x\right)^4+b\tan\left(d+e\,x\right)^2+a}\,dx$$

```
[In] int(tan(d + e*x)^2*(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)
[Out] int(tan(d + e*x)^2*(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)
```

3.33 $\int \sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 829

$$\int \sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \frac{\sqrt{a-b+c}\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} + \frac{\sqrt{c}\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{e\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)} - \frac{\sqrt[4]{a}\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{\left(b+\sqrt{a}\sqrt{c}-c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{2\sqrt[4]{a}\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{\sqrt[4]{c}(a-b+c)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{\left(\sqrt{a}+\sqrt{c}\right)(a-b+c)\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

```
[Out] 1/2*arctan((a-b+c)^(1/2)*tan(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)
)*(a-b+c)^(1/2)/e+c^(1/2)*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d
)/e/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)-a^(1/4)*c^(1/4)*(cos(2*arctan(c^(1/4)*ta
n(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*Ellip
ticE(sin(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1
```

```
/2))*((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(
1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^2)/e/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/
2)-1/2*c^(1/4)*(a-b+c)*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/
cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*ta
n(e*x+d)/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*((a+b*tan(e*x+d)^2+c*ta
n(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan(e*
x+d)^2)/a^(1/4)/e/(a^(1/2)-c^(1/2))/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)
+1/4*(a-b+c)*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*tan(e*x+d)/
a^(1/4))),-1/4*(a^(1/2)-c^(1/2))^2/a^(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c^(1/2)
)^(1/2))*(a^(1/2)+c^(1/2))*((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1
/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^2)/a^(1/4)/c^(1/4)/e
/(a^(1/2)-c^(1/2))/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)+1/2*(cos(2*arcta
n(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(
1/4)))*EllipticF(sin(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4))),1/2*(2-b/a^(1/2)
/c^(1/2))^(1/2))*(b-c+a^(1/2)*c^(1/2))*((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(
a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^2)/a^(1/
4)/c^(1/4)/e/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used

$= \{1222, 1211, 1117, 1209, 1230, 1720\}$

$$\int \sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$

$$= \frac{\sqrt{a-b+c}\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}\right)}{2e} + \frac{\sqrt{c}\tan(d+ex)\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}{e\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)} - \frac{\sqrt[4]{a}\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c\tan^4(d+ex)+b\tan^2(d+ex)+a}{\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)^2}}}{e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}} - \frac{\sqrt[4]{c}(a-b+c)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c\tan^4(d+ex)\cdots}{(\sqrt{c}\tan^2(\cdots}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}} + \frac{\left(b-c+\sqrt{a}\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c\tan^4(d+e\cdots}{(\sqrt{c}\tan\cdots}}}{2\sqrt[4]{a}\sqrt[4]{c}e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}} + \frac{\left(\sqrt{a}+\sqrt{c}\right)(a-b+c)\,\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex\cdots\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}$$

```
[In] Int[Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] (Sqrt[a - b + c]*ArcTan[(Sqrt[a - b + c]*Tan[d + e*x])/Sqrt[a + b*Tan[d + e
*x]^2 + c*Tan[d + e*x]^4]])/(2*e) + (Sqrt[c]*Tan[d + e*x]*Sqrt[a + b*Tan[d
+ e*x]^2 + c*Tan[d + e*x]^4])/(e*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)) - (a^(
1/4)*c^(1/4)*EllipticE[2*ArcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e
*x]^2 + c*Tan[d + e*x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(e*Sqrt[a
+ b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) + ((b + Sqrt[a]*Sqrt[c] - c)*Ellipt
icF[2*ArcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*
(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e
*x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(2*a^(1/4)*c^(1/4)*e*Sqrt[a +
 b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) - (c^(1/4)*(a - b + c)*EllipticF[2*A
rcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a
] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/
(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(2*a^(1/4)*(Sqrt[a] - Sqrt[c])*e*Sqr
t[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) + ((Sqrt[a] + Sqrt[c])*(a - b +
 c)*EllipticPi[-1/4*(Sqrt[a] - Sqrt[c])^2/(Sqrt[a]*Sqrt[c]), 2*ArcTan[(c^(1
/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt[c]
*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/(Sqrt[a] +
Sqrt[c]*Tan[d + e*x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[c])*c^(1/4)*e*Sqrt[a
 + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
```

```
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2+cx^4}}{1+x^2}\,dx, x, \tan(d+ex)\right)}{e} \\
&= -\frac{\text{Subst}\left(\int \frac{-b+c-cx^2}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e} \\
&\quad + \frac{(a-b+c)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{\left(b+\sqrt{a}\sqrt{c}-c\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e} \\
&\quad - \frac{\left(\sqrt{a}\sqrt{c}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e} \\
&\quad + \frac{\left(\sqrt{a}(a-b+c)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\left(\sqrt{a}-\sqrt{c}\right)e} \\
&\quad - \frac{\left(\sqrt{c}(a-b+c)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\left(\sqrt{a}-\sqrt{c}\right)e}
\end{aligned}$$

$$= \frac{\sqrt{a-b+c}\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} + \frac{\sqrt{c}\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{e\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)} - \frac{\sqrt[4]{a}\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan}{(\sqrt{a}+\sqrt{c}\tan^2(d+e}}}{e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{\left(b+\sqrt{a}\sqrt{c}-c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)}{2\sqrt[4]{a}\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{\sqrt[4]{c}(a-b+c)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{\left(\sqrt{a}+\sqrt{c}\right)(a-b+c)\text{EllipticPi}\left(-\frac{(\sqrt{a}-\sqrt{c})^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.27 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.52

$$\int \sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \frac{i\left(\left(-b+\sqrt{b^2-4ac}\right)E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\tan(d+ex)\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\left(b-2c+\sqrt{b^2-4ac}\right)\text{Elli}\right.}{}$$

```
[In] Integrate[Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] ((I/2)*((-b + Sqrt[b^2 - 4*a*c])*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sq
rt[b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*
a*c])] - (b - 2*c + Sqrt[b^2 - 4*a*c])*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^
2 - 4*a*c])] - 2*(a - b + c)*EllipticPi[(b + Sqrt[b^2 - 4*a*c])/(2*c), I*Ar
cSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2
- 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*Tan[d
 + e*x]^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*Tan[d + e*x]^2)/(-b + Sqr
t[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e*Sqrt[a + b*Tan
[d + e*x]^2 + c*Tan[d + e*x]^4])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 1497, normalized size of antiderivative = 1.81

method	result	size
derivativedivides	Expression too large to display	1497
default	Expression too large to display	1497

```
[In] int((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/4*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^
2-2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2+2/a*tan(
e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*
EllipticF(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+
2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*b-1/4*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^
2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2-2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^
(1/2)*(4+2/a*b*tan(e*x+d)^2+2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+b
*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*EllipticF(1/2*tan(e*x+d)*2^(1/2)*((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*
c-1/2*c*a*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)
^2-2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2+2/a*tan
(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)
/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2*c*a*2^(1
/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2-2/a*tan(e*x
+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2+2/a*tan(e*x+d)^2*(-4*
a*c+b^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)/(b+(-4*a*c+b^
2)^(1/2))*EllipticE(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+a*2^(1/2)/(-1/a*b+1/a*(-4*
a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*tan(e*x+d)^2-1/2/a*tan(e*x+d)^2*(-4*a*c+b^
2)^(1/2))^(1/2)*(1+1/2/a*b*tan(e*x+d)^2+1/2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/
2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*EllipticPi(1/2*tan(e*x+d)
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a,(-1
/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
))-b*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*tan(e*x+d)^2-
1/2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*tan(e*x+d)^2+1/2/a*
tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1
/2)*EllipticPi(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/
(-b+(-4*a*c+b^2)^(1/2))*a,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2))+c*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(
1/2)*(1+1/2/a*b*tan(e*x+d)^2-1/2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)*(
1+1/2/a*b*tan(e*x+d)^2+1/2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+b*ta
n(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*EllipticPi(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a,(-1/2*(b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx=\int \sqrt{a+b\tan^2{(d+ex)}+c\tan^4{(d+ex)}}\,dx$$

```
[In] integrate((a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2),x)
[Out] Integral(sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4), x)
```

Maxima [F]

$$\int \sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx=\int \sqrt{c\tan{(ex+d)}^4+b\tan{(ex+d)}^2+a}\,dx$$

```
[In] integrate((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a), x)
```

Giac [F]

$$\int \sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx=\int \sqrt{c\tan{(ex+d)}^4+b\tan{(ex+d)}^2+a}\,dx$$

```
[In] integrate((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx=\int \sqrt{c\tan\left(d+e\,x\right)^4+b\tan\left(d+e\,x\right)^2+a}\,dx$$

```
[In] int((a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)
[Out] int((a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)
```

3.34 $\int \cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 861

$$\begin{aligned}
&\int \cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx \\
&= -\frac{\sqrt{a-b+c}\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} \\
&\quad -\frac{\cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{e} \\
&\quad +\frac{\sqrt{c}\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{e\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)} \\
&\quad -\frac{\sqrt[4]{a}\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&\quad +\frac{\left(\sqrt{a}+\sqrt{c}\right)\sqrt[4]{c}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(\cdots}{(\sqrt{a}+\cdots}}}{2\sqrt[4]{a}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&\quad +\frac{\sqrt[4]{c}(a-b+c)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d\cdots}{(\sqrt{a}+\sqrt{\cdots}}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&\quad -\frac{\left(\sqrt{a}+\sqrt{c}\right)(a-b+c)\,\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2\cdots\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}
\end{aligned}$$

```
[Out] -1/2*arctan((a-b+c)^(1/2)*tan(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2
))*(a-b+c)^(1/2)/e-cot(e*x+d)*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)/e+c^(
1/2)*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)/e/(a^(1/2)+c^(1/2)*
```

```
tan(e*x+d)^2)-a^(1/4)*c^(1/4)*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticE(sin(2*arctan(c^(
1/4)*tan(e*x+d)/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*((a+b*tan(e*x+d)
^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)
*tan(e*x+d)^2)/e/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)+1/2*c^(1/4)*(a-b+c
)*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*
tan(e*x+d)/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4))),1/
2*(2-b/a^(1/2)/c^(1/2))^(1/2))*((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+
c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^2)/a^(1/4)/e/(a^
(1/2)-c^(1/2))/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)+1/2*c^(1/4)*(cos(2*a
rctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*tan(e*x+d)
/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4))),1/2*(2-b/a^(
1/2)/c^(1/2))^(1/2))*(a^(1/2)+c^(1/2))*((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(
a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^2)/a^(1/
4)/e/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)-1/4*(a-b+c)*(cos(2*arctan(c^(1
/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))
*EllipticPi(sin(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4))),-1/4*(a^(1/2)-c^(1/2)
)^2/a^(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+c^(1/2))*((a+
b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(
1/2)+c^(1/2)*tan(e*x+d)^2)/a^(1/4)/c^(1/4)/e/(a^(1/2)-c^(1/2))/(a+b*tan(e*x
+d)^2+c*tan(e*x+d)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 861, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used

$= \{3781, 1325, 1295, 1211, 1117, 1209, 1230, 1720\}$

$$\int \cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$

$$= -\frac{\sqrt{a-b+c}\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}\right)}{2e} - \frac{\cot(d+ex)\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}{e} + \frac{\sqrt{c}\tan(d+ex)\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}{e\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)} - \frac{\sqrt[4]{a}\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c\tan^4(d+ex)+b\tan^2(d+ex)+a}{\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)^2}}}{e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}} + \frac{\sqrt[4]{c}(a-b+c)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c\tan^4(d+ex)}{(\sqrt{c}\tan}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}} + \frac{\left(\sqrt{a}+\sqrt{c}\right)\sqrt[4]{c}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c\tan^4(d+e}{(\sqrt{c}\tan}}}{2\sqrt[4]{a}e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}} - \frac{\left(\sqrt{a}+\sqrt{c}\right)(a-b+c)\,\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+e\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}$$

```
[In] Int[Cot[d + e*x]^2*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] -1/2*(Sqrt[a - b + c]*ArcTan[(Sqrt[a - b + c]*Tan[d + e*x])/Sqrt[a + b*Tan[
d + e*x]^2 + c*Tan[d + e*x]^4]])/e - (Cot[d + e*x]*Sqrt[a + b*Tan[d + e*x]^
2 + c*Tan[d + e*x]^4])/e + (Sqrt[c]*Tan[d + e*x]*Sqrt[a + b*Tan[d + e*x]^2
+ c*Tan[d + e*x]^4])/(e*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)) - (a^(1/4)*c^(1
/4)*EllipticE[2*ArcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqr
t[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c
*Tan[d + e*x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(e*Sqrt[a + b*Tan[d
 + e*x]^2 + c*Tan[d + e*x]^4]) + ((Sqrt[a] + Sqrt[c])*c^(1/4)*EllipticF[2*A
rcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a
] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/
(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(2*a^(1/4)*e*Sqrt[a + b*Tan[d + e*x]
^2 + c*Tan[d + e*x]^4]) + (c^(1/4)*(a - b + c)*EllipticF[2*ArcTan[(c^(1/4)*
Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan
[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/(Sqrt[a] + Sqrt
[c]*Tan[d + e*x]^2)^2])/(2*a^(1/4)*(Sqrt[a] - Sqrt[c])*e*Sqrt[a + b*Tan[d +
 e*x]^2 + c*Tan[d + e*x]^4]) - ((Sqrt[a] + Sqrt[c])*(a - b + c)*EllipticPi[
-1/4*(Sqrt[a] - Sqrt[c])^2/(Sqrt[a]*Sqrt[c]), 2*ArcTan[(c^(1/4)*Tan[d + e*x
```

```
])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2
)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d +
 e*x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[c])*c^(1/4)*e*Sqrt[a + b*Tan[d + e*
x]^2 + c*Tan[d + e*x]^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1325

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] - Dist[(c*d^2 - b*d*e + a*e^2)/(d*e*f^2),
Int[(f*x)^(m + 2)*((a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2)), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2+cx^4}}{x^2(1+x^2)}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{a+cx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e} - \frac{(a-b+c)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= -\frac{\cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{e} - \frac{\text{Subst}\left(\int \frac{-ac-acx^2}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{ae} - \frac{\left(\sqrt{a}(a-b+c)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\left(\sqrt{a}-\sqrt{c}\right)e} + \frac{\left(\sqrt{c}(a-b+c)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\left(\sqrt{a}-\sqrt{c}\right)e}$$

$$= -\frac{\sqrt{a-b+c}\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} - \frac{\cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{e} + \frac{\sqrt[4]{c}(a-b+c)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+}{}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{\left(\sqrt{a}+\sqrt{c}\right)(a-b+c)\,\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}}, 2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{}\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+e}} - \frac{\left(\sqrt{a}\sqrt{c}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e} + \frac{\left(\left(\sqrt{a}+\sqrt{c}\right)\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= -\frac{\sqrt{a-b+c}\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} - \frac{\cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{e} + \frac{\sqrt{c}\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{e\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)} - \frac{\sqrt[4]{a}\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan}{(\sqrt{a}+\sqrt{c}\tan^2(d+e}}}{e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{\left(\sqrt{a}+\sqrt{c}\right)\sqrt[4]{c}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{}}{2\sqrt[4]{a}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{\sqrt[4]{c}(a-b+c)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{a}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{\left(\sqrt{a}+\sqrt{c}\right)(a-b+c)\,\text{EllipticPi}\left(-\frac{(\sqrt{a}-\sqrt{c})^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.25 (sec) , antiderivative size = 1258, normalized size of antiderivative = 1.46

$$\int \cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \frac{\sqrt{\frac{3a+b+3c+4a\cos(2(d+ex))-4c\cos(2(d+ex))+a\cos(4(d+ex))-b\cos(4(d+ex))+c\cos(4(d+ex))}{3+4\cos(2(d+ex))+\cos(4(d+ex))}}\left(-\cot(d+ex)+\frac{1}{2}\sin(2(d+ex)\right.}{e} + \frac{i\sqrt{2}\left(-b+\sqrt{b^2-4ac}\right)\left(E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\tan(d+ex)\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\text{EllipticF}\left(i\text{arcsinh}\right.\right.}{}$$

```
[In] Integrate[Cot[d + e*x]^2*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] (Sqrt[(3*a + b + 3*c + 4*a*Cos[2*(d + e*x)] - 4*c*Cos[2*(d + e*x)] + a*Cos[
4*(d + e*x)] - b*Cos[4*(d + e*x)] + c*Cos[4*(d + e*x)])/(3 + 4*Cos[2*(d + e
*x)] + Cos[4*(d + e*x)])]*(-Cot[d + e*x] + Sin[2*(d + e*x)]/2))/e + (I*Sqrt
[2]*(-b + Sqrt[b^2 - 4*a*c])*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[
b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c
])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Tan[d + e
```

```
*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])*(1 + Tan[d + e*x]^2
)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*Tan[d + e*x]^2)/(b + Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*Tan[d + e*x]^2)/(b - Sqrt[b^2 - 4*a*c])] - (2*I)*Sqrt[2]*c*
EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]],
(b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]*(1 + Tan[d + e*x]^2)*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*Tan[d + e*x]^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*Tan[d + e*x]^2)/(b - Sqrt[b^2 - 4*a*c])] + (2*I)*Sqrt[2]*a*Ellipti
cPi[(b + Sqrt[b^2 - 4*a*c])/(2*c), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]*(
1 + Tan[d + e*x]^2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*Tan[d + e*x]^2)/(b +
Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*Tan[d + e*x]^2)/(b - Sqrt[b^2 - 4*a*c])]
- (2*I)*Sqrt[2]*b*EllipticPi[(b + Sqrt[b^2 - 4*a*c])/(2*c), I*ArcSinh[Sqrt[
2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c])]*(1 + Tan[d + e*x]^2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*Tan[d + e*x]^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*Tan[d + e*x]^2)/
(b - Sqrt[b^2 - 4*a*c])] + (2*I)*Sqrt[2]*c*EllipticPi[(b + Sqrt[b^2 - 4*a*c
])/(2*c), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]],
(b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]*(1 + Tan[d + e*x]^2)*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*Tan[d + e*x]^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*Tan[d + e*x]^2)/(b - Sqrt[b^2 - 4*a*c])] - 4*Sqrt[c/(b + Sqrt[b^2
- 4*a*c])]*Tan[d + e*x]*(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4))/(4*Sqrt[
c/(b + Sqrt[b^2 - 4*a*c])]*e*(1 + Tan[d + e*x]^2)*Sqrt[a + b*Tan[d + e*x]^2
 + c*Tan[d + e*x]^4])
```

Maple [F]

$$\int \cot \left(ex+d\right)^{2} \sqrt{a+b\tan \left(ex+d\right)^{2}+c\tan \left(ex+d\right)^{4}}dx$$

```
[In] int(cot(e*x+d)^2*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x)
[Out] int(cot(e*x+d)^2*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate(cot(e*x+d)^2*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int \sqrt{a+b\tan^2{(d+ex)}+c\tan^4{(d+ex)}}\cot^2{(d+ex)}\,dx$$

```
[In] integrate(cot(e*x+d)**2*(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2),x)
[Out] Integral(sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4)*cot(d + e*x)**2, x
)
```

Maxima [F]

$$\int \cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int \sqrt{c\tan{(ex+d)}^4+b\tan{(ex+d)}^2+a}\cot{(ex+d)}^2\,dx$$

```
[In] integrate(cot(e*x+d)^2*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="maxima")
[Out] integrate(sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*cot(e*x + d)^2, x)
```

Giac [F]

$$\int \cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int \sqrt{c\tan{(ex+d)}^4+b\tan{(ex+d)}^2+a}\cot{(ex+d)}^2\,dx$$

```
[In] integrate(cot(e*x+d)^2*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="giac")
[Out] integrate(sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*cot(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int \cot(d+e\,x)^2\,\sqrt{c\tan\left(d+e\,x\right)^4+b\tan\left(d+e\,x\right)^2+a}\,dx$$

```
[In] int(cot(d + e*x)^2*(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)
[Out] int(cot(d + e*x)^2*(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)
```

3.35 $\int \cot^4(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 943

$$\int \cot^4(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$= \frac{\sqrt{a-b+c}\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e}$$
$$+ \frac{(3a-b)\cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{3ae}$$
$$- \frac{\cot^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{3e}$$
$$- \frac{(3a-b)\sqrt{c}\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{3ae\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)}$$
$$+ \frac{(3a-b)\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d-}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{3a^{3/4}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$- \frac{\left(3a-b+\sqrt{a}\sqrt{c}\right)\sqrt[4]{c}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a-}{}}}{6a^{3/4}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$- \frac{\sqrt[4]{c}(a-b+c)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d-}{(\sqrt{a}+\sqrt{}}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$+ \frac{\left(\sqrt{a}+\sqrt{c}\right)(a-b+c)\,\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

```
[Out] 1/2*arctan((a-b+c)^(1/2)*tan(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)
)*(a-b+c)^(1/2)/e+1/3*(3*a-b)*cot(e*x+d)*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^
```

```
(1/2)/a/e-1/3*cot(e*x+d)^3*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)/e-1/3*(3
*a-b)*c^(1/2)*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)/a/e/(a^(1/
2)+c^(1/2)*tan(e*x+d)^2)+1/3*(3*a-b)*c^(1/4)*(cos(2*arctan(c^(1/4)*tan(e*x+
d)/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticE(s
in(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(
(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(
a^(1/2)+c^(1/2)*tan(e*x+d)^2)/a^(3/4)/e/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(
1/2)-1/2*c^(1/4)*(a-b+c)*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*
tan(e*x+d)/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*((a+b*tan(e*x+d)^2+c*
tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan(
e*x+d)^2)/a^(1/4)/e/(a^(1/2)-c^(1/2))/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/
2)+1/4*(a-b+c)*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*tan(e*x+d
)/a^(1/4))),-1/4*(a^(1/2)-c^(1/2))^2/a^(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c^(1/
2))^(1/2))*(a^(1/2)+c^(1/2))*((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^
(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^2)/a^(1/4)/c^(1/4)
/e/(a^(1/2)-c^(1/2))/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)-1/6*c^(1/4)*(c
os(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*tan(
e*x+d)/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4))),1/2*(2
-b/a^(1/2)/c^(1/2))^(1/2))*(3*a-b+a^(1/2)*c^(1/2))*((a+b*tan(e*x+d)^2+c*tan
(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan(e*x
+d)^2)/a^(3/4)/e/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 943, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used

$= \{3781, 1323, 1295, 1211, 1117, 1209, 1230, 1720\}$

$$\int \cot^4(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$

$$= -\frac{\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}\cot^3(d+ex)}{3e}$$
$$+\frac{(3a-b)\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}\cot(d+ex)}{3ae}$$
$$+\frac{\sqrt{a-b+c}\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}\right)}{2e}$$
$$-\frac{(3a-b)\sqrt{c}\tan(d+ex)\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}{3ae\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)}$$
$$+\frac{(3a-b)\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c\tan^4(d+ex)+b\tan^2(d+ex}{(\sqrt{c}\tan^2(d+ex)+\sqrt{a})^2}}}{3a^{3/4}e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}$$
$$-\frac{\sqrt[4]{c}(a-b+c)\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c\tan^4(d+ex}{(\sqrt{c}\tan}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}$$
$$-\frac{\left(3a+\sqrt{c}\sqrt{a}-b\right)\sqrt[4]{c}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c}{}}}{6a^{3/4}e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}$$
$$+\frac{\left(\sqrt{a}+\sqrt{c}\right)(a-b+c)\,\mathrm{EllipticPi}\left(-\frac{(\sqrt{a}-\sqrt{c})^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+e\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}$$

```
[In] Int[Cot[d + e*x]^4*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] (Sqrt[a - b + c]*ArcTan[(Sqrt[a - b + c]*Tan[d + e*x])/Sqrt[a + b*Tan[d + e
*x]^2 + c*Tan[d + e*x]^4]])/(2*e) + ((3*a - b)*Cot[d + e*x]*Sqrt[a + b*Tan[
d + e*x]^2 + c*Tan[d + e*x]^4])/(3*a*e) - (Cot[d + e*x]^3*Sqrt[a + b*Tan[d
+ e*x]^2 + c*Tan[d + e*x]^4])/(3*e) - ((3*a - b)*Sqrt[c]*Tan[d + e*x]*Sqrt[
a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])/(3*a*e*(Sqrt[a] + Sqrt[c]*Tan[d +
 e*x]^2)) + ((3*a - b)*c^(1/4)*EllipticE[2*ArcTan[(c^(1/4)*Tan[d + e*x])/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sqr
t[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x]
^2)^2])/(3*a^(3/4)*e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) - ((3*a
 - b + Sqrt[a]*Sqrt[c])*c^(1/4)*EllipticF[2*ArcTan[(c^(1/4)*Tan[d + e*x])/a
^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sq
rt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x
]^2)^2])/(6*a^(3/4)*e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) - (c^(
1/4)*(a - b + c)*EllipticF[2*ArcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b
/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d
```

```
 + e*x]^2 + c*Tan[d + e*x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(2*a^(
1/4)*(Sqrt[a] - Sqrt[c])*e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) +
 ((Sqrt[a] + Sqrt[c])*(a - b + c)*EllipticPi[-1/4*(Sqrt[a] - Sqrt[c])^2/(Sq
rt[a]*Sqrt[c]), 2*ArcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 +
 c*Tan[d + e*x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(4*a^(1/4)*(Sqrt[
a] - Sqrt[c])*c^(1/4)*e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1323

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/d^2, Int[(f*x)^m*(a*d + (b*d - a*e)*x^
2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/(d^2*
f^4), Int[(f*x)^(m + 4)*((a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2)), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m
, -2]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2+cx^4}}{x^4(1+x^2)}\,dx, x, \tan(d+ex)\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{a+(-a+b)x^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e}\\
&\quad + \frac{(a-b+c)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e}
\end{aligned}$$

$$= -\frac{\cot^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{3e} - \frac{\text{Subst}\left(\int \frac{a(3a-b)+acx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{3ae} + \frac{\left(\sqrt{a}(a-b+c)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\left(\sqrt{a}-\sqrt{c}\right)e} - \frac{\left(\sqrt{c}(a-b+c)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\left(\sqrt{a}-\sqrt{c}\right)e}$$

$$= \frac{\sqrt{a-b+c}\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} + \frac{(3a-b)\cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{3ae} - \frac{\cot^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{3e} - \frac{\sqrt[4]{c}(a-b+c)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+}{}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{\left(\sqrt{a}+\sqrt{c}\right)(a-b+c)\,\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}}, 2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{}\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+e}} + \frac{\text{Subst}\left(\int \frac{-a^2c-a(3a-b)cx^2}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{3a^2e}$$

$$= \frac{\sqrt{a-b+c}\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} + \frac{(3a-b)\cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{3ae} - \frac{\cot^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{3e} - \frac{\sqrt[4]{c}(a-b+c)\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\cdots}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{\left(\sqrt{a}+\sqrt{c}\right)(a-b+c)\,\mathrm{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\cdots\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+\cdots}} + \frac{\left((3a-b)\sqrt{c}\right)\mathrm{Subst}\left(\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx,x,\tan(d+ex)\right)}{3\sqrt{a}e} - \frac{\left(\left(3a-b+\sqrt{a}\sqrt{c}\right)\sqrt{c}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{a+bx^2+cx^4}}\,dx,x,\tan(d+ex)\right)}{3\sqrt{a}e}$$

$$= \frac{\sqrt{a-b+c}\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e} + \frac{(3a-b)\cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{3ae} - \frac{\cot^3(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{3e} - \frac{(3a-b)\sqrt{c}\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{3ae\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)} + \frac{(3a-b)\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)\cdots}{(\sqrt{a}+\sqrt{c}\tan\cdots}}}{3a^{3/4}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{\left(3a-b+\sqrt{a}\sqrt{c}\right)\sqrt[4]{c}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+\cdots\right.}{6a^{3/4}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{\sqrt[4]{c}(a-b+c)\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\cdots}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{\left(\sqrt{a}+\sqrt{c}\right)(a-b+c)\,\mathrm{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\cdots\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+\cdots}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.59 (sec) , antiderivative size = 1590, normalized size of antiderivative = 1.69

$$\int \cot^4(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$

$$= \frac{\sqrt{\frac{3a+b+3c+4a\cos(2(d+ex))-4c\cos(2(d+ex))+a\cos(4(d+ex))-b\cos(4(d+ex))+c\cos(4(d+ex))}{3+4\cos(2(d+ex))+\cos(4(d+ex))}}\left(\frac{(4a\cos(d+ex)-b\cos(d+ex))\csc(d+ex)}{3a}-\cdots\right.}{e} + \frac{3i\sqrt{2}a\left(b-\sqrt{b^2-4ac}\right)\left(E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\tan(d+ex)\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\cdots\right.\right.\right.}{\cdots}$$

```
[In] Integrate[Cot[d + e*x]^4*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] (Sqrt[(3*a + b + 3*c + 4*a*Cos[2*(d + e*x)] - 4*c*Cos[2*(d + e*x)] + a*Cos[
4*(d + e*x)] - b*Cos[4*(d + e*x)] + c*Cos[4*(d + e*x)])/(3 + 4*Cos[2*(d + e
*x)] + Cos[4*(d + e*x)])]*(((4*a*Cos[d + e*x] - b*Cos[d + e*x])*Csc[d + e*x
])/(3*a) - (Cot[d + e*x]*Csc[d + e*x]^2)/3 - ((3*a - b)*Sin[2*(d + e*x)])/(
6*a)))/e + ((3*I)*Sqrt[2]*a*(b - Sqrt[b^2 - 4*a*c])*(EllipticE[I*ArcSinh[Sq
rt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c]
)/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b
^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)])*(1 + Tan[d + e*x]^2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*Tan[d + e*x]^2)/
(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*Tan[d + e*x]^2)/(b - Sqrt[b^2 - 4*a*
c])] + I*Sqrt[2]*b*(-b + Sqrt[b^2 - 4*a*c])*(EllipticE[I*ArcSinh[Sqrt[2]*Sq
rt[c/(b + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a
*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])*(1 +
 Tan[d + e*x]^2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*Tan[d + e*x]^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*Tan[d + e*x]^2)/(b - Sqrt[b^2 - 4*a*c])] + (
2*I)*Sqrt[2]*a*c*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]*(1 + Tan[
d + e*x]^2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*Tan[d + e*x]^2)/(b + Sqrt[b^2
 - 4*a*c])]*Sqrt[1 + (2*c*Tan[d + e*x]^2)/(b - Sqrt[b^2 - 4*a*c])] - (6*I)*
Sqrt[2]*a^2*EllipticPi[(b + Sqrt[b^2 - 4*a*c])/(2*c), I*ArcSinh[Sqrt[2]*Sqr
t[c/(b + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])]*(1 + Tan[d + e*x]^2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*Ta
n[d + e*x]^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*Tan[d + e*x]^2)/(b - S
qrt[b^2 - 4*a*c])] + (6*I)*Sqrt[2]*a*b*EllipticPi[(b + Sqrt[b^2 - 4*a*c])/(
2*c), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]], (b +
 Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]*(1 + Tan[d + e*x]^2)*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*Tan[d + e*x]^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
```

```
(2*c*Tan[d + e*x]^2)/(b - Sqrt[b^2 - 4*a*c])] - (6*I)*Sqrt[2]*a*c*EllipticP
i[(b + Sqrt[b^2 - 4*a*c])/(2*c), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]*(1
+ Tan[d + e*x]^2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*Tan[d + e*x]^2)/(b + Sq
rt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*Tan[d + e*x]^2)/(b - Sqrt[b^2 - 4*a*c])] -
4*(-3*a + b)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]*(a + b*Tan[d + e*
x]^2 + c*Tan[d + e*x]^4))/(12*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e*(1 + Tan[
d + e*x]^2)*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])
```

Maple [F]

$$\int \cot (ex+d)^4 \sqrt{a+b\tan (ex+d)^2+c\tan (ex+d)^4}dx$$

```
[In] int(cot(e*x+d)^4*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x)
[Out] int(cot(e*x+d)^4*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x)
```

Fricas [F]

$$\int \cot^4(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \int \sqrt{c\tan (ex+d)^4+b\tan (ex+d)^2+a}\cot (ex+d)^4\,dx$$

```
[In] integrate(cot(e*x+d)^4*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="fricas")
[Out] integral(sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*cot(e*x + d)^4, x)
```

Sympy [F]

$$\int \cot^4(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx = \int \sqrt{a+b\tan^2{(d+ex)}+c\tan^4{(d+ex)}}\cot^4{(d+ex)}\,dx$$

```
[In] integrate(cot(e*x+d)**4*(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2),x)
[Out] Integral(sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4)*cot(d + e*x)**4, x
)
```

Maxima [F]

$$\int \cot^4(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int \sqrt{c\tan{(ex+d)}^4+b\tan{(ex+d)}^2+a}\cot{(ex+d)}^4\,dx$$

```
[In] integrate(cot(e*x+d)^4*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*cot(e*x + d)^4, x)
```

Giac [F]

$$\int \cot^4(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int \sqrt{c\tan{(ex+d)}^4+b\tan{(ex+d)}^2+a}\cot{(ex+d)}^4\,dx$$

```
[In] integrate(cot(e*x+d)^4*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*cot(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}\,dx$$
$$=\int {\cot(d+e\,x)}^4\,\sqrt{c\,{\tan\left(d+e\,x\right)}^4+b\,{\tan\left(d+e\,x\right)}^2+a}\,dx$$

```
[In] int(cot(d + e*x)^4*(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)
```

```
[Out] int(cot(d + e*x)^4*(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)
```

3.36 $\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 182

$$\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e} - \frac{(b+2c)\operatorname{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4c^{3/2}e} + \frac{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2ce}$$

```
[Out] -1/4*(b+2*c)*arctanh(1/2*(b+2*c*tan(e*x+d)^2)/c^(1/2)/(a+b*tan(e*x+d)^2+c*t
an(e*x+d)^4)^(1/2))/c^(3/2)/e-1/2*arctanh(1/2*(2*a-b+(b-2*c)*tan(e*x+d)^2)/
(a-b+c)^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))/e/(a-b+c)^(1/2)+1/2*
(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)/c/e
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{3781, 1265, 1667, 857, 635, 212, 738\}$

$$\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$$
$$= -\frac{(b+2c)\operatorname{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4c^{3/2}e} - \frac{\operatorname{arctanh}\left(\frac{2a+(b-2c)\tan^2(d+ex)-b}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e\sqrt{a-b+c}} + \frac{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2ce}$$

```
[In] Int[Tan[d + e*x]^5/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] -1/2*ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a
 + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/(Sqrt[a - b + c]*e) - ((b + 2*c)*
ArcTanh[(b + 2*c*Tan[d + e*x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x]^2 + c*T
an[d + e*x]^4])])/(4*c^(3/2)*e) + Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x
]^4]/(2*c*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e} \\
&= \frac{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2ce} + \frac{\text{Subst}\left(\int \frac{-\frac{b}{2}-\frac{1}{2}(b+2c)x}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2ce} \\
&= \frac{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2ce} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e} \\
&\quad - \frac{(b+2c)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{4ce}
\end{aligned}
$$

$$
= \frac{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2ce}
- \frac{\text{Subst}\left(\int \frac{1}{4a-4b+4c-x^2}\,dx, x, \frac{2a-b-(-b+2c)\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{e}
- \frac{(b+2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2ce}
$$

$$
= -\frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e}
- \frac{(b+2c)\operatorname{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4c^{3/2}e}
+ \frac{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2ce}
$$

Mathematica [A] (verified)

Time = 3.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.95

$$
\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx =
-\frac{\frac{2\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{\sqrt{a-b+c}} + \frac{(b+2c)\operatorname{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{c^{3/2}} - \frac{2\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{c}}{4e}
$$

```
[In] Integrate[Tan[d + e*x]^5/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] -1/4*((2*ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sq
rt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])])/Sqrt[a - b + c] + ((b + 2*c)
*ArcTanh[(b + 2*c*Tan[d + e*x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x]^2 + c*
Tan[d + e*x]^4])])/c^(3/2) - (2*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^
4])/c)/e
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{\frac{\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}{2c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\tan(ex+d)^2}{\sqrt{c}}+\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}\right)}{4c^{\frac{3}{2}}}-\frac{\ln\left(\frac{\frac{b}{2}+c\tan(ex+d)^2}{\sqrt{c}}+\sqrt{a+b\tan(ex}\right.}{2\sqrt{c}}}{}$
default	$\frac{\frac{\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}{2c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\tan(ex+d)^2}{\sqrt{c}}+\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}\right)}{4c^{\frac{3}{2}}}-\frac{\ln\left(\frac{\frac{b}{2}+c\tan(ex+d)^2}{\sqrt{c}}+\sqrt{a+b\tan(ex}\right.}{2\sqrt{c}}}{}$

```
[In] int(tan(e*x+d)^5/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/e*(1/2/c*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)-1/4*b/c^(3/2)*ln((1/2*b+
c*tan(e*x+d)^2)/c^(1/2)+(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))-1/2*ln((1/
2*b+c*tan(e*x+d)^2)/c^(1/2)+(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))/c^(1/2
)-1/2/(a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b-2*c)*(1+tan(e*x+d)^2)+2*(a-b+c)^(1/2
)*(c*(1+tan(e*x+d)^2)^2+(b-2*c)*(1+tan(e*x+d)^2)+a-b+c)^(1/2))/(1+tan(e*x+d
)^2)))
```

Fricas [A] (verification not implemented)

none

Time = 1.32 (sec) , antiderivative size = 1226, normalized size of antiderivative = 6.74

$$\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(e*x+d)^5/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/8*(2*sqrt(a - b + c)*c^2*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x + d)
^4 + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 - 4*sqrt(c*tan(e*x + d)
^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a - b
+ c) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2 + 1)
) + (a*b - b^2 + (2*a - b)*c + 2*c^2)*sqrt(c)*log(8*c^2*tan(e*x + d)^4 + 8*
b*c*tan(e*x + d)^2 + b^2 - 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*
(2*c*tan(e*x + d)^2 + b)*sqrt(c) + 4*a*c) + 4*sqrt(c*tan(e*x + d)^4 + b*tan
(e*x + d)^2 + a)*((a - b)*c + c^2))/(((a - b)*c^2 + c^3)*e), 1/4*(sqrt(a -
b + c)*c^2*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x + d)^4 + 2*(4*a*b - 3
*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 - 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x +
d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*
a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2 + 1)) + (a*b - b^2 +
(2*a - b)*c + 2*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x
+ d)^2 + a)*(2*c*tan(e*x + d)^2 + b)*sqrt(-c)/(c^2*tan(e*x + d)^4 + b*c*tan
```

```
(e*x + d)^2 + a*c)) + 2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((a -
 b)*c + c^2))/(((a - b)*c^2 + c^3)*e), -1/8*(4*sqrt(-a + b - c)*c^2*arctan(
-1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^
2 + 2*a - b)*sqrt(-a + b - c)/(((a - b)*c + c^2)*tan(e*x + d)^4 + (a*b - b^
2 + b*c)*tan(e*x + d)^2 + a^2 - a*b + a*c)) - (a*b - b^2 + (2*a - b)*c + 2*
c^2)*sqrt(c)*log(8*c^2*tan(e*x + d)^4 + 8*b*c*tan(e*x + d)^2 + b^2 - 4*sqrt
(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*tan(e*x + d)^2 + b)*sqrt(c)
+ 4*a*c) - 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((a - b)*c + c^2
))/(((a - b)*c^2 + c^3)*e), -1/4*(2*sqrt(-a + b - c)*c^2*arctan(-1/2*sqrt(c
*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b
)*sqrt(-a + b - c)/(((a - b)*c + c^2)*tan(e*x + d)^4 + (a*b - b^2 + b*c)*ta
n(e*x + d)^2 + a^2 - a*b + a*c)) - (a*b - b^2 + (2*a - b)*c + 2*c^2)*sqrt(-
c)*arctan(1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*tan(e*x +
d)^2 + b)*sqrt(-c)/(c^2*tan(e*x + d)^4 + b*c*tan(e*x + d)^2 + a*c)) - 2*sqr
t(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((a - b)*c + c^2))/(((a - b)*c^2
 + c^3)*e)]
```

Sympy [F]

$$\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx=\int \frac{\tan^5{(d+ex)}}{\sqrt{a+b\tan^2{(d+ex)}+c\tan^4{(d+ex)}}}\,dx$$

```
[In] integrate(tan(e*x+d)**5/(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2),x)
```

```
[Out] Integral(tan(d + e*x)**5/sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4), x
)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx=\text{Timed out}$$

```
[In] integrate(tan(e*x+d)^5/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^5/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\tan(d+e\,x)^5}{\sqrt{c\tan\left(d+e\,x\right)^4+b\tan\left(d+e\,x\right)^2+a}}\,dx$$

```
[In] int(tan(d + e*x)^5/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)
```

```
[Out] int(tan(d + e*x)^5/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)
```

3.37 $\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 141

$$\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e} + \frac{\operatorname{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{c}e}$$

```
[Out] 1/2*arctanh(1/2*(b+2*c*tan(e*x+d)^2)/c^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)
^4)^(1/2))/e/c^(1/2)+1/2*arctanh(1/2*(2*a-b+(b-2*c)*tan(e*x+d)^2)/(a-b+c)^(
1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))/e/(a-b+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3781, 1265, 857, 635, 212, 738}

$$\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2a+(b-2c)\tan^2(d+ex)-b}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e\sqrt{a-b+c}} + \frac{\operatorname{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{c}e}$$

```
[In] Int[Tan[d + e*x]^3/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a + b*
Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/(2*Sqrt[a - b + c]*e) + ArcTanh[(b + 2
```

```
*c*Tan[d + e*x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]
)]/(2*Sqrt[c]*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{e} + \frac{\text{Subst}\left(\int \frac{1}{4a-4b+4c-x^2}\,dx, x, \frac{2a-b-(-b+2c)\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{e}$$

$$= \frac{\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e} + \frac{\text{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{c}e}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

$$\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \frac{\frac{\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{\sqrt{a-b+c}} + \frac{\text{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{\sqrt{c}}}{2e}$$

```
[In] Integrate[Tan[d + e*x]^3/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] (ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a + b
*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/Sqrt[a - b + c] + ArcTanh[(b + 2*c*Ta
n[d + e*x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/Sq
rt[c])/(2*e)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{\ln\left(\frac{\frac{b}{2}+c\tan(ex+d)^2}{\sqrt{c}}+\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}\right)}{2\sqrt{c}}+\frac{\ln\left(\frac{2a-2b+2c+(b-2c)\left(1+\tan(ex+d)^2\right)+2\sqrt{a-b+c}\,\sqrt{c\left(1+\tan(ex+d)^2\right)}}{1+\tan(ex+d)^2}\right)}{2\sqrt{a-b+c}}}{e}$
default	$\frac{\frac{\ln\left(\frac{\frac{b}{2}+c\tan(ex+d)^2}{\sqrt{c}}+\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}\right)}{2\sqrt{c}}+\frac{\ln\left(\frac{2a-2b+2c+(b-2c)\left(1+\tan(ex+d)^2\right)+2\sqrt{a-b+c}\,\sqrt{c\left(1+\tan(ex+d)^2\right)}}{1+\tan(ex+d)^2}\right)}{2\sqrt{a-b+c}}}{e}$

```
[In] int(tan(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/e*(1/2*ln((1/2*b+c*tan(e*x+d)^2)/c^(1/2)+(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4
)^(1/2))/c^(1/2)+1/2/(a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b-2*c)*(1+tan(e*x+d)^2)
+2*(a-b+c)^(1/2)*(c*(1+tan(e*x+d)^2)^2+(b-2*c)*(1+tan(e*x+d)^2)+a-b+c)^(1/2
))/(1+tan(e*x+d)^2)))
```

Fricas [A] (verification not implemented)

none

Time = 1.09 (sec) , antiderivative size = 993, normalized size of antiderivative = 7.04

$$\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/4*((a - b + c)*sqrt(c)*log(8*c^2*tan(e*x + d)^4 + 8*b*c*tan(e*x + d)^2 +
 b^2 + 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*tan(e*x + d)^2
+ b)*sqrt(c) + 4*a*c) + sqrt(a - b + c)*c*log(((b^2 + 4*(a - 2*b)*c + 8*c^2
)*tan(e*x + d)^4 + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 + 4*sqrt(
c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a -
b)*sqrt(a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e
*x + d)^2 + 1)))/(((a - b)*c + c^2)*e), -1/4*(2*(a - b + c)*sqrt(-c)*arctan
(1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*tan(e*x + d)^2 + b)
*sqrt(-c)/(c^2*tan(e*x + d)^4 + b*c*tan(e*x + d)^2 + a*c)) - sqrt(a - b + c
)*c*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x + d)^4 + 2*(4*a*b - 3*b^2 -
4*(a - b)*c)*tan(e*x + d)^2 + 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 +
a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b + b
^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2 + 1)))/(((a - b)*c + c^2)*e)
```

```
, 1/4*(2*sqrt(-a + b - c)*c*arctan(-1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x +
 d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(-a + b - c)/(((a - b)*
c + c^2)*tan(e*x + d)^4 + (a*b - b^2 + b*c)*tan(e*x + d)^2 + a^2 - a*b + a*
c)) + (a - b + c)*sqrt(c)*log(8*c^2*tan(e*x + d)^4 + 8*b*c*tan(e*x + d)^2 +
 b^2 + 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*tan(e*x + d)^2
+ b)*sqrt(c) + 4*a*c))/(((a - b)*c + c^2)*e), 1/2*(sqrt(-a + b - c)*c*arcta
n(-1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d
)^2 + 2*a - b)*sqrt(-a + b - c)/(((a - b)*c + c^2)*tan(e*x + d)^4 + (a*b -
b^2 + b*c)*tan(e*x + d)^2 + a^2 - a*b + a*c)) - (a - b + c)*sqrt(-c)*arctan
(1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*tan(e*x + d)^2 + b)
*sqrt(-c)/(c^2*tan(e*x + d)^4 + b*c*tan(e*x + d)^2 + a*c)))/(((a - b)*c + c
^2)*e)]
```

Sympy [F]

$$\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\tan^3{(d+ex)}}{\sqrt{a+b\tan^2{(d+ex)}+c\tan^4{(d+ex)}}}\,dx$$

```
[In] integrate(tan(e*x+d)**3/(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2),x)
```

```
[Out] Integral(tan(d + e*x)**3/sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4), x
)
```

Maxima [F]

$$\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\tan{(ex+d)}^3}{\sqrt{c\tan{(ex+d)}^4+b\tan{(ex+d)}^2+a}}\,dx$$

```
[In] integrate(tan(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(tan(e*x + d)^3/sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\tan(d+e\,x)^3}{\sqrt{c\tan\left(d+e\,x\right)^4+b\tan\left(d+e\,x\right)^2+a}}\,dx$$

```
[In] int(tan(d + e*x)^3/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)
```

```
[Out] int(tan(d + e*x)^3/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)
```

3.38 $\int \frac{\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 79

$$\int \frac{\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+ce}}$$

```
[Out] -1/2*arctanh(1/2*(2*a-b+(b-2*c)*tan(e*x+d)^2)/(a-b+c)^(1/2)/(a+b*tan(e*x+d)
^2+c*tan(e*x+d)^4)^(1/2))/e/(a-b+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3781, 1261, 738, 212}

$$\int \frac{\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+(b-2c)\tan^2(d+ex)-b}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e\sqrt{a-b+c}}$$

```
[In] Int[Tan[d + e*x]/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] -1/2*ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a
 + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/(Sqrt[a - b + c]*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{4a-4b+4c-x^2}\,dx, x, \frac{2a-b-(-b+2c)\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{e}\\
&= -\frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int \frac{\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e}$$

```
[In] Integrate[Tan[d + e*x]/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
[Out] -1/2*ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a
 + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/(Sqrt[a - b + c]*e)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.29

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{2a-2b+2c+(b-2c)\left(1+\tan(ex+d)^2\right)+2\sqrt{a-b+c}\,\sqrt{c\left(1+\tan(ex+d)^2\right)^2+(b-2c)\left(1+\tan(ex+d)^2\right)+a-b+c}}{1+\tan(ex+d)^2}\right)}{2e\sqrt{a-b+c}}$	102
default	$-\frac{\ln\left(\frac{2a-2b+2c+(b-2c)\left(1+\tan(ex+d)^2\right)+2\sqrt{a-b+c}\,\sqrt{c\left(1+\tan(ex+d)^2\right)^2+(b-2c)\left(1+\tan(ex+d)^2\right)+a-b+c}}{1+\tan(ex+d)^2}\right)}{2e\sqrt{a-b+c}}$	102

```
[In] int(tan(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/2/e/(a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b-2*c)*(1+tan(e*x+d)^2)+2*(a-b+c)^(1/
2)*(c*(1+tan(e*x+d)^2)^2+(b-2*c)*(1+tan(e*x+d)^2)+a-b+c)^(1/2))/(1+tan(e*x+
d)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 299, normalized size of antiderivative = 3.78

$$\int \frac{\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$$
$$= \left[\frac{\log\left(\frac{(b^2+4\,(a-2\,b)c+8\,c^2)\tan(ex+d)^4+2\,(4\,ab-3\,b^2-4\,(a-b)c)\tan(ex+d)^2-4\,\sqrt{c\tan(ex+d)^4+b\tan(ex+d)^2+a}\left((b-2\,c)\tan(ex+d)^2+2\,a-b\right.}{\tan(ex+d)^4+2\,\tan(ex+d)^2+1}\right.}{4\,\sqrt{a-b+c}e}\right.$$
$$\left.-\frac{\sqrt{-a+b-c}\arctan\left(-\frac{\sqrt{c\tan(ex+d)^4+b\tan(ex+d)^2+a}\left((b-2\,c)\tan(ex+d)^2+2\,a-b\right)\sqrt{-a+b-c}}{2\left(((a-b)c+c^2)\tan(ex+d)^4+(ab-b^2+bc)\tan(ex+d)^2+a^2-ab+ac\right)}\right)}{2\,(a-b+c)e}\right]$$

```
[In] integrate(tan(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x + d)^4 + 2*(4*a*b - 3*b^2 -
 4*(a - b)*c)*tan(e*x + d)^2 - 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 +
 a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b +
b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2 + 1))/(sqrt(a - b + c)*e),
```

```
-1/2*sqrt(-a + b - c)*arctan(-1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2
+ a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(-a + b - c)/(((a - b)*c + c^
2)*tan(e*x + d)^4 + (a*b - b^2 + b*c)*tan(e*x + d)^2 + a^2 - a*b + a*c))/((
a - b + c)*e)]
```

Sympy [F]

$$\int \frac{\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\tan\left(d+ex\right)}{\sqrt{a+b\tan^2\left(d+ex\right)+c\tan^4\left(d+ex\right)}}\,dx$$

```
[In] integrate(tan(e*x+d)/(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2),x)
[Out] Integral(tan(d + e*x)/sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\tan\left(ex+d\right)}{\sqrt{c\tan\left(ex+d\right)^4+b\tan\left(ex+d\right)^2+a}}\,dx$$

```
[In] integrate(tan(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm="
maxima")
[Out] integrate(tan(e*x + d)/sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm="
giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx=\int \frac{\tan(d+e\,x)}{\sqrt{c\tan\left(d+e\,x\right)^4+b\tan\left(d+e\,x\right)^2+a}}\,dx$$

```
[In] int(tan(d + e*x)/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)
[Out] int(tan(d + e*x)/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)
```

3.39 $\int \frac{\cot(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 142

$$\int \frac{\cot(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a}e} + \frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e}$$

```
[Out] -1/2*arctanh(1/2*(2*a+b*tan(e*x+d)^2)/a^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d
)^4)^(1/2))/e/a^(1/2)+1/2*arctanh(1/2*(2*a-b+(b-2*c)*tan(e*x+d)^2)/(a-b+c)^
(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))/e/(a-b+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3781, 1265, 974, 738, 212}

$$\int \frac{\cot(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2a+(b-2c)\tan^2(d+ex)-b}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e\sqrt{a-b+c}} - \frac{\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a}e}$$

```
[In] Int[Cot[d + e*x]/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] -1/2*ArcTanh[(2*a + b*Tan[d + e*x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x]^2
+ c*Tan[d + e*x]^4])]/(Sqrt[a]*e) + ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*
```

```
x]^2)/(2*Sqrt[a - b + c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/(2
*Sqrt[a - b + c]*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(-1-x)\sqrt{a+bx+cx^2}} + \frac{1}{x\sqrt{a+bx+cx^2}}\right) dx, x, \tan^2(d+ex)\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1-x)\sqrt{a+bx+cx^2}} dx, x, \tan^2(d+ex)\right)}{2e} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x, \tan^2(d+ex)\right)}{2e}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+b\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{e}$$
$$- \frac{\text{Subst}\left(\int \frac{1}{4a-4b+4c-x^2} dx, x, \frac{-2a+b-(b-2c)\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{e}$$

$$= -\frac{\text{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a}e} + \frac{\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.98

$$\int \frac{\cot(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} dx$$
$$= \frac{-\frac{\text{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a}} - \frac{\text{arctanh}\left(\frac{-2a+b-(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}}}{e}$$

```
[In] Integrate[Cot[d + e*x]/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] (-1/2*ArcTanh[(2*a + b*Tan[d + e*x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x]^2
 + c*Tan[d + e*x]^4])]/Sqrt[a] - ArcTanh[(-2*a + b - (b - 2*c)*Tan[d + e*x]
^2)/(2*Sqrt[a - b + c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/(2*S
qrt[a - b + c]))/e
```

Maple [F]

$$\int \frac{\cot(ex+d)}{\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}dx$$

```
[In] int(cot(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x)
```

```
[Out] int(cot(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.96 (sec) , antiderivative size = 1015, normalized size of antiderivative = 7.15

$$\int \frac{\cot(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*(sqrt(a - b + c)*a*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x + d)^4 +
 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 + 4*sqrt(c*tan(e*x + d)^4 +
 b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a - b + c)
 + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2 + 1)) +
(a - b + c)*sqrt(a)*log(((b^2 + 4*a*c)*tan(e*x + d)^4 + 8*a*b*tan(e*x + d)^
2 - 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(b*tan(e*x + d)^2 + 2*a
)*sqrt(a) + 8*a^2)/tan(e*x + d)^4))/((a^2 - a*b + a*c)*e), 1/4*(2*sqrt(-a)*
(a - b + c)*arctan(1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(b*tan
(e*x + d)^2 + 2*a)*sqrt(-a)/(a*c*tan(e*x + d)^4 + a*b*tan(e*x + d)^2 + a^2)
) + sqrt(a - b + c)*a*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x + d)^4 + 2
*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 + 4*sqrt(c*tan(e*x + d)^4 + b
*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a - b + c) +
 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2 + 1)))/((a
^2 - a*b + a*c)*e), 1/4*(2*a*sqrt(-a + b - c)*arctan(-1/2*sqrt(c*tan(e*x +
d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(-a +
 b - c)/(((a - b)*c + c^2)*tan(e*x + d)^4 + (a*b - b^2 + b*c)*tan(e*x + d)^
2 + a^2 - a*b + a*c)) + (a - b + c)*sqrt(a)*log(((b^2 + 4*a*c)*tan(e*x + d)
^4 + 8*a*b*tan(e*x + d)^2 - 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)
*(b*tan(e*x + d)^2 + 2*a)*sqrt(a) + 8*a^2)/tan(e*x + d)^4))/((a^2 - a*b + a
*c)*e), 1/2*(sqrt(-a)*(a - b + c)*arctan(1/2*sqrt(c*tan(e*x + d)^4 + b*tan(
e*x + d)^2 + a)*(b*tan(e*x + d)^2 + 2*a)*sqrt(-a)/(a*c*tan(e*x + d)^4 + a*b
*tan(e*x + d)^2 + a^2)) + a*sqrt(-a + b - c)*arctan(-1/2*sqrt(c*tan(e*x + d
)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(-a +
b - c)/(((a - b)*c + c^2)*tan(e*x + d)^4 + (a*b - b^2 + b*c)*tan(e*x + d)^2
 + a^2 - a*b + a*c)))/((a^2 - a*b + a*c)*e)]
```

Sympy [F]

$$\int \frac{\cot(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\cot(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$$

```
[In] integrate(cot(e*x+d)/(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2),x)
[Out] Integral(cot(d + e*x)/sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{\cot(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\cot(ex+d)}{\sqrt{c\tan(ex+d)^4+b\tan(ex+d)^2+a}}\,dx$$

```
[In] integrate(cot(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm="
maxima")
[Out] integrate(cot(e*x + d)/sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a), x)
```

Giac [F]

$$\int \frac{\cot(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\cot(ex+d)}{\sqrt{c\tan(ex+d)^4+b\tan(ex+d)^2+a}}\,dx$$

```
[In] integrate(cot(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm="
giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\cot(d+e\,x)}{\sqrt{c\tan(d+e\,x)^4+b\tan(d+e\,x)^2+a}}\,dx$$

```
[In] int(cot(d + e*x)/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)
[Out] int(cot(d + e*x)/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)
```

3.40 $\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 249

$$\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a}e} + \frac{b\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4a^{3/2}e} - \frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e} - \frac{\cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2ae}$$

```
[Out] 1/4*b*arctanh(1/2*(2*a+b*tan(e*x+d)^2)/a^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+
d)^4)^(1/2))/a^(3/2)/e+1/2*arctanh(1/2*(2*a+b*tan(e*x+d)^2)/a^(1/2)/(a+b*ta
n(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))/e/a^(1/2)-1/2*arctanh(1/2*(2*a-b+(b-2*c)*
tan(e*x+d)^2)/(a-b+c)^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))/e/(a-b
+c)^(1/2)-1/2*cot(e*x+d)^2*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)/a/e
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used

$= \{3781, 1265, 974, 744, 738, 212\}$

$$\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$$
$$= \frac{b\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4a^{3/2}e} + \frac{\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a}e}$$
$$- \frac{\operatorname{arctanh}\left(\frac{2a+(b-2c)\tan^2(d+ex)-b}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e\sqrt{a-b+c}}$$
$$- \frac{\cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2ae}$$

```
[In] Int[Cot[d + e*x]^3/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] ArcTanh[(2*a + b*Tan[d + e*x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x]^2 + c*T
an[d + e*x]^4])]/(2*Sqrt[a]*e) + (b*ArcTanh[(2*a + b*Tan[d + e*x]^2)/(2*Sqr
t[a]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])])/(4*a^(3/2)*e) - ArcTa
nh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a + b*Tan[d
 + e*x]^2 + c*Tan[d + e*x]^4])]/(2*Sqrt[a - b + c]*e) - (Cot[d + e*x]^2*Sqr
t[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])/(2*a*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
```

```
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x^2\sqrt{a+bx+cx^2}}-\frac{1}{x\sqrt{a+bx+cx^2}}+\frac{1}{(1+x)\sqrt{a+bx+cx^2}}\right)dx, x, \tan^2(d+ex)\right)}{2e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e}\\
&\quad -\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e}\\
&\quad +\frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2e}
\end{aligned}$$

$$= -\frac{\cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2ae} + \frac{\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{4a-4b+4c-x^2}\,dx, x, \frac{2a-b-(-b+2c)\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{e} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{4ae}$$

$$= \frac{\text{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a}e} - \frac{\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e} - \frac{\cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2ae} + \frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2ae}$$

$$= \frac{\text{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a}e} + \frac{b\text{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4a^{3/2}e} - \frac{\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e} - \frac{\cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2ae}$$

Mathematica [A] (verified)

Time = 4.42 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.76

$$\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$$

$$= \frac{(2a+b)\text{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right) + 2\sqrt{a}\left(-\frac{a\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{\sqrt{a-b+c}} - \cot^2 \right.}{4a^{3/2}e}$$

```
[In] Integrate[Cot[d + e*x]^3/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] ((2*a + b)*ArcTanh[(2*a + b*Tan[d + e*x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e
*x]^2 + c*Tan[d + e*x]^4])] + 2*Sqrt[a]*(-((a*ArcTanh[(2*a - b + (b - 2*c)*
```

```
Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*
x]^4])])/Sqrt[a - b + c]) - Cot[d + e*x]^2*Sqrt[a + b*Tan[d + e*x]^2 + c*Ta
n[d + e*x]^4]))/(4*a^(3/2)*e)
```

Maple [F]

$$\int \frac{\cot(ex+d)^3}{\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}dx$$

```
[In] int(cot(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x)
[Out] int(cot(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 1.29 (sec) , antiderivative size = 1350, normalized size of antiderivative = 5.42

$$\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="fricas")
[Out] [1/8*(2*sqrt(a - b + c)*a^2*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x + d)
^4 + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 - 4*sqrt(c*tan(e*x + d)
^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a - b
+ c) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2 + 1)
)*tan(e*x + d)^2 + (2*a^2 - a*b - b^2 + (2*a + b)*c)*sqrt(a)*log(((b^2 + 4*
a*c)*tan(e*x + d)^4 + 8*a*b*tan(e*x + d)^2 + 4*sqrt(c*tan(e*x + d)^4 + b*ta
n(e*x + d)^2 + a)*(b*tan(e*x + d)^2 + 2*a)*sqrt(a) + 8*a^2)/tan(e*x + d)^4)
*tan(e*x + d)^2 - 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(a^2 - a*
b + a*c))/((a^3 - a^2*b + a^2*c)*e*tan(e*x + d)^2), 1/4*(sqrt(a - b + c)*a^
2*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x + d)^4 + 2*(4*a*b - 3*b^2 - 4*
(a - b)*c)*tan(e*x + d)^2 - 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)
*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b + b^2
 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2 + 1))*tan(e*x + d)^2 - (2*a^2
- a*b - b^2 + (2*a + b)*c)*sqrt(-a)*arctan(1/2*sqrt(c*tan(e*x + d)^4 + b*ta
n(e*x + d)^2 + a)*(b*tan(e*x + d)^2 + 2*a)*sqrt(-a)/(a*c*tan(e*x + d)^4 + a
*b*tan(e*x + d)^2 + a^2))*tan(e*x + d)^2 - 2*sqrt(c*tan(e*x + d)^4 + b*tan(
e*x + d)^2 + a)*(a^2 - a*b + a*c))/((a^3 - a^2*b + a^2*c)*e*tan(e*x + d)^2)
, -1/8*(4*a^2*sqrt(-a + b - c)*arctan(-1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*
x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(-a + b - c)/(((a -
b)*c + c^2)*tan(e*x + d)^4 + (a*b - b^2 + b*c)*tan(e*x + d)^2 + a^2 - a*b +
```

```
a*c))*tan(e*x + d)^2 - (2*a^2 - a*b - b^2 + (2*a + b)*c)*sqrt(a)*log(((b^2
 + 4*a*c)*tan(e*x + d)^4 + 8*a*b*tan(e*x + d)^2 + 4*sqrt(c*tan(e*x + d)^4 +
 b*tan(e*x + d)^2 + a)*(b*tan(e*x + d)^2 + 2*a)*sqrt(a) + 8*a^2)/tan(e*x +
d)^4)*tan(e*x + d)^2 + 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(a^2
 - a*b + a*c))/((a^3 - a^2*b + a^2*c)*e*tan(e*x + d)^2), -1/4*(2*a^2*sqrt(-
a + b - c)*arctan(-1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b -
2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(-a + b - c)/(((a - b)*c + c^2)*tan(e*x
+ d)^4 + (a*b - b^2 + b*c)*tan(e*x + d)^2 + a^2 - a*b + a*c))*tan(e*x + d)^
2 + (2*a^2 - a*b - b^2 + (2*a + b)*c)*sqrt(-a)*arctan(1/2*sqrt(c*tan(e*x +
d)^4 + b*tan(e*x + d)^2 + a)*(b*tan(e*x + d)^2 + 2*a)*sqrt(-a)/(a*c*tan(e*x
 + d)^4 + a*b*tan(e*x + d)^2 + a^2))*tan(e*x + d)^2 + 2*sqrt(c*tan(e*x + d)
^4 + b*tan(e*x + d)^2 + a)*(a^2 - a*b + a*c))/((a^3 - a^2*b + a^2*c)*e*tan(
e*x + d)^2)]
```

Sympy [F]

$$\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$$

```
[In] integrate(cot(e*x+d)**3/(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2),x)
```

```
[Out] Integral(cot(d + e*x)**3/sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4), x
)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\cot\left(ex+d\right)^3}{\sqrt{c\tan\left(ex+d\right)^4+b\tan\left(ex+d\right)^2+a}}\,dx$$

```
[In] integrate(cot(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\cot(d+e\,x)^3}{\sqrt{c\tan\left(d+e\,x\right)^4+b\tan\left(d+e\,x\right)^2+a}}\,dx$$

```
[In] int(cot(d + e*x)^3/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)
```

```
[Out] int(cot(d + e*x)^3/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)
```

3.41 $\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 662

$$\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$$

$$= \frac{\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e} + \frac{\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{\sqrt{c}e\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)}$$

$$- \frac{\sqrt[4]{a}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{c^{3/4}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

$$+ \frac{\sqrt[4]{a}(\sqrt{a}-2\sqrt{c})\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2}{(\sqrt{a}+}}}{2\left(\sqrt{a}-\sqrt{c}\right)c^{3/4}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

$$+ \frac{\left(\sqrt{a}+\sqrt{c}\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{}}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

```
[Out] 1/2*arctan((a-b+c)^(1/2)*tan(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)
)/e/(a-b+c)^(1/2)+(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)/e/c^(1
/2)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)-a^(1/4)*(cos(2*arctan(c^(1/4)*tan(e*x+d)
/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticE(sin
(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*((a
+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^
(1/2)+c^(1/2)*tan(e*x+d)^2)/c^(3/4)/e/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/
2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*tan(e*x+d)
/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)-2*c^(1/2))*((a+b*tan(e
```

```
*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^
(1/2)*tan(e*x+d)^2)/c^(3/4)/e/(a^(1/2)-c^(1/2))/(a+b*tan(e*x+d)^2+c*tan(e*x
+d)^4)^(1/2)+1/4*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*tan(e*x
+d)/a^(1/4))),-1/4*(a^(1/2)-c^(1/2))^2/a^(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c^(
1/2))^(1/2))*(a^(1/2)+c^(1/2))*((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+
c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^2)/a^(1/4)/c^(1/
4)/e/(a^(1/2)-c^(1/2))/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 662, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3781, 1339, 1117, 1209, 1720}

$$\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$$
$$= \frac{\sqrt[4]{a}(\sqrt{a}-2\sqrt{c})\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\right.\right.}{2c^{3/4}e\left(\sqrt{a}-\sqrt{c}\right)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$-\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}\tan^2(d+ex))\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$+\frac{\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e\sqrt{a-b+c}}$$
$$+\frac{\left(\sqrt{a}+\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(\right.\right.}{\sqrt[4]{a}}\right.}{4\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}-\sqrt{c}\right)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$+\frac{\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{\sqrt{c}e\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)}$$

```
[In] Int[Tan[d + e*x]^4/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] ArcTan[(Sqrt[a - b + c]*Tan[d + e*x])/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d +
 e*x]^4]]/(2*Sqrt[a - b + c]*e) + (Tan[d + e*x]*Sqrt[a + b*Tan[d + e*x]^2 +
 c*Tan[d + e*x]^4])/(Sqrt[c]*e*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)) - (a^(1/
4)*EllipticE[2*ArcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt
[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*
Tan[d + e*x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(c^(3/4)*e*Sqrt[a +
b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) + (a^(1/4)*(Sqrt[a] - 2*Sqrt[c])*Elli
pticF[2*ArcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4
]*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d +
```

```
 e*x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(2*(Sqrt[a] - Sqrt[c])*c^(3
/4)*e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) + ((Sqrt[a] + Sqrt[c])
*EllipticPi[-1/4*(Sqrt[a] - Sqrt[c])^2/(Sqrt[a]*Sqrt[c]), 2*ArcTan[(c^(1/4)
*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt[c]*Ta
n[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/(Sqrt[a] + Sqr
t[c]*Tan[d + e*x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[c])*c^(1/4)*e*Sqrt[a +
b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1339

```
Int[(x_)^4/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[-(2*c*d - a*e*q)/(c*e*(e - d*q))
, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + (-Dist[1/(e*q), Int[(1 - q*x^2)/S
qrt[a + b*x^2 + c*x^4], x], x] + Dist[d^2/(e*(e - d*q)), Int[(1 + q*x^2)/((
d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x])] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\left(1-\frac{\sqrt{c}}{\sqrt{a}}\right)e} \\
&\quad - \frac{\sqrt{a}\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\sqrt{c}e} \\
&\quad + \frac{\left(\sqrt{a}\left(\sqrt{a}-2\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\left(\sqrt{a}-\sqrt{c}\right)\sqrt{c}e} \\
&= \frac{\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e} + \frac{\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{\sqrt{c}e\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)} \\
&\quad - \frac{\sqrt[4]{a}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+e}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{c^{3/4}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&\quad + \frac{\sqrt[4]{a}\left(\sqrt{a}-2\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{}}{2\left(\sqrt{a}-\sqrt{c}\right)c^{3/4}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&\quad + \frac{\left(\sqrt{a}+\sqrt{c}\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}}, 2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.75 (sec) , antiderivative size = 533, normalized size of antiderivative = 0.81

$$\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$$

$$= \frac{\frac{\sqrt{(3a+b+3c+4(a-c)\cos(2(d+ex))+(a-b+c)\cos(4(d+ex)))\sec^4(d+ex)}\sin(2(d+ex))}{\sqrt{2}} + \frac{i\sqrt{2}\left(\left(-b+\sqrt{b^2-4ac}\right)E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,\text{ta}\right.\right.\right.}{}}{}$$

```
[In] Integrate[Tan[d + e*x]^4/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] ((Sqrt[(3*a + b + 3*c + 4*(a - c)*Cos[2*(d + e*x)] + (a - b + c)*Cos[4*(d +
 e*x)])*Sec[d + e*x]^4]*Sin[2*(d + e*x)])/Sqrt[2] + ((I*Sqrt[2]*((-b + Sqrt
[b^2 - 4*a*c])*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*
Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (b + 2*c
- Sqrt[b^2 - 4*a*c])*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a
*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - 2*c
*EllipticPi[(b + Sqrt[b^2 - 4*a*c])/(2*c), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sq
rt[b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*
a*c])])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*Tan[d + e*x]^2)/(b + Sqrt[b^2 - 4
*a*c])]*Sqrt[1 + (2*c*Tan[d + e*x]^2)/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[c/(b +
 Sqrt[b^2 - 4*a*c])] - 4*Cos[d + e*x]*Sin[d + e*x]*(a + b*Tan[d + e*x]^2 +
c*Tan[d + e*x]^4))/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])/(4*c*e)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 646, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)\tan(ex+d)^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)\tan(ex+d)^2}{a}}\operatorname{EllipticF}\left(\frac{\tan(ex+d)\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}, \sqrt{-4+\ldots}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}$
default	$-\frac{\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)\tan(ex+d)^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)\tan(ex+d)^2}{a}}\operatorname{EllipticF}\left(\frac{\tan(ex+d)\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}, \sqrt{-4+\ldots}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}$

```
[In] int(tan(e*x+d)^4/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/e*(-1/4*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(
1/2))/a*tan(e*x+d)^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*tan(e*x+d)^2)^(1/
2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*EllipticF(1/2*tan(e*x+d)*2^(1/2)
*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^
(1/2))-1/2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2
)^(1/2))/a*tan(e*x+d)^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*tan(e*x+d)^2)^
(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ellip
ticF(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+2
^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*tan(e*x+d)^2-1/2/a*
tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*tan(e*x+d)^2+1/2/a*tan(e*
x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*El
lipticPi(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-
4*a*c+b^2)^(1/2))*a,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2)))
```

Fricas [F]

$$\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx=\int \frac{\tan\left(ex+d\right)^4}{\sqrt{c\tan\left(ex+d\right)^4+b\tan\left(ex+d\right)^2+a}}\,dx$$

```
[In] integrate(tan(e*x+d)^4/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(tan(e*x + d)^4/sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a), x)
```

Sympy [F]

$$\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx=\int \frac{\tan^4\left(d+ex\right)}{\sqrt{a+b\tan^2\left(d+ex\right)+c\tan^4\left(d+ex\right)}}\,dx$$

```
[In] integrate(tan(e*x+d)**4/(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2),x)
```

```
[Out] Integral(tan(d + e*x)**4/sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4), x
)
```

Maxima [F]

$$\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\tan(ex+d)^4}{\sqrt{c\tan(ex+d)^4+b\tan(ex+d)^2+a}}\,dx$$

```
[In] integrate(tan(e*x+d)^4/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(tan(e*x + d)^4/sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^4/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\tan(d+e\,x)^4}{\sqrt{c\tan(d+e\,x)^4+b\tan(d+e\,x)^2+a}}\,dx$$

```
[In] int(tan(d + e*x)^4/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)
```

```
[Out] int(tan(d + e*x)^4/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)
```

3.42 $\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 436

$$\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e}$$
$$+\frac{\sqrt[4]{a}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+e\cdots}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}\cdots}{2\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$-\frac{\left(\sqrt{a}+\sqrt{c}\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\cdots}}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

```
[Out] -1/2*arctan((a-b+c)^(1/2)*tan(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2
))/e/(a-b+c)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2
)^(1/2)/cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticF(sin(2*arctan(c^
(1/4)*tan(e*x+d)/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*((a+b*tan(e*x+d
)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2
)*tan(e*x+d)^2)/c^(1/4)/e/(a^(1/2)-c^(1/2))/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^
4)^(1/2)-1/4*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*tan(e*x+d)/
a^(1/4))),-1/4*(a^(1/2)-c^(1/2))^2/a^(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c^(1/2)
)^(1/2))*(a^(1/2)+c^(1/2))*((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1
/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^2)/a^(1/4)/c^(1/4)/e
/(a^(1/2)-c^(1/2))/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3781, 1333, 1117, 1720}

$$\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e\sqrt{a-b+c}} + \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right.\right.}{2\sqrt[4]{c}e\left(\sqrt{a}-\sqrt{c}\right)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{\left(\sqrt{a}+\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan}{\sqrt[4]{}}\right.\right.}{4\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}-\sqrt{c}\right)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

```
[In] Int[Tan[d + e*x]^2/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] -1/2*ArcTan[(Sqrt[a - b + c]*Tan[d + e*x])/Sqrt[a + b*Tan[d + e*x]^2 + c*Ta
n[d + e*x]^4]]/(Sqrt[a - b + c]*e) + (a^(1/4)*EllipticF[2*ArcTan[(c^(1/4)*T
an[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan[
d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/(Sqrt[a] + Sqrt[
c]*Tan[d + e*x]^2)^2])/(2*(Sqrt[a] - Sqrt[c])*c^(1/4)*e*Sqrt[a + b*Tan[d +
e*x]^2 + c*Tan[d + e*x]^4]) - ((Sqrt[a] + Sqrt[c])*EllipticPi[-1/4*(Sqrt[a]
 - Sqrt[c])^2/(Sqrt[a]*Sqrt[c]), 2*ArcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)],
(2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b
*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/
(4*a^(1/4)*(Sqrt[a] - Sqrt[c])*c^(1/4)*e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[
d + e*x]^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1333

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(-a)*((e + d*q)/(c*d^2 - a*e^2))
, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[a*d*((e + d*q)/(c*d^2 - a*e^
2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\left(\sqrt{a}-\sqrt{c}\right)e} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\left(\sqrt{a}-\sqrt{c}\right)e}$$

$$= -\frac{\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e} + \frac{\sqrt[4]{a}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+}{(\sqrt{a}+\sqrt{c}\tan^2}}}{2\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{\left(\sqrt{a}+\sqrt{c}\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}}, 2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.11 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.71

$$\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = -\frac{i\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\tan(d+ex)\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\text{EllipticPi}\left(\frac{b+\sqrt{b^2-4ac}}{2c},i\text{arcsinh}\left(\sqrt{\right.\right.\right.}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}e\sqrt{a+b\tan^2(d+ex)}}$$

[In] Integrate[Tan[d + e*x]^2/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]

[Out] ((-I)*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticPi[(b + Sqrt[b^2 - 4*a*c])/(2*c), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*Tan[d + e*x]^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*Tan[d + e*x]^2)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)\tan(ex+d)^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)\tan(ex+d)^2}{a}}\,\text{EllipticF}\left(\frac{\tan(ex+d)\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\sqrt{-4+\frac{2b(}{}}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}$
default	$\frac{\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)\tan(ex+d)^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)\tan(ex+d)^2}{a}}\,\text{EllipticF}\left(\frac{\tan(ex+d)\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\sqrt{-4+\frac{2b(}{}}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}$

[In] int(tan(e*x+d)^2/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/e*(1/4*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*tan(e*x+d)^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*tan(e*x+d)^2)^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*EllipticF(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*tan(e*x+d)^2

```
-1/2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*tan(e*x+d)^2+1/2/a
*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(
1/2)*EllipticPi(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2
/(-b+(-4*a*c+b^2)^(1/2))*a,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

Fricas [F]

$$\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\tan(ex+d)^2}{\sqrt{c\tan(ex+d)^4+b\tan(ex+d)^2+a}}\,dx$$

```
[In] integrate(tan(e*x+d)^2/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(tan(e*x + d)^2/sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a), x)
```

Sympy [F]

$$\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$$

```
[In] integrate(tan(e*x+d)**2/(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2),x)
```

```
[Out] Integral(tan(d + e*x)**2/sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4), x
)
```

Maxima [F]

$$\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\tan(ex+d)^2}{\sqrt{c\tan(ex+d)^4+b\tan(ex+d)^2+a}}\,dx$$

```
[In] integrate(tan(e*x+d)^2/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(tan(e*x + d)^2/sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^2/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\tan(d+e\,x)^2}{\sqrt{c\tan\left(d+e\,x\right)^4+b\tan\left(d+e\,x\right)^2+a}}\,dx$$

```
[In] int(tan(d + e*x)^2/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)
```

```
[Out] int(tan(d + e*x)^2/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)
```

3.43 $\int \frac{1}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 436

$$\int \frac{1}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e} - \frac{\sqrt[4]{c}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+e\ldots}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{\left(\sqrt{a}+\sqrt{c}\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\ldots}}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

```
[Out] 1/2*arctan((a-b+c)^(1/2)*tan(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)
)/e/(a-b+c)^(1/2)-1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticF(sin(2*arctan(c^(
1/4)*tan(e*x+d)/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*((a+b*tan(e*x+d)
^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)
*tan(e*x+d)^2)/a^(1/4)/e/(a^(1/2)-c^(1/2))/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4
)^(1/2)+1/4*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*tan(e*x+d)/a
^(1/4))),-1/4*(a^(1/2)-c^(1/2))^2/a^(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c^(1/2))
^(1/2))*(a^(1/2)+c^(1/2))*((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/
2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^2)/a^(1/4)/c^(1/4)/e/
(a^(1/2)-c^(1/2))/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1230, 1117, 1720}

$$\int \frac{1}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e\sqrt{a-b+c}}$$
$$-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right.\right.}{2\sqrt[4]{a}e\left(\sqrt{a}-\sqrt{c}\right)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$+\frac{\left(\sqrt{a}+\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan}{\sqrt[4]{}}\right.\right.}{4\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}-\sqrt{c}\right)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

```
[In] Int[1/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] ArcTan[(Sqrt[a - b + c]*Tan[d + e*x])/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d +
 e*x]^4]]/(2*Sqrt[a - b + c]*e) - (c^(1/4)*EllipticF[2*ArcTan[(c^(1/4)*Tan[
d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan[d +
 e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/(Sqrt[a] + Sqrt[c]*
Tan[d + e*x]^2)^2])/(2*a^(1/4)*(Sqrt[a] - Sqrt[c])*e*Sqrt[a + b*Tan[d + e*x
]^2 + c*Tan[d + e*x]^4]) + ((Sqrt[a] + Sqrt[c])*EllipticPi[-1/4*(Sqrt[a] -
Sqrt[c])^2/(Sqrt[a]*Sqrt[c]), 2*ArcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2
- b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Ta
n[d + e*x]^2 + c*Tan[d + e*x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(4*
a^(1/4)*(Sqrt[a] - Sqrt[c])*c^(1/4)*e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d +
 e*x]^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\sqrt{a}\text{Subst}\left(\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\left(\sqrt{a}-\sqrt{c}\right)e} - \frac{\sqrt{c}\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\left(\sqrt{a}-\sqrt{c}\right)e}$$

$$= \frac{\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e} - \frac{\sqrt[4]{c}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+\cdots}{(\sqrt{a}+\sqrt{c}\tan^2\cdots}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{\left(\sqrt{a}+\sqrt{c}\right)\text{EllipticPi}\left(-\frac{(\sqrt{a}-\sqrt{c})^2}{4\sqrt{a}\sqrt{c}}, 2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+\cdots\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = -\frac{i\,\text{EllipticPi}\left(\frac{b+\sqrt{b^2-4ac}}{2c}, i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\tan(d+ex)\right), \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2c\tan^2(d+ex)}{b+\sqrt{b^2-4ac}}}\sqrt{1-\cdots}}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

```
[In] Integrate[1/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] ((-I)*EllipticPi[(b + Sqrt[b^2 - 4*a*c])/(2*c), I*ArcSinh[Sqrt[2]*Sqrt[c/(b
 + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*Tan[d + e*x]^2)/(b + Sqrt[b^2
 - 4*a*c])]*Sqrt[1 - (2*c*Tan[d + e*x]^2)/(-b + Sqrt[b^2 - 4*a*c])])/(Sqrt[
2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d +
e*x]^4])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.53

method	result
derivativedivides	$\frac{\sqrt{2}\,\sqrt{1+\frac{b\tan(ex+d)^2}{2a}-\frac{\tan(ex+d)^2\sqrt{-4ac+b^2}}{2a}}\,\sqrt{1+\frac{b\tan(ex+d)^2}{2a}+\frac{\tan(ex+d)^2\sqrt{-4ac+b^2}}{2a}}\,\operatorname{EllipticPi}\left(\frac{\tan(ex+d)\sqrt{2}\,\sqrt{\frac{-b+\sqrt{}}{}}}{2}\right.}{e\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\,\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}$
default	$\frac{\sqrt{2}\,\sqrt{1+\frac{b\tan(ex+d)^2}{2a}-\frac{\tan(ex+d)^2\sqrt{-4ac+b^2}}{2a}}\,\sqrt{1+\frac{b\tan(ex+d)^2}{2a}+\frac{\tan(ex+d)^2\sqrt{-4ac+b^2}}{2a}}\,\operatorname{EllipticPi}\left(\frac{\tan(ex+d)\sqrt{2}\,\sqrt{\frac{-b+\sqrt{}}{}}}{2}\right.}{e\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\,\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}$

```
[In] int(1/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*tan(e*x+d)^2-1
/2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*tan(e*x+d)^2+1/2/a*t
an(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/
2)*EllipticPi(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(
-b+(-4*a*c+b^2)^(1/2))*a,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx=\int \frac{1}{\sqrt{a+b\tan^2\left(d+ex\right)+c\tan^4\left(d+ex\right)}}\,dx$$

```
[In] integrate(1/(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2),x)
[Out] Integral(1/sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx=\int \frac{1}{\sqrt{c\tan\left(ex+d\right)^4+b\tan\left(ex+d\right)^2+a}}\,dx$$

```
[In] integrate(1/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx=\int \frac{1}{\sqrt{c\tan\left(ex+d\right)^4+b\tan\left(ex+d\right)^2+a}}\,dx$$

```
[In] integrate(1/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx=\int \frac{1}{\sqrt{c\,{\tan\left(d+e\,x\right)}^4+b\,{\tan\left(d+e\,x\right)}^2+a}}\,dx$$

```
[In] int(1/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)
[Out] int(1/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)
```

3.44 $\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 707

$$\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e} - \frac{\cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{ae}$$
$$+ \frac{\sqrt{c}\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{ae\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)}$$
$$- \frac{\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{a^{3/4}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$+ \frac{\left(2\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2}{(\sqrt{a}-}}}{2a^{3/4}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$- \frac{\left(\sqrt{a}+\sqrt{c}\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{}}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

```
[Out] -1/2*arctan((a-b+c)^(1/2)*tan(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2
))/e/(a-b+c)^(1/2)-cot(e*x+d)*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)/a/e+c
^(1/2)*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)/a/e/(a^(1/2)+c^(1
/2)*tan(e*x+d)^2)-c^(1/4)*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/
2)/cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)
*tan(e*x+d)/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*((a+b*tan(e*x+d)^2+c
*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan
(e*x+d)^2)/a^(3/4)/e/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)+1/2*c^(1/4)*(c
```

```
os(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*tan(
e*x+d)/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4))),1/2*(2
-b/a^(1/2)/c^(1/2))^(1/2))*(2*a^(1/2)-c^(1/2))*((a+b*tan(e*x+d)^2+c*tan(e*x
+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^
2)/a^(3/4)/e/(a^(1/2)-c^(1/2))/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)-1/4*
(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*ta
n(e*x+d)/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4))),-1/
4*(a^(1/2)-c^(1/2))^2/a^(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(
1/2)+c^(1/2))*((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d
)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^2)/a^(1/4)/c^(1/4)/e/(a^(1/2)-c^(
1/2))/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3781, 1343, 1728, 1209, 1722, 1117, 1720}

$$\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$$
$$= \frac{\sqrt[4]{c}(2\sqrt{a}-\sqrt{c})\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\right.\right.}{2a^{3/4}e\left(\sqrt{a}-\sqrt{c}\right)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$-\frac{\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e\sqrt{a-b+c}}$$
$$-\frac{\left(\sqrt{a}+\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d}{\sqrt[4]{a}}\right.\right.}{4\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}-\sqrt{c}\right)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$+\frac{\sqrt{c}\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{ae\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)}$$
$$-\frac{\cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{ae}$$

```
[In] Int[Cot[d + e*x]^2/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] -1/2*ArcTan[(Sqrt[a - b + c]*Tan[d + e*x])/Sqrt[a + b*Tan[d + e*x]^2 + c*Ta
n[d + e*x]^4]]/(Sqrt[a - b + c]*e) - (Cot[d + e*x]*Sqrt[a + b*Tan[d + e*x]^
2 + c*Tan[d + e*x]^4])/(a*e) + (Sqrt[c]*Tan[d + e*x]*Sqrt[a + b*Tan[d + e*x
]^2 + c*Tan[d + e*x]^4])/(a*e*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)) - (c^(1/4
```

```
)*EllipticE[2*ArcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[
c]))/4]*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*T
an[d + e*x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(a^(3/4)*e*Sqrt[a + b
*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) + ((2*Sqrt[a] - Sqrt[c])*c^(1/4)*Ellip
ticF[2*ArcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
*(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d +
e*x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(2*a^(3/4)*(Sqrt[a] - Sqrt[c
])*e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) - ((Sqrt[a] + Sqrt[c])*
EllipticPi[-1/4*(Sqrt[a] - Sqrt[c])^2/(Sqrt[a]*Sqrt[c]), 2*ArcTan[(c^(1/4)*
Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan
[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/(Sqrt[a] + Sqrt
[c]*Tan[d + e*x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[c])*c^(1/4)*e*Sqrt[a + b
*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1343

```
Int[(x_)^(m_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4]), x_Symbol] :> Simp[x^(m + 1)*(Sqrt[a + b*x^2 + c*x^4]/(a*d*(m + 1))), x
] - Dist[1/(a*d*(m + 1)), Int[(x^(m + 2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^
4]))*Simp[a*e*(m + 1) + b*d*(m + 2) + (b*e*(m + 2) + c*d*(m + 3))*x^2 + c*e
*(m + 3)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && ILtQ[m/2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
```

```
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{e} \\
&= -\frac{\cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{ae} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{-a+cx^2+cx^4}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{ae}
\end{aligned}
$$

$$= -\frac{\cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{ae} + \frac{\text{Subst}\left(\int \frac{-ac+\sqrt{a}c^{3/2}+(c^2-c(-\sqrt{a}\sqrt{c}+c))x^2}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx,x,\tan(d+ex)\right)}{ace} - \frac{\sqrt{c}\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx,x,\tan(d+ex)\right)}{\sqrt{a}e}$$

$$= -\frac{\cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{ae} + \frac{\sqrt{c}\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{ae\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)} - \frac{\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{a^{3/4}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{\sqrt{a}\text{Subst}\left(\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx,x,\tan(d+ex)\right)}{\left(\sqrt{a}-\sqrt{c}\right)e} + \frac{\left(\left(2-\frac{\sqrt{c}}{\sqrt{a}}\right)\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx,x,\tan(d+ex)\right)}{\left(\sqrt{a}-\sqrt{c}\right)e}$$

$$= -\frac{\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2\sqrt{a-b+c}e} - \frac{\cot(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{ae} + \frac{\sqrt{c}\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{ae\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)} - \frac{\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{a^{3/4}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{\left(2\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{}}{2a^{3/4}\left(\sqrt{a}-\sqrt{c}\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{\left(\sqrt{a}+\sqrt{c}\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d-\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.21 (sec) , antiderivative size = 683, normalized size of antiderivative = 0.97

$$\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$$
$$= \frac{\sqrt{\frac{3a+b+3c+4a\cos(2(d+ex))-4c\cos(2(d+ex))+a\cos(4(d+ex))-b\cos(4(d+ex))+c\cos(4(d+ex))}{3+4\cos(2(d+ex))+\cos(4(d+ex))}}\left(-\frac{\cot(d+ex)}{a}+\frac{\sin(2(d+ex))}{2a}\right)}{e}$$
$$+ \frac{\frac{i\sqrt{2}\left(-b+\sqrt{b^2-4ac}\right)\left(E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\tan(d+ex)\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\tan(d+ex)\right),\frac{b+\sqrt{}}{b-\sqrt{}}\right.\right.}{\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}}}{}$$

```
[In] Integrate[Cot[d + e*x]^2/Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4],x]
```

```
[Out] (Sqrt[(3*a + b + 3*c + 4*a*Cos[2*(d + e*x)] - 4*c*Cos[2*(d + e*x)] + a*Cos[
4*(d + e*x)] - b*Cos[4*(d + e*x)] + c*Cos[4*(d + e*x)])/(3 + 4*Cos[2*(d + e
*x)] + Cos[4*(d + e*x)])]*(-(Cot[d + e*x]/a) + Sin[2*(d + e*x)]/(2*a)))/e +
 ((I*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c])*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^
2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*
Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*Tan[d + e*x]^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (
2*c*Tan[d + e*x]^2)/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)] + ((2*I)*Sqrt[2]*a*EllipticPi[(b + Sqrt[b^2 - 4*a*c])/(2*c), I*ArcSinh[S
qrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c
])/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*Tan[d + e*x]^
2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*Tan[d + e*x]^2)/(b - Sqrt[b^2 - 4
*a*c])])/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])] - (4*Tan[d + e*x]*(a + b*Tan[d + e
*x]^2 + c*Tan[d + e*x]^4))/(1 + Tan[d + e*x]^2))/(4*a*e*Sqrt[a + b*Tan[d +
e*x]^2 + c*Tan[d + e*x]^4])
```

Maple [F]

$$\int \frac{\cot(ex+d)^2}{\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}dx$$

```
[In] int(cot(e*x+d)^2/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x)
```

```
[Out] int(cot(e*x+d)^2/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(e*x+d)^2/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx$$

```
[In] integrate(cot(e*x+d)**2/(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(1/2),x)
```

```
[Out] Integral(cot(d + e*x)**2/sqrt(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4), x
)
```

Maxima [F]

$$\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\cot(ex+d)^2}{\sqrt{c\tan(ex+d)^4+b\tan(ex+d)^2+a}}\,dx$$

```
[In] integrate(cot(e*x+d)^2/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(cot(e*x + d)^2/sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a), x)
```

Giac [F]

$$\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx = \int \frac{\cot(ex+d)^2}{\sqrt{c\tan(ex+d)^4+b\tan(ex+d)^2+a}}\,dx$$

```
[In] integrate(cot(e*x+d)^2/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2),x, algorithm
="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\,dx=\int \frac{\cot(d+e\,x)^2}{\sqrt{c\tan\left(d+e\,x\right)^4+b\tan\left(d+e\,x\right)^2+a}}\,dx$$

```
[In] int(cot(d + e*x)^2/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2),x)
[Out] int(cot(d + e*x)^2/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(1/2), x)
```

3.45 $\int \frac{\tan^7(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 235

$$\int \frac{\tan^7(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2(a-b+c)^{3/2}e} + \frac{\operatorname{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2c^{3/2}e} + \frac{a(b^2-a(b+2c))+(b^3+2a^2c-ab(b+3c))\tan^2(d+ex)}{c(a-b+c)(b^2-4ac)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

```
[Out] 1/2*arctanh(1/2*(2*a-b+(b-2*c)*tan(e*x+d)^2)/(a-b+c)^(1/2)/(a+b*tan(e*x+d)^
2+c*tan(e*x+d)^4)^(1/2))/(a-b+c)^(3/2)/e+1/2*arctanh(1/2*(b+2*c*tan(e*x+d)^
2)/c^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))/c^(3/2)/e+(a*(b^2-a*(b+
2*c))+(b^3+2*a^2*c-a*b*(b+3*c))*tan(e*x+d)^2)/c/(a-b+c)/(-4*a*c+b^2)/e/(a+b
*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3781, 1265, 1660, 857, 635, 212, 738}

$$\int \frac{\tan^7(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \frac{(2a^2c-ab(b+3c)+b^3)\tan^2(d+ex)+a(b^2-a(b+2c}{ce(a-b+c)(b^2-4ac)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+}} + \frac{\operatorname{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2c^{3/2}e} + \frac{\operatorname{arctanh}\left(\frac{2a+(b-2c)\tan^2(d+ex)-b}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e(a-b+c)^{3/2}}$$

```
[In] Int[Tan[d + e*x]^7/(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)^(3/2),x]
```

```
[Out] ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a + b*
Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/(2*(a - b + c)^(3/2)*e) + ArcTanh[(b +
 2*c*Tan[d + e*x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^
4])]/(2*c^(3/2)*e) + (a*(b^2 - a*(b + 2*c)) + (b^3 + 2*a^2*c - a*b*(b + 3*c
))*Tan[d + e*x]^2)/(c*(a - b + c)*(b^2 - 4*a*c)*e*Sqrt[a + b*Tan[d + e*x]^2
 + c*Tan[d + e*x]^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
```

```
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^7}{(1+x^2)(a+bx^2+cx^4)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{x^3}{(1+x)(a+bx+cx^2)^{3/2}}\,dx, x, \tan^2(d+ex)\right)}{2e}\\
&= \frac{a(b^2-a(b+2c))+(b^3+2a^2c-ab(b+3c))\tan^2(d+ex)}{c(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\\
&\quad - \frac{\text{Subst}\left(\int \frac{-\frac{(a-b)\left(b^2-4ac\right)}{2c(a-b+c)}-\frac{\left(b^2-4ac\right)x}{2c}}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{\left(b^2-4ac\right)e}\\
&= \frac{a(b^2-a(b+2c))+(b^3+2a^2c-ab(b+3c))\tan^2(d+ex)}{c(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2ce}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2(a-b+c)e}
\end{aligned}$$

$$= \frac{a(b^2-a(b+2c))+(b^3+2a^2c-ab(b+3c))\tan^2(d+ex)}{c(a-b+c)\,(b^2-4ac)\,e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{ce} + \frac{\text{Subst}\left(\int \frac{1}{4a-4b+4c-x^2}\,dx, x, \frac{2a-b-(-b+2c)\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{(a-b+c)e}$$

$$= \frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2(a-b+c)^{3/2}e} + \frac{\operatorname{arctanh}\left(\frac{b+2c\tan^2(d+ex)}{2\sqrt{c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2c^{3/2}e} + \frac{a(b^2-a(b+2c))+(b^3+2a^2c-ab(b+3c))\tan^2(d+ex)}{c(a-b+c)\,(b^2-4ac)\,e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

Mathematica [A] (verified)

Time = 7.14 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.17

$$\int \frac{\tan^7(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx = \frac{\frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{(a-b+c)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{b+}{2\sqrt{c}\sqrt{a+b\tan}}\right.}{c^3}}{}$$

```
[In] Integrate[Tan[d + e*x]^7/(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)^(3/2),x]
```

```
[Out] (ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a + b
*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/(a - b + c)^(3/2) + ArcTanh[(b + 2*c*
Tan[d + e*x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/
c^(3/2) + (2*Sqrt[2]*(b*(a^2 - b^2 + 3*a*c) + (b^3 - a*b*(2*b + 3*c) + a^2*
(b + 4*c))*Cos[2*(d + e*x)])*Sec[d + e*x]^2)/(c*(a - b + c)*(-b^2 + 4*a*c)*
Sqrt[(3*a + b + 3*c + 4*(a - c)*Cos[2*(d + e*x)] + (a - b + c)*Cos[4*(d + e
*x)])*Sec[d + e*x]^4]))/(2*e)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. 2(213) = 426.

Time = 0.52 (sec) , antiderivative size = 684, normalized size of antiderivative = 2.91

method	result
derivativedivides	$\frac{\frac{b+2c\tan(ex+d)^2}{\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}\,(4ac-b^2)}-\frac{\tan(ex+d)^2}{2c\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}-\frac{b\left(-\frac{1}{c\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}-\frac{}{c\sqrt{a+b}}\right.}{4c}}{}$
default	$\frac{\frac{b+2c\tan(ex+d)^2}{\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}\,(4ac-b^2)}-\frac{\tan(ex+d)^2}{2c\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}-\frac{b\left(-\frac{1}{c\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}}-\frac{}{c\sqrt{a+b}}\right.}{4c}}{}$

```
[In] int(tan(e*x+d)^7/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/e*(1/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*(b+2*c*tan(e*x+d)^2)/(4*a*c-
b^2)-1/2*tan(e*x+d)^2/c/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)-1/4*b/c*(-1
/c/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)-b/c/(a+b*tan(e*x+d)^2+c*tan(e*x+
d)^4)^(1/2)*(b+2*c*tan(e*x+d)^2)/(4*a*c-b^2))+1/2/c^(3/2)*ln((1/2*b+c*tan(e
*x+d)^2)/c^(1/2)+(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))+1/(a+b*tan(e*x+d)
^2+c*tan(e*x+d)^4)^(1/2)*(2*a+b*tan(e*x+d)^2)/(4*a*c-b^2)-2*c/((-4*a*c+b^2)
^(1/2)-b+2*c)/((-4*a*c+b^2)^(1/2)+b-2*c)/(a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b-2
*c)*(1+tan(e*x+d)^2)+2*(a-b+c)^(1/2)*(c*(1+tan(e*x+d)^2)^2+(b-2*c)*(1+tan(e
*x+d)^2)+a-b+c)^(1/2))/(1+tan(e*x+d)^2))+2*c/((-4*a*c+b^2)^(1/2)-b+2*c)/(-4
*a*c+b^2)/(tan(e*x+d)^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)*(c*(tan(e*x+d)^2-1/2
*(-b+(-4*a*c+b^2)^(1/2))/c)^2+(-4*a*c+b^2)^(1/2)*(tan(e*x+d)^2-1/2*(-b+(-4*
a*c+b^2)^(1/2))/c))^(1/2)-2*c/((-4*a*c+b^2)^(1/2)+b-2*c)/(-4*a*c+b^2)/(tan(
e*x+d)^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(c*(tan(e*x+d)^2+1/2*(b+(-4*a*c+b^2)
^(1/2))/c)^2-(-4*a*c+b^2)^(1/2)*(tan(e*x+d)^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(216) = 432$.

Time = 2.49 (sec) , antiderivative size = 3773, normalized size of antiderivative = 16.06

$$\int \frac{\tan^7(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(e*x+d)^7/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm
="fricas")
```

```
[Out] [-1/4*((a^3*b^2 - 2*a^2*b^3 + a*b^4 - 4*a^2*c^3 - (4*a*c^4 + (8*a^2 - 8*a*b
 - b^2)*c^3 + 2*(2*a^3 - 4*a^2*b + a*b^2 + b^3)*c^2 - (a^2*b^2 - 2*a*b^3 +
b^4)*c)*tan(e*x + d)^4 - (8*a^3 - 8*a^2*b - a*b^2)*c^2 + (a^2*b^3 - 2*a*b^4
 + b^5 - 4*a*b*c^3 - (8*a^2*b - 8*a*b^2 - b^3)*c^2 - 2*(2*a^3*b - 4*a^2*b^2
```

```
 + a*b^3 + b^4)*c)*tan(e*x + d)^2 - 2*(2*a^4 - 4*a^3*b + a^2*b^2 + a*b^3)*c
)*sqrt(c)*log(8*c^2*tan(e*x + d)^4 + 8*b*c*tan(e*x + d)^2 + b^2 + 4*sqrt(c*
tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*tan(e*x + d)^2 + b)*sqrt(c) + 4
*a*c) + (a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*tan(e*x + d)^4 + (b^3*
c^2 - 4*a*b*c^3)*tan(e*x + d)^2)*sqrt(a - b + c)*log(((b^2 + 4*(a - 2*b)*c
+ 8*c^2)*tan(e*x + d)^4 + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 +
4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 +
 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 +
2*tan(e*x + d)^2 + 1)) - 4*(2*a^2*c^3 + (2*a^3 - a^2*b - a*b^2)*c^2 - ((2*a
^2 - 3*a*b)*c^3 + (2*a^3 - 5*a^2*b + 2*a*b^2 + b^3)*c^2 - (a^2*b^2 - 2*a*b^
3 + b^4)*c)*tan(e*x + d)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*c)*sqrt(c*tan(e*x
+ d)^4 + b*tan(e*x + d)^2 + a))/((4*a*c^6 + (8*a^2 - 8*a*b - b^2)*c^5 + 2*(
2*a^3 - 4*a^2*b + a*b^2 + b^3)*c^4 - (a^2*b^2 - 2*a*b^3 + b^4)*c^3)*e*tan(e
*x + d)^4 + (4*a*b*c^5 + (8*a^2*b - 8*a*b^2 - b^3)*c^4 + 2*(2*a^3*b - 4*a^2
*b^2 + a*b^3 + b^4)*c^3 - (a^2*b^3 - 2*a*b^4 + b^5)*c^2)*e*tan(e*x + d)^2 +
 (4*a^2*c^5 + (8*a^3 - 8*a^2*b - a*b^2)*c^4 + 2*(2*a^4 - 4*a^3*b + a^2*b^2
+ a*b^3)*c^3 - (a^3*b^2 - 2*a^2*b^3 + a*b^4)*c^2)*e), 1/4*(2*(a^3*b^2 - 2*a
^2*b^3 + a*b^4 - 4*a^2*c^3 - (4*a*c^4 + (8*a^2 - 8*a*b - b^2)*c^3 + 2*(2*a^
3 - 4*a^2*b + a*b^2 + b^3)*c^2 - (a^2*b^2 - 2*a*b^3 + b^4)*c)*tan(e*x + d)^
4 - (8*a^3 - 8*a^2*b - a*b^2)*c^2 + (a^2*b^3 - 2*a*b^4 + b^5 - 4*a*b*c^3 -
(8*a^2*b - 8*a*b^2 - b^3)*c^2 - 2*(2*a^3*b - 4*a^2*b^2 + a*b^3 + b^4)*c)*ta
n(e*x + d)^2 - 2*(2*a^4 - 4*a^3*b + a^2*b^2 + a*b^3)*c)*sqrt(-c)*arctan(1/2
*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*c*tan(e*x + d)^2 + b)*sqr
t(-c)/(c^2*tan(e*x + d)^4 + b*c*tan(e*x + d)^2 + a*c)) - (a*b^2*c^2 - 4*a^2
*c^3 + (b^2*c^3 - 4*a*c^4)*tan(e*x + d)^4 + (b^3*c^2 - 4*a*b*c^3)*tan(e*x +
 d)^2)*sqrt(a - b + c)*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x + d)^4 +
2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 + 4*sqrt(c*tan(e*x + d)^4 +
b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a - b + c)
+ 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2 + 1)) + 4
*(2*a^2*c^3 + (2*a^3 - a^2*b - a*b^2)*c^2 - ((2*a^2 - 3*a*b)*c^3 + (2*a^3 -
 5*a^2*b + 2*a*b^2 + b^3)*c^2 - (a^2*b^2 - 2*a*b^3 + b^4)*c)*tan(e*x + d)^2
 + (a^3*b - 2*a^2*b^2 + a*b^3)*c)*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2
+ a))/((4*a*c^6 + (8*a^2 - 8*a*b - b^2)*c^5 + 2*(2*a^3 - 4*a^2*b + a*b^2 +
b^3)*c^4 - (a^2*b^2 - 2*a*b^3 + b^4)*c^3)*e*tan(e*x + d)^4 + (4*a*b*c^5 + (
8*a^2*b - 8*a*b^2 - b^3)*c^4 + 2*(2*a^3*b - 4*a^2*b^2 + a*b^3 + b^4)*c^3 -
(a^2*b^3 - 2*a*b^4 + b^5)*c^2)*e*tan(e*x + d)^2 + (4*a^2*c^5 + (8*a^3 - 8*a
^2*b - a*b^2)*c^4 + 2*(2*a^4 - 4*a^3*b + a^2*b^2 + a*b^3)*c^3 - (a^3*b^2 -
2*a^2*b^3 + a*b^4)*c^2)*e), -1/4*(2*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a
*c^4)*tan(e*x + d)^4 + (b^3*c^2 - 4*a*b*c^3)*tan(e*x + d)^2)*sqrt(-a + b -
c)*arctan(-1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan
(e*x + d)^2 + 2*a - b)*sqrt(-a + b - c)/(((a - b)*c + c^2)*tan(e*x + d)^4 +
 (a*b - b^2 + b*c)*tan(e*x + d)^2 + a^2 - a*b + a*c)) + (a^3*b^2 - 2*a^2*b^
3 + a*b^4 - 4*a^2*c^3 - (4*a*c^4 + (8*a^2 - 8*a*b - b^2)*c^3 + 2*(2*a^3 - 4
*a^2*b + a*b^2 + b^3)*c^2 - (a^2*b^2 - 2*a*b^3 + b^4)*c)*tan(e*x + d)^4 - (
8*a^3 - 8*a^2*b - a*b^2)*c^2 + (a^2*b^3 - 2*a*b^4 + b^5 - 4*a*b*c^3 - (8*a^
```

```
2*b - 8*a*b^2 - b^3)*c^2 - 2*(2*a^3*b - 4*a^2*b^2 + a*b^3 + b^4)*c)*tan(e*x
 + d)^2 - 2*(2*a^4 - 4*a^3*b + a^2*b^2 + a*b^3)*c)*sqrt(c)*log(8*c^2*tan(e*
x + d)^4 + 8*b*c*tan(e*x + d)^2 + b^2 + 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x
 + d)^2 + a)*(2*c*tan(e*x + d)^2 + b)*sqrt(c) + 4*a*c) - 4*(2*a^2*c^3 + (2*
a^3 - a^2*b - a*b^2)*c^2 - ((2*a^2 - 3*a*b)*c^3 + (2*a^3 - 5*a^2*b + 2*a*b^
2 + b^3)*c^2 - (a^2*b^2 - 2*a*b^3 + b^4)*c)*tan(e*x + d)^2 + (a^3*b - 2*a^2
*b^2 + a*b^3)*c)*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a))/((4*a*c^6 +
 (8*a^2 - 8*a*b - b^2)*c^5 + 2*(2*a^3 - 4*a^2*b + a*b^2 + b^3)*c^4 - (a^2*b
^2 - 2*a*b^3 + b^4)*c^3)*e*tan(e*x + d)^4 + (4*a*b*c^5 + (8*a^2*b - 8*a*b^2
 - b^3)*c^4 + 2*(2*a^3*b - 4*a^2*b^2 + a*b^3 + b^4)*c^3 - (a^2*b^3 - 2*a*b^
4 + b^5)*c^2)*e*tan(e*x + d)^2 + (4*a^2*c^5 + (8*a^3 - 8*a^2*b - a*b^2)*c^4
 + 2*(2*a^4 - 4*a^3*b + a^2*b^2 + a*b^3)*c^3 - (a^3*b^2 - 2*a^2*b^3 + a*b^4
)*c^2)*e), -1/2*((a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*tan(e*x + d)^
4 + (b^3*c^2 - 4*a*b*c^3)*tan(e*x + d)^2)*sqrt(-a + b - c)*arctan(-1/2*sqrt
(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a -
 b)*sqrt(-a + b - c)/(((a - b)*c + c^2)*tan(e*x + d)^4 + (a*b - b^2 + b*c)*
tan(e*x + d)^2 + a^2 - a*b + a*c)) - (a^3*b^2 - 2*a^2*b^3 + a*b^4 - 4*a^2*c
^3 - (4*a*c^4 + (8*a^2 - 8*a*b - b^2)*c^3 + 2*(2*a^3 - 4*a^2*b + a*b^2 + b^
3)*c^2 - (a^2*b^2 - 2*a*b^3 + b^4)*c)*tan(e*x + d)^4 - (8*a^3 - 8*a^2*b - a
*b^2)*c^2 + (a^2*b^3 - 2*a*b^4 + b^5 - 4*a*b*c^3 - (8*a^2*b - 8*a*b^2 - b^3
)*c^2 - 2*(2*a^3*b - 4*a^2*b^2 + a*b^3 + b^4)*c)*tan(e*x + d)^2 - 2*(2*a^4
- 4*a^3*b + a^2*b^2 + a*b^3)*c)*sqrt(-c)*arctan(1/2*sqrt(c*tan(e*x + d)^4 +
 b*tan(e*x + d)^2 + a)*(2*c*tan(e*x + d)^2 + b)*sqrt(-c)/(c^2*tan(e*x + d)^
4 + b*c*tan(e*x + d)^2 + a*c)) - 2*(2*a^2*c^3 + (2*a^3 - a^2*b - a*b^2)*c^2
 - ((2*a^2 - 3*a*b)*c^3 + (2*a^3 - 5*a^2*b + 2*a*b^2 + b^3)*c^2 - (a^2*b^2
- 2*a*b^3 + b^4)*c)*tan(e*x + d)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*c)*sqrt(c*
tan(e*x + d)^4 + b*tan(e*x + d)^2 + a))/((4*a*c^6 + (8*a^2 - 8*a*b - b^2)*c
^5 + 2*(2*a^3 - 4*a^2*b + a*b^2 + b^3)*c^4 - (a^2*b^2 - 2*a*b^3 + b^4)*c^3)
*e*tan(e*x + d)^4 + (4*a*b*c^5 + (8*a^2*b - 8*a*b^2 - b^3)*c^4 + 2*(2*a^3*b
 - 4*a^2*b^2 + a*b^3 + b^4)*c^3 - (a^2*b^3 - 2*a*b^4 + b^5)*c^2)*e*tan(e*x
+ d)^2 + (4*a^2*c^5 + (8*a^3 - 8*a^2*b - a*b^2)*c^4 + 2*(2*a^4 - 4*a^3*b +
a^2*b^2 + a*b^3)*c^3 - (a^3*b^2 - 2*a^2*b^3 + a*b^4)*c^2)*e)]
```

Sympy [F]

$$\int \frac{\tan^7(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx=\int \frac{\tan^7\left(d+ex\right)}{\left(a+b\tan^2\left(d+ex\right)+c\tan^4\left(d+ex\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(e*x+d)**7/(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(3/2),x)
```

```
[Out] Integral(tan(d + e*x)**7/(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4)**(3/2)
, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^7(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(tan(e*x+d)^7/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^7(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^7/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^7(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \int \frac{\tan(d+e\,x)^7}{\left(c\tan\left(d+e\,x\right)^4+b\tan\left(d+e\,x\right)^2+a\right)^{3/2}}\,dx$$

```
[In] int(tan(d + e*x)^7/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(3/2),x)
```

```
[Out] int(tan(d + e*x)^7/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(3/2), x)
```

3.46 $\int \frac{\tan^5(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 159

$$\int \frac{\tan^5(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2(a-b+c)^{3/2}e} + \frac{a(2a-b)+((a-b)b+2ac)\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

```
[Out] -1/2*arctanh(1/2*(2*a-b+(b-2*c)*tan(e*x+d)^2)/(a-b+c)^(1/2)/(a+b*tan(e*x+d)
^2+c*tan(e*x+d)^4)^(1/2))/(a-b+c)^(3/2)/e+(a*(2*a-b)+((a-b)*b+2*a*c)*tan(e*
x+d)^2)/(a-b+c)/(-4*a*c+b^2)/e/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3781, 1265, 1660, 12, 738, 212}

$$\int \frac{\tan^5(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx = \frac{(b(a-b)+2ac)\tan^2(d+ex)+a(2a-b)}{e(a-b+c)\left(b^2-4ac\right)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{\operatorname{arctanh}\left(\frac{2a+(b-2c)\tan^2(d+ex)-b}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e(a-b+c)^{3/2}}$$

```
[In] Int[Tan[d + e*x]^5/(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)^(3/2),x]
```

```
[Out] -1/2*ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a
 + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/((a - b + c)^(3/2)*e) + (a*(2*a -
 b) + ((a - b)*b + 2*a*c)*Tan[d + e*x]^2)/((a - b + c)*(b^2 - 4*a*c)*e*Sqrt
[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
```

```
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)(a+bx^2+cx^4)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)(a+bx+cx^2)^{3/2}}\,dx, x, \tan^2(d+ex)\right)}{2e}\\
&= \frac{a(2a-b)+((a-b)b+2ac)\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\\
&\quad-\frac{\text{Subst}\left(\int -\frac{b^2-4ac}{2(a-b+c)(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{\left(b^2-4ac\right)e}\\
&= \frac{a(2a-b)+((a-b)b+2ac)\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2(a-b+c)e}\\
&= \frac{a(2a-b)+((a-b)b+2ac)\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\\
&\quad-\frac{\text{Subst}\left(\int \frac{1}{4a-4b+4c-x^2}\,dx, x, \frac{2a-b-(-b+2c)\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{(a-b+c)e}\\
&= -\frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2(a-b+c)^{3/2}e}\\
&\quad+\frac{a(2a-b)+((a-b)b+2ac)\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 4.90 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.30

$$\int \frac{\tan^5(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx =$$

$$-\frac{\frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{(a-b+c)^{3/2}}+\frac{2\sqrt{2}\left(2a^2-b^2+2ac+\left(2a^2+b^2-2a(b+c)\right)\cos(2(d+ex))\right)\sec^2(d+ex)}{(a-b+c)(-b^2+4ac)\sqrt{(3a+b+3c+4(a-c)\cos(2(d+ex))+(a-b+c)\cos(4(d+ex)))\sec^4(\cdots}}}{2e}$$

[In] Integrate[Tan[d + e*x]^5/(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)^(3/2),x]

[Out] -1/2*(ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/(a - b + c)^(3/2) + (2*Sqrt[2]*(2*a^2 - b^2 + 2*a*c + (2*a^2 + b^2 - 2*a*(b + c))*Cos[2*(d + e*x)])*Sec[d + e*x]^2)/((a - b + c)*(-b^2 + 4*a*c)*Sqrt[(3*a + b + 3*c + 4*(a - c)*Cos[2*(d + e*x)] + (a - b + c)*Cos[4*(d + e*x)])*Sec[d + e*x]^4]))/e

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(147) = 294$.

Time = 0.10 (sec) , antiderivative size = 509, normalized size of antiderivative = 3.20

method	result
derivativedivides	$\frac{-\frac{2a+b\tan(ex+d)^2}{\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}\,\left(4ac-b^2\right)}-\frac{b+2c\tan(ex+d)^2}{\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}\,\left(4ac-b^2\right)}+\frac{2c\ln\left(\frac{2a-2b+2c+(b-2c)\left(1+\tan(ex+d)\cdots\right.}{\cdots}\right)}{\left(\sqrt{-4ac\cdots}\right.\cdots}}{\cdots}$
default	$\frac{-\frac{2a+b\tan(ex+d)^2}{\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}\,\left(4ac-b^2\right)}-\frac{b+2c\tan(ex+d)^2}{\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}\,\left(4ac-b^2\right)}+\frac{2c\ln\left(\frac{2a-2b+2c+(b-2c)\left(1+\tan(ex+d)\cdots\right.}{\cdots}\right)}{\left(\sqrt{-4ac\cdots}\right.\cdots}}{\cdots}$

[In] int(tan(e*x+d)^5/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/e*(-1/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*(2*a+b*tan(e*x+d)^2)/(4*a*c-b^2)-1/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*(b+2*c*tan(e*x+d)^2)/(4*a*c-b^2)+2*c/((-4*a*c+b^2)^(1/2)-b+2*c)/((-4*a*c+b^2)^(1/2)+b-2*c)/(a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b-2*c)*(1+tan(e*x+d)^2)+2*(a-b+c)^(1/2)*(c*(1+tan(e*x+d)^2)^2+(b-2*c)*(1+tan(e*x+d)^2)+a-b+c)^(1/2))/(1+tan(e*x+d)^2))-2*c/((-4*a*c+b^2)^(1/2)-b+2*c)/(-4*a*c+b^2)/(tan(e*x+d)^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)*(c*(tan(e*x+d)^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2+(-4*a*c+b^2)^(1/2)*(tan(e*x+d)^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2)+2*c/((-4*a*c+b^2)^(1/2)+b-2*c)/(-4*a*c+b^2)/(tan(e*x+d)^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(c*(tan(e*x+d)^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2-(-4*a*c+b^2)^(1/2)*(tan(e*x+d)^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(147) = 294.

Time = 0.65 (sec) , antiderivative size = 1095, normalized size of antiderivative = 6.89

$$\int \frac{\tan^5(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(e*x+d)^5/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm
="fricas")
```

```
[Out] [-1/4*(((b^2*c - 4*a*c^2)*tan(e*x + d)^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c
)*tan(e*x + d)^2)*sqrt(a - b + c)*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*
x + d)^4 + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 - 4*sqrt(c*tan(e*
x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(
a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^
2 + 1)) + 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(2*a^3 - 3*a^2*b
+ a*b^2 + (a^2*b - 2*a*b^2 + b^3 + 2*a*c^2 + (2*a^2 - a*b - b^2)*c)*tan(e*x
 + d)^2 + (2*a^2 - a*b)*c))/((4*a*c^4 + (8*a^2 - 8*a*b - b^2)*c^3 + 2*(2*a^
3 - 4*a^2*b + a*b^2 + b^3)*c^2 - (a^2*b^2 - 2*a*b^3 + b^4)*c)*e*tan(e*x + d
)^4 - (a^2*b^3 - 2*a*b^4 + b^5 - 4*a*b*c^3 - (8*a^2*b - 8*a*b^2 - b^3)*c^2
- 2*(2*a^3*b - 4*a^2*b^2 + a*b^3 + b^4)*c)*e*tan(e*x + d)^2 - (a^3*b^2 - 2*
a^2*b^3 + a*b^4 - 4*a^2*c^3 - (8*a^3 - 8*a^2*b - a*b^2)*c^2 - 2*(2*a^4 - 4*
a^3*b + a^2*b^2 + a*b^3)*c)*e), 1/2*(((b^2*c - 4*a*c^2)*tan(e*x + d)^4 + a*
b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*tan(e*x + d)^2)*sqrt(-a + b - c)*arctan(-1/
2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 +
 2*a - b)*sqrt(-a + b - c)/(((a - b)*c + c^2)*tan(e*x + d)^4 + (a*b - b^2 +
 b*c)*tan(e*x + d)^2 + a^2 - a*b + a*c)) - 2*sqrt(c*tan(e*x + d)^4 + b*tan(
e*x + d)^2 + a)*(2*a^3 - 3*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3 + 2*a*c^2
 + (2*a^2 - a*b - b^2)*c)*tan(e*x + d)^2 + (2*a^2 - a*b)*c))/((4*a*c^4 + (8
*a^2 - 8*a*b - b^2)*c^3 + 2*(2*a^3 - 4*a^2*b + a*b^2 + b^3)*c^2 - (a^2*b^2
- 2*a*b^3 + b^4)*c)*e*tan(e*x + d)^4 - (a^2*b^3 - 2*a*b^4 + b^5 - 4*a*b*c^3
 - (8*a^2*b - 8*a*b^2 - b^3)*c^2 - 2*(2*a^3*b - 4*a^2*b^2 + a*b^3 + b^4)*c)
*e*tan(e*x + d)^2 - (a^3*b^2 - 2*a^2*b^3 + a*b^4 - 4*a^2*c^3 - (8*a^3 - 8*a
^2*b - a*b^2)*c^2 - 2*(2*a^4 - 4*a^3*b + a^2*b^2 + a*b^3)*c)*e)]
```

Sympy [F]

$$\int \frac{\tan^5(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \int \frac{\tan^5(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(e*x+d)**5/(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(3/2),x)
```

```
[Out] Integral(tan(d + e*x)**5/(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4)**(3/2)
, x)
```

Maxima [F]

$$\int \frac{\tan^5(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \int \frac{\tan(ex+d)^5}{\left(c\tan(ex+d)^4+b\tan(ex+d)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(e*x+d)^5/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(tan(e*x + d)^5/(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)^(3/2), x
)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^5(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^5/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \int \frac{\tan(d+e\,x)^5}{\left(c\tan(d+e\,x)^4+b\tan(d+e\,x)^2+a\right)^{3/2}}\,dx$$

```
[In] int(tan(d + e*x)^5/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(3/2),x)
```

```
[Out] int(tan(d + e*x)^5/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(3/2), x)
```

3.47 $\int \frac{\tan^3(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}} dx$

Optimal result

Integrand size = 35, antiderivative size = 154

$$\int \frac{\tan^3(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}} dx = \frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2(a-b+c)^{3/2}e} - \frac{a(b-2c)+(2a-b)c\tan^2(d+ex)}{(a-b+c)(b^2-4ac)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

```
[Out] 1/2*arctanh(1/2*(2*a-b+(b-2*c)*tan(e*x+d)^2)/(a-b+c)^(1/2)/(a+b*tan(e*x+d)^
2+c*tan(e*x+d)^4)^(1/2))/(a-b+c)^(3/2)/e+(-a*(b-2*c)-(2*a-b)*c*tan(e*x+d)^2
)/(a-b+c)/(-4*a*c+b^2)/e/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3781, 1265, 836, 12, 738, 212}

$$\int \frac{\tan^3(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}} dx = \frac{\operatorname{arctanh}\left(\frac{2a+(b-2c)\tan^2(d+ex)-b}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e(a-b+c)^{3/2}} - \frac{c(2a-b)\tan^2(d+ex)+a(b-2c)}{e(a-b+c)(b^2-4ac)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

```
[In] Int[Tan[d + e*x]^3/(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)^(3/2),x]
```

```
[Out] ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a + b*
Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/(2*(a - b + c)^(3/2)*e) - (a*(b - 2*c)
```

```
 + (2*a - b)*c*Tan[d + e*x]^2)/((a - b + c)*(b^2 - 4*a*c)*e*Sqrt[a + b*Tan[
d + e*x]^2 + c*Tan[d + e*x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
```

```
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)(a+bx^2+cx^4)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{x}{(1+x)(a+bx+cx^2)^{3/2}}\,dx, x, \tan^2(d+ex)\right)}{2e} \\
&= -\frac{a(b-2c)+(2a-b)c\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&\quad -\frac{\text{Subst}\left(\int \frac{b^2-4ac}{2(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{(a-b+c)\left(b^2-4ac\right)e} \\
&= -\frac{a(b-2c)+(2a-b)c\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&\quad -\frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2(a-b+c)e} \\
&= -\frac{a(b-2c)+(2a-b)c\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&\quad +\frac{\text{Subst}\left(\int \frac{1}{4a-4b+4c-x^2}\,dx, x, \frac{2a-b-(-b+2c)\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{(a-b+c)e} \\
&= \frac{\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2(a-b+c)^{3/2}e} \\
&\quad -\frac{a(b-2c)+(2a-b)c\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.17 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.01

$$\int \frac{\tan^3(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx = \frac{\frac{\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{(a-b+c)^{3/2}} + \frac{2a(b-2c)+}{(a-b+c)(-b^2+4ac)\sqrt{}}}{2e}$$

```
[In] Integrate[Tan[d + e*x]^3/(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)^(3/2),x]
```

```
[Out] (ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a + b
*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/(a - b + c)^(3/2) + (2*a*(b - 2*c) +
2*(2*a - b)*c*Tan[d + e*x]^2)/((a - b + c)*(-b^2 + 4*a*c)*Sqrt[a + b*Tan[d
+ e*x]^2 + c*Tan[d + e*x]^4]))/(2*e)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(143) = 286$.

Time = 0.09 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.96

method	result
derivativedivides	$\frac{\frac{b+2c\tan(ex+d)^2}{\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}\,(4ac-b^2)} - \frac{2c\ln\left(\frac{2a-2b+2c+(b-2c)\left(1+\tan(ex+d)^2\right)+2\sqrt{a-b+c}\sqrt{c\left(1+\tan(ex+d)^2\right)^2+(b-2c)\left(1+\right.}}{1+\tan(ex+d)^2}\right.}{\left(\sqrt{-4ac+b^2}-b+2c\right)\left(\sqrt{-4ac+b^2}+b-2c\right)\sqrt{a-b+c}}}{}$
default	$\frac{\frac{b+2c\tan(ex+d)^2}{\sqrt{a+b\tan(ex+d)^2+c\tan(ex+d)^4}\,(4ac-b^2)} - \frac{2c\ln\left(\frac{2a-2b+2c+(b-2c)\left(1+\tan(ex+d)^2\right)+2\sqrt{a-b+c}\sqrt{c\left(1+\tan(ex+d)^2\right)^2+(b-2c)\left(1+\right.}}{1+\tan(ex+d)^2}\right.}{\left(\sqrt{-4ac+b^2}-b+2c\right)\left(\sqrt{-4ac+b^2}+b-2c\right)\sqrt{a-b+c}}}{}$

```
[In] int(tan(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/e*(1/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*(b+2*c*tan(e*x+d)^2)/(4*a*c-
b^2)-2*c/((-4*a*c+b^2)^(1/2)-b+2*c)/((-4*a*c+b^2)^(1/2)+b-2*c)/(a-b+c)^(1/2
)*ln((2*a-2*b+2*c+(b-2*c)*(1+tan(e*x+d)^2)+2*(a-b+c)^(1/2)*(c*(1+tan(e*x+d)
^2)^2+(b-2*c)*(1+tan(e*x+d)^2)+a-b+c)^(1/2))/(1+tan(e*x+d)^2))+2*c/((-4*a*c
+b^2)^(1/2)-b+2*c)/(-4*a*c+b^2)/(tan(e*x+d)^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c
)*(c*(tan(e*x+d)^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2+(-4*a*c+b^2)^(1/2)*(tan
(e*x+d)^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2)-2*c/((-4*a*c+b^2)^(1/2)+b-2
*c)/(-4*a*c+b^2)/(tan(e*x+d)^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(c*(tan(e*x+d)
^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2-(-4*a*c+b^2)^(1/2)*(tan(e*x+d)^2+1/2*(b+
(-4*a*c+b^2)^(1/2))/c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(142) = 284$.

Time = 0.63 (sec) , antiderivative size = 1077, normalized size of antiderivative = 6.99

$$\int \frac{\tan^3(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm
="fricas")
```

```
[Out] [-1/4*(((b^2*c - 4*a*c^2)*tan(e*x + d)^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c
)*tan(e*x + d)^2)*sqrt(a - b + c)*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*
x + d)^4 + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 + 4*sqrt(c*tan(e*
x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(
a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^
2 + 1)) - 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(a^2*b - a*b^2 -
2*a*c^2 + ((2*a - b)*c^2 + (2*a^2 - 3*a*b + b^2)*c)*tan(e*x + d)^2 - (2*a^2
 - 3*a*b)*c))/((4*a*c^4 + (8*a^2 - 8*a*b - b^2)*c^3 + 2*(2*a^3 - 4*a^2*b +
a*b^2 + b^3)*c^2 - (a^2*b^2 - 2*a*b^3 + b^4)*c)*e*tan(e*x + d)^4 - (a^2*b^3
 - 2*a*b^4 + b^5 - 4*a*b*c^3 - (8*a^2*b - 8*a*b^2 - b^3)*c^2 - 2*(2*a^3*b -
 4*a^2*b^2 + a*b^3 + b^4)*c)*e*tan(e*x + d)^2 - (a^3*b^2 - 2*a^2*b^3 + a*b^
4 - 4*a^2*c^3 - (8*a^3 - 8*a^2*b - a*b^2)*c^2 - 2*(2*a^4 - 4*a^3*b + a^2*b^
2 + a*b^3)*c)*e), -1/2*(((b^2*c - 4*a*c^2)*tan(e*x + d)^4 + a*b^2 - 4*a^2*c
 + (b^3 - 4*a*b*c)*tan(e*x + d)^2)*sqrt(-a + b - c)*arctan(-1/2*sqrt(c*tan(
e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqr
t(-a + b - c)/(((a - b)*c + c^2)*tan(e*x + d)^4 + (a*b - b^2 + b*c)*tan(e*x
 + d)^2 + a^2 - a*b + a*c)) - 2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 +
a)*(a^2*b - a*b^2 - 2*a*c^2 + ((2*a - b)*c^2 + (2*a^2 - 3*a*b + b^2)*c)*tan
(e*x + d)^2 - (2*a^2 - 3*a*b)*c))/((4*a*c^4 + (8*a^2 - 8*a*b - b^2)*c^3 + 2
*(2*a^3 - 4*a^2*b + a*b^2 + b^3)*c^2 - (a^2*b^2 - 2*a*b^3 + b^4)*c)*e*tan(e
*x + d)^4 - (a^2*b^3 - 2*a*b^4 + b^5 - 4*a*b*c^3 - (8*a^2*b - 8*a*b^2 - b^3
)*c^2 - 2*(2*a^3*b - 4*a^2*b^2 + a*b^3 + b^4)*c)*e*tan(e*x + d)^2 - (a^3*b^
2 - 2*a^2*b^3 + a*b^4 - 4*a^2*c^3 - (8*a^3 - 8*a^2*b - a*b^2)*c^2 - 2*(2*a^
4 - 4*a^3*b + a^2*b^2 + a*b^3)*c)*e)]
```

Sympy [F]

$$\int \frac{\tan^3(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx=\int \frac{\tan^3(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(e*x+d)**3/(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(3/2),x)
```

```
[Out] Integral(tan(d + e*x)**3/(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4)**(3/2)
, x)
```

Maxima [F]

$$\int \frac{\tan^3(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \int \frac{\tan(ex+d)^3}{\left(c\tan(ex+d)^4+b\tan(ex+d)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(tan(e*x + d)^3/(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)^(3/2), x
)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^3(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \int \frac{\tan(d+e\,x)^3}{\left(c\tan(d+e\,x)^4+b\tan(d+e\,x)^2+a\right)^{3/2}}\,dx$$

```
[In] int(tan(d + e*x)^3/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(3/2),x)
```

```
[Out] int(tan(d + e*x)^3/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(3/2), x)
```

3.48 $\int \frac{\tan(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 155

$$\int \frac{\tan(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2(a-b+c)^{3/2}e} + \frac{b^2-2ac-bc+(b-2c)c\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

```
[Out] -1/2*arctanh(1/2*(2*a-b+(b-2*c)*tan(e*x+d)^2)/(a-b+c)^(1/2)/(a+b*tan(e*x+d)
^2+c*tan(e*x+d)^4)^(1/2))/(a-b+c)^(3/2)/e+(b^2-2*a*c-b*c+(b-2*c)*c*tan(e*x+
d)^2)/(a-b+c)/(-4*a*c+b^2)/e/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3781, 1261, 754, 12, 738, 212}

$$\int \frac{\tan(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx = \frac{-2ac+b^2+c(b-2c)\tan^2(d+ex)-bc}{e(a-b+c)\left(b^2-4ac\right)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+e}} - \frac{\operatorname{arctanh}\left(\frac{2a+(b-2c)\tan^2(d+ex)-b}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e(a-b+c)^{3/2}}$$

```
[In] Int[Tan[d + e*x]/(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)^(3/2),x]
```

```
[Out] -1/2*ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a
 + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/((a - b + c)^(3/2)*e) + (b^2 - 2*
a*c - b*c + (b - 2*c)*c*Tan[d + e*x]^2)/((a - b + c)*(b^2 - 4*a*c)*e*Sqrt[a
 + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
```

```
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)(a+bx^2+cx^4)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx+cx^2)^{3/2}}\,dx, x, \tan^2(d+ex)\right)}{2e} \\
&= \frac{b^2-2ac-bc+(b-2c)c\tan^2(d+ex)}{(a-b+c)\,(b^2-4ac)\,e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{(a-b+c)\,(b^2-4ac)\,e} \\
&= \frac{b^2-2ac-bc+(b-2c)c\tan^2(d+ex)}{(a-b+c)\,(b^2-4ac)\,e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2(a-b+c)e} \\
&= \frac{b^2-2ac-bc+(b-2c)c\tan^2(d+ex)}{(a-b+c)\,(b^2-4ac)\,e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{4a-4b+4c-x^2}\,dx, x, \frac{2a-b-(-b+2c)\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{(a-b+c)e} \\
&= -\frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2(a-b+c)^{3/2}e} \\
&\quad + \frac{b^2-2ac-bc+(b-2c)c\tan^2(d+ex)}{(a-b+c)\,(b^2-4ac)\,e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01

$$
\int \frac{\tan(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = -\frac{\frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{(a-b+c)^{3/2}} + \frac{2(-b^2+2ac+bc-(b-2c)c\tan^2(d+ex))}{(a-b+c)(b^2-4ac)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}}{2e}
$$

```
[In] Integrate[Tan[d + e*x]/(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)^(3/2),x]
```

```
[Out] -1/2*(ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[
a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/(a - b + c)^(3/2) + (2*(-b^2 + 2
*a*c + b*c - (b - 2*c)*c*Tan[d + e*x]^2))/((a - b + c)*(b^2 - 4*a*c)*Sqrt[a
 + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]))/e
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(143) = 286$.

Time = 0.06 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.62

method	result
derivativedivides	$\frac{\frac{2c\ln\left(\frac{2a-2b+2c+(b-2c)\left(1+\tan(ex+d)^2\right)+2\sqrt{a-b+c}\sqrt{c\left(1+\tan(ex+d)^2\right)^2+(b-2c)\left(1+\tan(ex+d)^2\right)+a-b+c}}{1+\tan(ex+d)^2}\right)}{\left(\sqrt{-4ac+b^2}-b+2c\right)\left(\sqrt{-4ac+b^2}+b-2c\right)\sqrt{a-b+c}}-\frac{2c\sqrt{c\left(\tan(ex+d)^2-\right.}}{\left(\sqrt{-4ac\right.}}}{}$
default	$\frac{\frac{2c\ln\left(\frac{2a-2b+2c+(b-2c)\left(1+\tan(ex+d)^2\right)+2\sqrt{a-b+c}\sqrt{c\left(1+\tan(ex+d)^2\right)^2+(b-2c)\left(1+\tan(ex+d)^2\right)+a-b+c}}{1+\tan(ex+d)^2}\right)}{\left(\sqrt{-4ac+b^2}-b+2c\right)\left(\sqrt{-4ac+b^2}+b-2c\right)\sqrt{a-b+c}}-\frac{2c\sqrt{c\left(\tan(ex+d)^2-\right.}}{\left(\sqrt{-4ac\right.}}}{}$

```
[In] int(tan(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/e*(2*c/((-4*a*c+b^2)^(1/2)-b+2*c)/((-4*a*c+b^2)^(1/2)+b-2*c)/(a-b+c)^(1/2
)*ln((2*a-2*b+2*c+(b-2*c)*(1+tan(e*x+d)^2)+2*(a-b+c)^(1/2)*(c*(1+tan(e*x+d)
^2)^2+(b-2*c)*(1+tan(e*x+d)^2)+a-b+c)^(1/2))/(1+tan(e*x+d)^2))-2*c/((-4*a*c
+b^2)^(1/2)-b+2*c)/(-4*a*c+b^2)/(tan(e*x+d)^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c
)*(c*(tan(e*x+d)^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2+(-4*a*c+b^2)^(1/2)*(tan
(e*x+d)^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2)+2*c/((-4*a*c+b^2)^(1/2)+b-2
*c)/(-4*a*c+b^2)/(tan(e*x+d)^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(c*(tan(e*x+d)
^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2-(-4*a*c+b^2)^(1/2)*(tan(e*x+d)^2+1/2*(b+
(-4*a*c+b^2)^(1/2))/c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(143) = 286$.

Time = 0.66 (sec) , antiderivative size = 1099, normalized size of antiderivative = 7.09

$$\int \frac{\tan(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm="
fricas")
```

```
[Out] [-1/4*(((b^2*c - 4*a*c^2)*tan(e*x + d)^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c
)*tan(e*x + d)^2)*sqrt(a - b + c)*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*
x + d)^4 + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 - 4*sqrt(c*tan(e*
x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(
a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^
2 + 1)) + 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(a*b^2 - b^3 - (2
*a + b)*c^2 - ((2*a - 3*b)*c^2 + 2*c^3 - (a*b - b^2)*c)*tan(e*x + d)^2 - (2
*a^2 - a*b - 2*b^2)*c))/((4*a*c^4 + (8*a^2 - 8*a*b - b^2)*c^3 + 2*(2*a^3 -
4*a^2*b + a*b^2 + b^3)*c^2 - (a^2*b^2 - 2*a*b^3 + b^4)*c)*e*tan(e*x + d)^4
- (a^2*b^3 - 2*a*b^4 + b^5 - 4*a*b*c^3 - (8*a^2*b - 8*a*b^2 - b^3)*c^2 - 2*
(2*a^3*b - 4*a^2*b^2 + a*b^3 + b^4)*c)*e*tan(e*x + d)^2 - (a^3*b^2 - 2*a^2*
b^3 + a*b^4 - 4*a^2*c^3 - (8*a^3 - 8*a^2*b - a*b^2)*c^2 - 2*(2*a^4 - 4*a^3*
b + a^2*b^2 + a*b^3)*c)*e), 1/2*(((b^2*c - 4*a*c^2)*tan(e*x + d)^4 + a*b^2
- 4*a^2*c + (b^3 - 4*a*b*c)*tan(e*x + d)^2)*sqrt(-a + b - c)*arctan(-1/2*sq
rt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a
 - b)*sqrt(-a + b - c)/(((a - b)*c + c^2)*tan(e*x + d)^4 + (a*b - b^2 + b*c
)*tan(e*x + d)^2 + a^2 - a*b + a*c)) - 2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x
+ d)^2 + a)*(a*b^2 - b^3 - (2*a + b)*c^2 - ((2*a - 3*b)*c^2 + 2*c^3 - (a*b
- b^2)*c)*tan(e*x + d)^2 - (2*a^2 - a*b - 2*b^2)*c))/((4*a*c^4 + (8*a^2 - 8
*a*b - b^2)*c^3 + 2*(2*a^3 - 4*a^2*b + a*b^2 + b^3)*c^2 - (a^2*b^2 - 2*a*b^
3 + b^4)*c)*e*tan(e*x + d)^4 - (a^2*b^3 - 2*a*b^4 + b^5 - 4*a*b*c^3 - (8*a^
2*b - 8*a*b^2 - b^3)*c^2 - 2*(2*a^3*b - 4*a^2*b^2 + a*b^3 + b^4)*c)*e*tan(e
*x + d)^2 - (a^3*b^2 - 2*a^2*b^3 + a*b^4 - 4*a^2*c^3 - (8*a^3 - 8*a^2*b - a
*b^2)*c^2 - 2*(2*a^4 - 4*a^3*b + a^2*b^2 + a*b^3)*c)*e)]
```

Sympy [F]

$$\int \frac{\tan(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx=\int \frac{\tan\left(d+ex\right)}{\left(a+b\tan^2\left(d+ex\right)+c\tan^4\left(d+ex\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(e*x+d)/(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(3/2),x)
```

```
[Out] Integral(tan(d + e*x)/(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4)**(3/2), x
)
```

Maxima [F]

$$\int \frac{\tan(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \int \frac{\tan{(ex+d)}}{\left(c\tan{(ex+d)}^4+b\tan{(ex+d)}^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(tan(e*x + d)/(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \int \frac{\tan(d+e\,x)}{\left(c\tan{(d+e\,x)}^4+b\tan{(d+e\,x)}^2+a\right)^{3/2}}\,dx$$

```
[In] int(tan(d + e*x)/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(3/2),x)
```

```
[Out] int(tan(d + e*x)/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(3/2), x)
```

3.49 $\int \frac{\cot(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 280

$$\begin{aligned}\int \frac{\cot(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx =& -\frac{\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2a^{3/2}e} \\ &+\frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2(a-b+c)^{3/2}e} \\ &+\frac{b^2-2ac+bc\tan^2(d+ex)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\ &-\frac{b^2-2ac-bc+(b-2c)c\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\end{aligned}$$

```
[Out] -1/2*arctanh(1/2*(2*a+b*tan(e*x+d)^2)/a^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d
)^4)^(1/2))/a^(3/2)/e+1/2*arctanh(1/2*(2*a-b+(b-2*c)*tan(e*x+d)^2)/(a-b+c)^
(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))/(a-b+c)^(3/2)/e+(b^2-2*a*c+b
*c*tan(e*x+d)^2)/a/(-4*a*c+b^2)/e/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)+(
-b^2+2*a*c+b*c-(b-2*c)*c*tan(e*x+d)^2)/(a-b+c)/(-4*a*c+b^2)/e/(a+b*tan(e*x+
d)^2+c*tan(e*x+d)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3781, 1265, 974, 754, 12, 738, 212}

$$\int \frac{\cot(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2a^{3/2}e} + \frac{\operatorname{arctanh}\left(\frac{2a+(b-2c)\tan^2(d+ex)-b}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e(a-b+c)^{3/2}} + \frac{-2ac+b^2+bc\tan^2(d+ex)}{ae\,(b^2-4ac)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{-2ac+b^2+c(b-2c)\tan^2(d+ex)-bc}{e(a-b+c)\,(b^2-4ac)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

```
[In] Int[Cot[d + e*x]/(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)^(3/2),x]
```

```
[Out] -1/2*ArcTanh[(2*a + b*Tan[d + e*x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x]^2
+ c*Tan[d + e*x]^4])]/(a^(3/2)*e) + ArcTanh[(2*a - b + (b - 2*c)*Tan[d + e*
x]^2)/(2*Sqrt[a - b + c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])]/(2
*(a - b + c)^(3/2)*e) + (b^2 - 2*a*c + b*c*Tan[d + e*x]^2)/(a*(b^2 - 4*a*c)
*e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) - (b^2 - 2*a*c - b*c + (b
 - 2*c)*c*Tan[d + e*x]^2)/((a - b + c)*(b^2 - 4*a*c)*e*Sqrt[a + b*Tan[d + e
*x]^2 + c*Tan[d + e*x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)(a+bx^2+cx^4)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx+cx^2)^{3/2}}\,dx, x, \tan^2(d+ex)\right)}{2e}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{(-1-x)(a+bx+cx^2)^{3/2}}+\frac{1}{x(a+bx+cx^2)^{3/2}}\right)dx, x, \tan^2(d+ex)\right)}{2e}\end{aligned}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1-x)(a+bx+cx^2)^{3/2}}\,dx, x, \tan^2(d+ex)\right)}{2e} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^{3/2}}\,dx, x, \tan^2(d+ex)\right)}{2e}$$

$$= \frac{b^2-2ac+bc\tan^2(d+ex)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{b^2-2ac-bc+(b-2c)c\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{a\left(b^2-4ac\right)e} - \frac{\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{(-1-x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{(a-b+c)\left(b^2-4ac\right)e}$$

$$= \frac{b^2-2ac+bc\tan^2(d+ex)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{b^2-2ac-bc+(b-2c)c\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2ae} + \frac{\text{Subst}\left(\int \frac{1}{(-1-x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2(a-b+c)e}$$

$$= \frac{b^2-2ac+bc\tan^2(d+ex)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{b^2-2ac-bc+(b-2c)c\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{ae} - \frac{\text{Subst}\left(\int \frac{1}{4a-4b+4c-x^2}\,dx, x, \frac{-2a+b-(b-2c)\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{(a-b+c)e}$$

$$= -\frac{\text{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2a^{3/2}e} + \frac{\text{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2(a-b+c)^{3/2}e} + \frac{b^2-2ac+bc\tan^2(d+ex)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{b^2-2ac-bc+(b-2c)c\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

Mathematica [A] (verified)

Time = 3.89 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.99

$$\int \frac{\cot(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \frac{\frac{\left(-\frac{b^2}{2}+2ac\right)\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{a^{3/2}} - \frac{(b^2-4ac)\operatorname{arc}}{}}{}$$

```
[In] Integrate[Cot[d + e*x]/(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)^(3/2),x]
```

```
[Out] (((-1/2*b^2 + 2*a*c)*ArcTanh[(2*a + b*Tan[d + e*x]^2)/(2*Sqrt[a]*Sqrt[a + b
*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])])/a^(3/2) - ((b^2 - 4*a*c)*ArcTanh[(-2
*a + b - (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a + b*Tan[d + e*
x]^2 + c*Tan[d + e*x]^4])])/(2*(a - b + c)^(3/2)) + (b^2 - 2*a*c + b*c*Tan[
d + e*x]^2)/(a*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) - (b^2 - 2*a*
c - b*c + (b - 2*c)*c*Tan[d + e*x]^2)/((a - b + c)*Sqrt[a + b*Tan[d + e*x]^
2 + c*Tan[d + e*x]^4]))/((b^2 - 4*a*c)*e)
```

Maple [F]

$$\int \frac{\cot(ex+d)}{\left(a+b\tan(ex+d)^2+c\tan(ex+d)^4\right)^{\frac{3}{2}}}dx$$

```
[In] int(cot(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x)
```

```
[Out] int(cot(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 963 vs. 2(256) = 512.

Time = 2.62 (sec) , antiderivative size = 3951, normalized size of antiderivative = 14.11

$$\int \frac{\cot(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm="
fricas")
```

```
[Out] [-1/4*((a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*tan(e*x + d)^4 + (a^2*b
^3 - 4*a^3*b*c)*tan(e*x + d)^2)*sqrt(a - b + c)*log(((b^2 + 4*(a - 2*b)*c +
 8*c^2)*tan(e*x + d)^4 + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*tan(e*x + d)^2 + 4
*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 +
2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c)/(tan(e*x + d)^4 + 2
*tan(e*x + d)^2 + 1)) + (a^3*b^2 - 2*a^2*b^3 + a*b^4 - 4*a^2*c^3 - (4*a*c^4
```

+ (8*a^2 - 8*a*b - b^2)*c^3 + 2*(2*a^3 - 4*a^2*b + a*b^2 + b^3)*c^2 - (a^2
*b^2 - 2*a*b^3 + b^4)*c)*tan(e*x + d)^4 - (8*a^3 - 8*a^2*b - a*b^2)*c^2 + (
a^2*b^3 - 2*a*b^4 + b^5 - 4*a*b*c^3 - (8*a^2*b - 8*a*b^2 - b^3)*c^2 - 2*(2*
a^3*b - 4*a^2*b^2 + a*b^3 + b^4)*c)*tan(e*x + d)^2 - 2*(2*a^4 - 4*a^3*b + a
^2*b^2 + a*b^3)*c)*sqrt(a)*log(((b^2 + 4*a*c)*tan(e*x + d)^4 + 8*a*b*tan(e*
x + d)^2 - 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(b*tan(e*x + d)^
2 + 2*a)*sqrt(a) + 8*a^2)/tan(e*x + d)^4) - 4*(a^2*b^3 - a*b^4 + 2*a^2*c^3
+ (2*a^3 - 5*a^2*b - a*b^2)*c^2 - ((2*a^2 + a*b)*c^3 + (2*a^3 - a^2*b - 2*a
*b^2)*c^2 - (a^2*b^2 - a*b^3)*c)*tan(e*x + d)^2 - (3*a^3*b - 2*a^2*b^2 - 2*
a*b^3)*c)*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a))/((4*a^3*c^4 + (8*a
^4 - 8*a^3*b - a^2*b^2)*c^3 + 2*(2*a^5 - 4*a^4*b + a^3*b^2 + a^2*b^3)*c^2 -
(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*c)*e*tan(e*x + d)^4 - (a^4*b^3 - 2*a^3*b^4
+ a^2*b^5 - 4*a^3*b*c^3 - (8*a^4*b - 8*a^3*b^2 - a^2*b^3)*c^2 - 2*(2*a^5*b
- 4*a^4*b^2 + a^3*b^3 + a^2*b^4)*c)*e*tan(e*x + d)^2 - (a^5*b^2 - 2*a^4*b^
3 + a^3*b^4 - 4*a^4*c^3 - (8*a^5 - 8*a^4*b - a^3*b^2)*c^2 - 2*(2*a^6 - 4*a^
5*b + a^4*b^2 + a^3*b^3)*c)*e), -1/4*(2*(a^3*b^2 - 2*a^2*b^3 + a*b^4 - 4*a^
2*c^3 - (4*a*c^4 + (8*a^2 - 8*a*b - b^2)*c^3 + 2*(2*a^3 - 4*a^2*b + a*b^2 +
b^3)*c^2 - (a^2*b^2 - 2*a*b^3 + b^4)*c)*tan(e*x + d)^4 - (8*a^3 - 8*a^2*b
- a*b^2)*c^2 + (a^2*b^3 - 2*a*b^4 + b^5 - 4*a*b*c^3 - (8*a^2*b - 8*a*b^2 -
b^3)*c^2 - 2*(2*a^3*b - 4*a^2*b^2 + a*b^3 + b^4)*c)*tan(e*x + d)^2 - 2*(2*a
^4 - 4*a^3*b + a^2*b^2 + a*b^3)*c)*sqrt(-a)*arctan(1/2*sqrt(c*tan(e*x + d)^
4 + b*tan(e*x + d)^2 + a)*(b*tan(e*x + d)^2 + 2*a)*sqrt(-a)/(a*c*tan(e*x +
d)^4 + a*b*tan(e*x + d)^2 + a^2)) + (a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3
*c^2)*tan(e*x + d)^4 + (a^2*b^3 - 4*a^3*b*c)*tan(e*x + d)^2)*sqrt(a - b + c
)*log(((b^2 + 4*(a - 2*b)*c + 8*c^2)*tan(e*x + d)^4 + 2*(4*a*b - 3*b^2 - 4*
(a - b)*c)*tan(e*x + d)^2 + 4*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)
*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b + b^2
+ 4*a*c)/(tan(e*x + d)^4 + 2*tan(e*x + d)^2 + 1)) - 4*(a^2*b^3 - a*b^4 + 2
*a^2*c^3 + (2*a^3 - 5*a^2*b - a*b^2)*c^2 - ((2*a^2 + a*b)*c^3 + (2*a^3 - a^
2*b - 2*a*b^2)*c^2 - (a^2*b^2 - a*b^3)*c)*tan(e*x + d)^2 - (3*a^3*b - 2*a^2
*b^2 - 2*a*b^3)*c)*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a))/((4*a^3*c
^4 + (8*a^4 - 8*a^3*b - a^2*b^2)*c^3 + 2*(2*a^5 - 4*a^4*b + a^3*b^2 + a^2*b
^3)*c^2 - (a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*c)*e*tan(e*x + d)^4 - (a^4*b^3 -
2*a^3*b^4 + a^2*b^5 - 4*a^3*b*c^3 - (8*a^4*b - 8*a^3*b^2 - a^2*b^3)*c^2 - 2
*(2*a^5*b - 4*a^4*b^2 + a^3*b^3 + a^2*b^4)*c)*e*tan(e*x + d)^2 - (a^5*b^2 -
2*a^4*b^3 + a^3*b^4 - 4*a^4*c^3 - (8*a^5 - 8*a^4*b - a^3*b^2)*c^2 - 2*(2*a
^6 - 4*a^5*b + a^4*b^2 + a^3*b^3)*c)*e), -1/4*(2*(a^3*b^2 - 4*a^4*c + (a^2*
b^2*c - 4*a^3*c^2)*tan(e*x + d)^4 + (a^2*b^3 - 4*a^3*b*c)*tan(e*x + d)^2)*s
qrt(-a + b - c)*arctan(-1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(
(b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(-a + b - c)/(((a - b)*c + c^2)*tan
(e*x + d)^4 + (a*b - b^2 + b*c)*tan(e*x + d)^2 + a^2 - a*b + a*c)) + (a^3*b
^2 - 2*a^2*b^3 + a*b^4 - 4*a^2*c^3 - (4*a*c^4 + (8*a^2 - 8*a*b - b^2)*c^3 +
2*(2*a^3 - 4*a^2*b + a*b^2 + b^3)*c^2 - (a^2*b^2 - 2*a*b^3 + b^4)*c)*tan(e
*x + d)^4 - (8*a^3 - 8*a^2*b - a*b^2)*c^2 + (a^2*b^3 - 2*a*b^4 + b^5 - 4*a*
b*c^3 - (8*a^2*b - 8*a*b^2 - b^3)*c^2 - 2*(2*a^3*b - 4*a^2*b^2 + a*b^3 + b^

```
4)*c)*tan(e*x + d)^2 - 2*(2*a^4 - 4*a^3*b + a^2*b^2 + a*b^3)*c)*sqrt(a)*log
(((b^2 + 4*a*c)*tan(e*x + d)^4 + 8*a*b*tan(e*x + d)^2 - 4*sqrt(c*tan(e*x +
d)^4 + b*tan(e*x + d)^2 + a)*(b*tan(e*x + d)^2 + 2*a)*sqrt(a) + 8*a^2)/tan(
e*x + d)^4) - 4*(a^2*b^3 - a*b^4 + 2*a^2*c^3 + (2*a^3 - 5*a^2*b - a*b^2)*c^
2 - ((2*a^2 + a*b)*c^3 + (2*a^3 - a^2*b - 2*a*b^2)*c^2 - (a^2*b^2 - a*b^3)*
c)*tan(e*x + d)^2 - (3*a^3*b - 2*a^2*b^2 - 2*a*b^3)*c)*sqrt(c*tan(e*x + d)^
4 + b*tan(e*x + d)^2 + a))/((4*a^3*c^4 + (8*a^4 - 8*a^3*b - a^2*b^2)*c^3 +
2*(2*a^5 - 4*a^4*b + a^3*b^2 + a^2*b^3)*c^2 - (a^4*b^2 - 2*a^3*b^3 + a^2*b^
4)*c)*e*tan(e*x + d)^4 - (a^4*b^3 - 2*a^3*b^4 + a^2*b^5 - 4*a^3*b*c^3 - (8*
a^4*b - 8*a^3*b^2 - a^2*b^3)*c^2 - 2*(2*a^5*b - 4*a^4*b^2 + a^3*b^3 + a^2*b
^4)*c)*e*tan(e*x + d)^2 - (a^5*b^2 - 2*a^4*b^3 + a^3*b^4 - 4*a^4*c^3 - (8*a
^5 - 8*a^4*b - a^3*b^2)*c^2 - 2*(2*a^6 - 4*a^5*b + a^4*b^2 + a^3*b^3)*c)*e)
, -1/2*((a^3*b^2 - 2*a^2*b^3 + a*b^4 - 4*a^2*c^3 - (4*a*c^4 + (8*a^2 - 8*a*
b - b^2)*c^3 + 2*(2*a^3 - 4*a^2*b + a*b^2 + b^3)*c^2 - (a^2*b^2 - 2*a*b^3 +
 b^4)*c)*tan(e*x + d)^4 - (8*a^3 - 8*a^2*b - a*b^2)*c^2 + (a^2*b^3 - 2*a*b^
4 + b^5 - 4*a*b*c^3 - (8*a^2*b - 8*a*b^2 - b^3)*c^2 - 2*(2*a^3*b - 4*a^2*b^
2 + a*b^3 + b^4)*c)*tan(e*x + d)^2 - 2*(2*a^4 - 4*a^3*b + a^2*b^2 + a*b^3)*
c)*sqrt(-a)*arctan(1/2*sqrt(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)*(b*tan
(e*x + d)^2 + 2*a)*sqrt(-a)/(a*c*tan(e*x + d)^4 + a*b*tan(e*x + d)^2 + a^2)
) + (a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*tan(e*x + d)^4 + (a^2*b^3
- 4*a^3*b*c)*tan(e*x + d)^2)*sqrt(-a + b - c)*arctan(-1/2*sqrt(c*tan(e*x +
d)^4 + b*tan(e*x + d)^2 + a)*((b - 2*c)*tan(e*x + d)^2 + 2*a - b)*sqrt(-a +
 b - c)/(((a - b)*c + c^2)*tan(e*x + d)^4 + (a*b - b^2 + b*c)*tan(e*x + d)^
2 + a^2 - a*b + a*c)) - 2*(a^2*b^3 - a*b^4 + 2*a^2*c^3 + (2*a^3 - 5*a^2*b -
 a*b^2)*c^2 - ((2*a^2 + a*b)*c^3 + (2*a^3 - a^2*b - 2*a*b^2)*c^2 - (a^2*b^2
 - a*b^3)*c)*tan(e*x + d)^2 - (3*a^3*b - 2*a^2*b^2 - 2*a*b^3)*c)*sqrt(c*tan
(e*x + d)^4 + b*tan(e*x + d)^2 + a))/((4*a^3*c^4 + (8*a^4 - 8*a^3*b - a^2*b
^2)*c^3 + 2*(2*a^5 - 4*a^4*b + a^3*b^2 + a^2*b^3)*c^2 - (a^4*b^2 - 2*a^3*b^
3 + a^2*b^4)*c)*e*tan(e*x + d)^4 - (a^4*b^3 - 2*a^3*b^4 + a^2*b^5 - 4*a^3*b
*c^3 - (8*a^4*b - 8*a^3*b^2 - a^2*b^3)*c^2 - 2*(2*a^5*b - 4*a^4*b^2 + a^3*b
^3 + a^2*b^4)*c)*e*tan(e*x + d)^2 - (a^5*b^2 - 2*a^4*b^3 + a^3*b^4 - 4*a^4*
c^3 - (8*a^5 - 8*a^4*b - a^3*b^2)*c^2 - 2*(2*a^6 - 4*a^5*b + a^4*b^2 + a^3*
b^3)*c)*e)]
```

Sympy [F]

$$\int \frac{\cot(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx=\int \frac{\cot\left(d+ex\right)}{\left(a+b\tan^2\left(d+ex\right)+c\tan^4\left(d+ex\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(e*x+d)/(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(3/2),x)
```

```
[Out] Integral(cot(d + e*x)/(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4)**(3/2), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cot(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm="
maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \int \frac{\cot(d+e\,x)}{\left(c\tan\left(d+e\,x\right)^4+b\tan\left(d+e\,x\right)^2+a\right)^{3/2}}\,dx$$

```
[In] int(cot(d + e*x)/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(3/2),x)
```

```
[Out] int(cot(d + e*x)/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(3/2), x)
```

3.50 $\int \frac{\cot^3(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 477

$$\begin{aligned}
\int \frac{\cot^3(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx &= \frac{\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2a^{3/2}e} \\
&+ \frac{3b\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4a^{5/2}e} - \frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2(a-b+c)^{3/2}e} \\
&- \frac{b^2-2ac+bc\tan^2(d+ex)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&+ \frac{\cot^2(d+ex)\left(b^2-2ac+bc\tan^2(d+ex)\right)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&+ \frac{b^2-2ac-bc+(b-2c)c\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&- \frac{\left(3b^2-8ac\right)\cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2a^2\left(b^2-4ac\right)e}
\end{aligned}$$

```
[Out] 1/2*arctanh(1/2*(2*a+b*tan(e*x+d)^2)/a^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)
^4)^(1/2))/a^(3/2)/e+3/4*b*arctanh(1/2*(2*a+b*tan(e*x+d)^2)/a^(1/2)/(a+b*ta
n(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))/a^(5/2)/e-1/2*arctanh(1/2*(2*a-b+(b-2*c)*
tan(e*x+d)^2)/(a-b+c)^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2))/(a-b+c
)^(3/2)/e-1/2*(-8*a*c+3*b^2)*cot(e*x+d)^2*(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)
^(1/2)/a^2/(-4*a*c+b^2)/e+(-b^2+2*a*c-b*c*tan(e*x+d)^2)/a/(-4*a*c+b^2)/e/(a
+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)+cot(e*x+d)^2*(b^2-2*a*c+b*c*tan(e*x+d
)^2)/a/(-4*a*c+b^2)/e/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)+(b^2-2*a*c-b*
c+(b-2*c)*c*tan(e*x+d)^2)/(a-b+c)/(-4*a*c+b^2)/e/(a+b*tan(e*x+d)^2+c*tan(e*
x+d)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3781, 1265, 974, 754, 820, 738, 212, 12}

$$\int \frac{\cot^3(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx = \frac{3b\,\text{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4a^{5/2}e}$$
$$+\frac{\text{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2a^{3/2}e}$$
$$-\frac{(3b^2-8ac)\cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2a^2e\,(b^2-4ac)}$$
$$-\frac{\text{arctanh}\left(\frac{2a+(b-2c)\tan^2(d+ex)-b}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2e(a-b+c)^{3/2}}$$
$$-\frac{-2ac+b^2+bc\tan^2(d+ex)}{ae\,(b^2-4ac)\,\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$+\frac{-2ac+b^2+c(b-2c)\tan^2(d+ex)-bc}{e(a-b+c)\,(b^2-4ac)\,\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$+\frac{\cot^2(d+ex)\,(-2ac+b^2+bc\tan^2(d+ex))}{ae\,(b^2-4ac)\,\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$

```
[In] Int[Cot[d + e*x]^3/(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)^(3/2),x]
```

```
[Out] ArcTanh[(2*a + b*Tan[d + e*x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x]^2 + c*T
an[d + e*x]^4])]/(2*a^(3/2)*e) + (3*b*ArcTanh[(2*a + b*Tan[d + e*x]^2)/(2*S
qrt[a]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])])/(4*a^(5/2)*e) - Arc
Tanh[(2*a - b + (b - 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b + c]*Sqrt[a + b*Tan
[d + e*x]^2 + c*Tan[d + e*x]^4])]/(2*(a - b + c)^(3/2)*e) - (b^2 - 2*a*c +
b*c*Tan[d + e*x]^2)/(a*(b^2 - 4*a*c)*e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d
+ e*x]^4]) + (Cot[d + e*x]^2*(b^2 - 2*a*c + b*c*Tan[d + e*x]^2))/(a*(b^2 -
4*a*c)*e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) + (b^2 - 2*a*c - b*
c + (b - 2*c)*c*Tan[d + e*x]^2)/((a - b + c)*(b^2 - 4*a*c)*e*Sqrt[a + b*Tan
[d + e*x]^2 + c*Tan[d + e*x]^4]) - ((3*b^2 - 8*a*c)*Cot[d + e*x]^2*Sqrt[a +
 b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])/(2*a^2*(b^2 - 4*a*c)*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)(a+bx^2+cx^4)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)(a+bx+cx^2)^{3/2}}\,dx, x, \tan^2(d+ex)\right)}{2e}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x^2(a+bx+cx^2)^{3/2}} - \frac{1}{x(a+bx+cx^2)^{3/2}} + \frac{1}{(1+x)(a+bx+cx^2)^{3/2}}\right)dx, x, \tan^2(d+ex)\right)}{2e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^{3/2}}\,dx, x, \tan^2(d+ex)\right)}{2e}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^{3/2}}\,dx, x, \tan^2(d+ex)\right)}{2e}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx+cx^2)^{3/2}}\,dx, x, \tan^2(d+ex)\right)}{2e}\\
&= -\frac{b^2-2ac+bc\tan^2(d+ex)}{a(b^2-4ac)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\\
&\quad + \frac{\cot^2(d+ex)\left(b^2-2ac+bc\tan^2(d+ex)\right)}{a(b^2-4ac)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\\
&\quad + \frac{b^2-2ac-bc+(b-2c)c\tan^2(d+ex)}{(a-b+c)(b^2-4ac)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{a(b^2-4ac)e}\\
&\quad - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-3b^2+8ac)-bcx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{a(b^2-4ac)e}\\
&\quad - \frac{\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{(a-b+c)(b^2-4ac)e}
\end{aligned}$$

$$= -\frac{b^2 - 2ac + bc\tan^2(d+ex)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{\cot^2(d+ex)\left(b^2-2ac+bc\tan^2(d+ex)\right)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{b^2-2ac-bc+(b-2c)c\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{\left(3b^2-8ac\right)\cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2a^2\left(b^2-4ac\right)e} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2ae} - \frac{(3b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{4a^2e} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx+cx^2}}\,dx, x, \tan^2(d+ex)\right)}{2(a-b+c)e}$$

$$= -\frac{b^2 - 2ac + bc\tan^2(d+ex)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{\cot^2(d+ex)\left(b^2-2ac+bc\tan^2(d+ex)\right)}{a\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{b^2-2ac-bc+(b-2c)c\tan^2(d+ex)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{\left(3b^2-8ac\right)\cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2a^2\left(b^2-4ac\right)e} + \frac{\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{ae} + \frac{(3b)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2a^2e} - \frac{\text{Subst}\left(\int \frac{1}{4a-4b+4c-x^2}\,dx, x, \frac{2a-b-(-b+2c)\tan^2(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{(a-b+c)e}$$

$$= \frac{\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2a^{3/2}e} + \frac{3b\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{4a^{5/2}e}$$
$$- \frac{\operatorname{arctanh}\left(\frac{2a-b+(b-2c)\tan^2(d+ex)}{2\sqrt{a-b+c}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2(a-b+c)^{3/2}e}$$
$$- \frac{b^2-2ac+bc\tan^2(d+ex)}{a(b^2-4ac)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$+ \frac{\cot^2(d+ex)(b^2-2ac+bc\tan^2(d+ex))}{a(b^2-4ac)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$+ \frac{b^2-2ac-bc+(b-2c)c\tan^2(d+ex)}{(a-b+c)(b^2-4ac)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$- \frac{(3b^2-8ac)\cot^2(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{2a^2(b^2-4ac)e}$$

Mathematica [A] (verified)

Time = 6.10 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.16

$$\int \frac{\cot^3(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \frac{-\frac{2\left(-\frac{b^2}{2}+2ac\right)\operatorname{arctanh}\left(\frac{2a+b\tan^2(d+ex)}{2\sqrt{a}\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{a^{3/2}(b^2-4ac)} + \frac{8\left(-\frac{b^2}{2}+2ac\right.}{}}{}$$

```
[In] Integrate[Cot[d + e*x]^3/(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)^(3/2),x]
```

```
[Out] ((-2*(-1/2*b^2 + 2*a*c)*ArcTanh[(2*a + b*Tan[d + e*x]^2)/(2*Sqrt[a]*Sqrt[a
+ b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])])/(a^(3/2)*(b^2 - 4*a*c)) + (8*(-1/
2*b^2 + 2*a*c)*ArcTanh[(2*a - b - (-b + 2*c)*Tan[d + e*x]^2)/(2*Sqrt[a - b
+ c]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])])/(Sqrt[a - b + c]*(4*a
 - 4*b + 4*c)*(b^2 - 4*a*c)) + (2*(-b^2 + 2*a*c - b*c*Tan[d + e*x]^2))/(a*(
b^2 - 4*a*c)*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) - (2*Cot[d + e*
x]^2*(-b^2 + 2*a*c - b*c*Tan[d + e*x]^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*Tan[d
 + e*x]^2 + c*Tan[d + e*x]^4]) - (2*(-b^2 + 2*a*c + b*c + c*(-b + 2*c)*Tan[
d + e*x]^2))/((a - b + c)*(b^2 - 4*a*c)*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d
 + e*x]^4]) - (2*(((2*a*b*c + (b*(-3*b^2 + 8*a*c))/2)*ArcTanh[(2*a + b*Tan[
d + e*x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])])/(2*
a^(3/2)) + ((3*b^2 - 8*a*c)*Cot[d + e*x]^2*Sqrt[a + b*Tan[d + e*x]^2 + c*Ta
n[d + e*x]^4])/(2*a)))/(a*(b^2 - 4*a*c)))/(2*e)
```

Maple [F]

$$\int \frac{\cot\left(ex+d\right)^{3}}{\left(a+b\tan\left(ex+d\right)^{2}+c\tan\left(ex+d\right)^{4}\right)^{\frac{3}{2}}}dx$$

```
[In] int(cot(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x)
[Out] int(cot(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1273 vs. $2(437) = 874$.

Time = 3.27 (sec) , antiderivative size = 5189, normalized size of antiderivative = 10.88

$$\int \frac{\cot^3(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm
="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\cot^3(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \int \frac{\cot^{3}\left(d+ex\right)}{\left(a+b\tan^{2}\left(d+ex\right)+c\tan^{4}\left(d+ex\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(e*x+d)**3/(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(3/2),x)
[Out] Integral(cot(d + e*x)**3/(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4)**(3/2)
, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^3(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm
="maxima")
[Out] Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^3(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(e*x+d)^3/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \text{Hanged}$$

```
[In] int(cot(d + e*x)^3/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(3/2),x)
```

```
[Out] \text{Hanged}
```

3.51 $\int \frac{\tan^2(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 981

$$\begin{aligned}
\int \frac{\tan^2(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx &= -\frac{\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2(a-b+c)^{3/2}e} \\
&+ \frac{\tan(d+ex)\left(b^2-2ac-bc+(b-2c)c\tan^2(d+ex)\right)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&- \frac{(b-2c)\sqrt{c}\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{(a-b+c)\left(b^2-4ac\right)e\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)} \\
&+ \frac{\sqrt[4]{a}(b-2c)\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d-}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&+ \frac{\sqrt[4]{c}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+c\tan^4(d+ex)}{\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)^2}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)(a-b+c)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&- \frac{\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+e}{(\sqrt{a}+\sqrt{c}\mathrm{t}}}}{2\sqrt[4]{a}\left(b-2\sqrt{a}\sqrt{c}\right)(a-b+c)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} \\
&- \frac{\left(\sqrt{a}+\sqrt{c}\right)\mathrm{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a-}{}}}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}(a-b+c)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}
\end{aligned}$$

```
[Out] -1/2*arctan((a-b+c)^(1/2)*tan(e*x+d)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2
))/(a-b+c)^(3/2)/e+tan(e*x+d)*(b^2-2*a*c-b*c+(b-2*c)*c*tan(e*x+d)^2)/(a-b+c
)/(-4*a*c+b^2)/e/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)-(b-2*c)*c^(1/2)*(a
+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*tan(e*x+d)/(a-b+c)/(-4*a*c+b^2)/e/(a^
```

```
(1/2)+c^(1/2)*tan(e*x+d)^2)+a^(1/4)*(b-2*c)*c^(1/4)*(cos(2*arctan(c^(1/4)*t
an(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*Elli
pticE(sin(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(
1/2))*((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^
(1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^2)/(a-b+c)/(-4*a*c+b^2)/e/(a+b*tan(e*x+d)
^2+c*tan(e*x+d)^4)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/
4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticF(sin(2*arc
tan(c^(1/4)*tan(e*x+d)/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*((a+b*tan
(e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+
c^(1/2)*tan(e*x+d)^2)/a^(1/4)/(a-b+c)/e/(a^(1/2)-c^(1/2))/(a+b*tan(e*x+d)^2
+c*tan(e*x+d)^4)^(1/2)-1/4*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/
4)*tan(e*x+d)/a^(1/4))),-1/4*(a^(1/2)-c^(1/2))^2/a^(1/2)/c^(1/2),1/2*(2-b/a
^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+c^(1/2))*((a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)
/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c^(1/2)*tan(e*x+d)^2)/a^(
1/4)/c^(1/4)/(a-b+c)/e/(a^(1/2)-c^(1/2))/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^
(1/2)-1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*tan(e*x+d)/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*tan(e*x
+d)/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)-c^(1/2))*((a+b*tan(
e*x+d)^2+c*tan(e*x+d)^4)/(a^(1/2)+c^(1/2)*tan(e*x+d)^2)^2)^(1/2)*(a^(1/2)+c
^(1/2)*tan(e*x+d)^2)/a^(1/4)/(a-b+c)/e/(b-2*a^(1/2)*c^(1/2))/(a+b*tan(e*x+d
)^2+c*tan(e*x+d)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 981, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used

$= \{3781, 1329, 1192, 1211, 1117, 1209, 1230, 1720\}$

$$\int \frac{\tan^2(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}\right)}{2(a-b+c)^{3/2}e}$$

$$-\frac{(b-2c)\sqrt{c}\tan(d+ex)\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}{(a-b+c)(b^2-4ac)e\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)}$$

$$+\frac{\sqrt[4]{a}(b-2c)\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c\tan^4(d+ex)+b\tan^2(d+ex)+a}{(\sqrt{c}\tan^2(d+ex)+\sqrt{a})^2}}}{(a-b+c)(b^2-4ac)e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}$$

$$+\frac{\sqrt[4]{c}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c\tan^4(d+ex)+b\tan^2(d+ex)+a}{(\sqrt{c}\tan^2(d+ex)+\sqrt{a})^2}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)(a-b+c)e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}$$

$$-\frac{\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c\tan^4(d+ex)+b\tan^2(d+ex)+a}{(\sqrt{c}\tan^2(d+ex)+\sqrt{a})^2}}}{2\sqrt[4]{a}\left(b-2\sqrt{a}\sqrt{c}\right)(a-b+c)e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}$$

$$-\frac{\left(\sqrt{a}+\sqrt{c}\right)\mathrm{EllipticPi}\left(-\frac{(\sqrt{a}-\sqrt{c})^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{c}\tan^2(d+ex)+\sqrt{a}\right)\sqrt{\frac{c\tan^4(d+ex)+b\tan^2(d+ex)+a}{(\sqrt{c}\tan^2(d+ex)+\sqrt{a})^2}}}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}(a-b+c)e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}$$

$$+\frac{\tan(d+ex)\left(b^2-cb+(b-2c)c\tan^2(d+ex)-2ac\right)}{(a-b+c)(b^2-4ac)e\sqrt{c\tan^4(d+ex)+b\tan^2(d+ex)+a}}$$

```
[In] Int[Tan[d + e*x]^2/(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)^(3/2),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[a - b + c]*Tan[d + e*x])/Sqrt[a + b*Tan[d + e*x]^2 + c*Ta
n[d + e*x]^4]]/((a - b + c)^(3/2)*e) + (Tan[d + e*x]*(b^2 - 2*a*c - b*c + (
b - 2*c)*c*Tan[d + e*x]^2))/((a - b + c)*(b^2 - 4*a*c)*e*Sqrt[a + b*Tan[d +
 e*x]^2 + c*Tan[d + e*x]^4]) - ((b - 2*c)*Sqrt[c]*Tan[d + e*x]*Sqrt[a + b*T
an[d + e*x]^2 + c*Tan[d + e*x]^4])/((a - b + c)*(b^2 - 4*a*c)*e*(Sqrt[a] +
Sqrt[c]*Tan[d + e*x]^2)) + (a^(1/4)*(b - 2*c)*c^(1/4)*EllipticE[2*ArcTan[(c
^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt
[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/(Sqrt[a]
 + Sqrt[c]*Tan[d + e*x]^2)^2])/((a - b + c)*(b^2 - 4*a*c)*e*Sqrt[a + b*Tan[
d + e*x]^2 + c*Tan[d + e*x]^4]) + (c^(1/4)*EllipticF[2*ArcTan[(c^(1/4)*Tan[
d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a] + Sqrt[c]*Tan[d +
 e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/(Sqrt[a] + Sqrt[c]*
Tan[d + e*x]^2)^2])/(2*a^(1/4)*(Sqrt[a] - Sqrt[c])*(a - b + c)*e*Sqrt[a + b
*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) - ((Sqrt[a] - Sqrt[c])*c^(1/4)*Ellipti
cF[2*ArcTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(
Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*
x]^4)/(Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(2*a^(1/4)*(b - 2*Sqrt[a]*Sqrt
[c])*(a - b + c)*e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4]) - ((Sqrt[
```

```
a] + Sqrt[c])*EllipticPi[-1/4*(Sqrt[a] - Sqrt[c])^2/(Sqrt[a]*Sqrt[c]), 2*Ar
cTan[(c^(1/4)*Tan[d + e*x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]*(Sqrt[a]
 + Sqrt[c]*Tan[d + e*x]^2)*Sqrt[(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)/(
Sqrt[a] + Sqrt[c]*Tan[d + e*x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[c])*c^(1/4
)*(a - b + c)*e*Sqrt[a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1329

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[f^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^(
m - 2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^p, x], x] - Dist[d*e*(f^2/(c*d^2
 - b*d*e + a*e^2)), Int[(f*x)^(m - 2)*((a + b*x^2 + c*x^4)^(p + 1)/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
p, -1] && GtQ[m, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+bx^2+cx^4)^{3/2}}\,dx, x, \tan(d+ex)\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{a+cx^2}{(a+bx^2+cx^4)^{3/2}}\,dx, x, \tan(d+ex)\right)}{(a-b+c)e} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{(a-b+c)e}$$

$$= \frac{\tan(d+ex)\left(b^2-2ac-bc+(b-2c)c\tan^2(d+ex)\right)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{\sqrt{a}\text{Subst}\left(\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(1+x^2)\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\left(\sqrt{a}-\sqrt{c}\right)(a-b+c)e} + \frac{\sqrt{c}\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\left(\sqrt{a}-\sqrt{c}\right)(a-b+c)e} - \frac{\text{Subst}\left(\int \frac{a(2a-b)c+a(b-2c)cx^2}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{a(a-b+c)\left(b^2-4ac\right)e}$$

$$= -\frac{\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2(a-b+c)^{3/2}e} + \frac{\tan(d+ex)\left(b^2-2ac-bc+(b-2c)c\tan^2(d+ex)\right)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} + \frac{\sqrt[4]{c}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)+}{\left(\sqrt{a}+\sqrt{c}\tan^2\right.}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)(a-b+c)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}} - \frac{\left(\sqrt{a}+\sqrt{c}\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}}, 2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}(a-b+c)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+e}} - \frac{\left(\left(\sqrt{a}-\sqrt{c}\right)\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{\left(b-2\sqrt{a}\sqrt{c}\right)(a-b+c)e} + \frac{\left(\sqrt{a}(b-2c)\sqrt{c}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx, x, \tan(d+ex)\right)}{(a-b+c)\left(b^2-4ac\right)e}$$

$$= -\frac{\arctan\left(\frac{\sqrt{a-b+c}\tan(d+ex)}{\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}\right)}{2(a-b+c)^{3/2}e}$$
$$+ \frac{\tan(d+ex)\left(b^2-2ac-bc+(b-2c)c\tan^2(d+ex)\right)}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$- \frac{(b-2c)\sqrt{c}\tan(d+ex)\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}{(a-b+c)\left(b^2-4ac\right)e\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)}$$
$$+ \frac{\sqrt[4]{a}(b-2c)\sqrt[4]{c}E\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+}{(\sqrt{a}+\sqrt{}}}}{(a-b+c)\left(b^2-4ac\right)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$+ \frac{\sqrt[4]{c}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{\frac{a+b\tan^2(d+ex)}{(\sqrt{a}+\sqrt{c}\tan}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)(a-b+c)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$- \frac{\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d+ex)\right)\sqrt{}}{2\sqrt[4]{a}\left(b-2\sqrt{a}\sqrt{c}\right)(a-b+c)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+ex)}}$$
$$- \frac{\left(\sqrt{a}+\sqrt{c}\right)\mathrm{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}},2\arctan\left(\frac{\sqrt[4]{c}\tan(d+ex)}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}+\sqrt{c}\tan^2(d-\right.}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{c}\right)\sqrt[4]{c}(a-b+c)e\sqrt{a+b\tan^2(d+ex)+c\tan^4(d+}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 19.77 (sec) , antiderivative size = 831, normalized size of antiderivative = 0.85

$$\int \frac{\tan^2(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx = \frac{\sqrt{\frac{3a+b+3c+4a\cos(2(d+ex))-4c\cos(2(d+ex))+a\cos(4(d+ex))-b\cos(4(d+ex}{3+4\cos(2(d+ex))+\cos(4(d+ex))}}}{}$$
$$+ \frac{i\sqrt{2}\left((b-2c)\left(-b+\sqrt{b^2-4ac}\right)E\left(i\,\mathrm{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\tan(d+ex)\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+\left(b^2-b\sqrt{b^2-4ac}+2c\left(-2a+\sqrt{b^2-4ac}\right)\right)\mathrm{EllipticF}\left(i\mathrm{ar}\right.\right.}{}$$

```
[In] Integrate[Tan[d + e*x]^2/(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4)^(3/2),x]
```

```
[Out] (Sqrt[(3*a + b + 3*c + 4*a*Cos[2*(d + e*x)] - 4*c*Cos[2*(d + e*x)] + a*Cos[
4*(d + e*x)] - b*Cos[4*(d + e*x)] + c*Cos[4*(d + e*x)])/(3 + 4*Cos[2*(d + e
*x)] + Cos[4*(d + e*x)])]*(((b - 2*c)*Sin[2*(d + e*x)])/(2*(-a + b - c)*(b^
2 - 4*a*c)) + (2*b^2*Sin[2*(d + e*x)] - 4*a*c*Sin[2*(d + e*x)] - 4*c^2*Sin[
2*(d + e*x)] + b^2*Sin[4*(d + e*x)] - 2*a*c*Sin[4*(d + e*x)] - 2*b*c*Sin[4*
(d + e*x)] + 2*c^2*Sin[4*(d + e*x)])/((a - b + c)*(-b^2 + 4*a*c)*(-3*a - b
- 3*c - 4*a*Cos[2*(d + e*x)] + 4*c*Cos[2*(d + e*x)] - a*Cos[4*(d + e*x)] +
```

```
b*Cos[4*(d + e*x)] - c*Cos[4*(d + e*x)]))))/e + ((I*Sqrt[2]*((b - 2*c)*(-b
+ Sqrt[b^2 - 4*a*c])*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a
*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (b^
2 - b*Sqrt[b^2 - 4*a*c] + 2*c*(-2*a + Sqrt[b^2 - 4*a*c]))*EllipticF[I*ArcSi
nh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Tan[d + e*x]], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])] - 2*(b^2 - 4*a*c)*EllipticPi[(b + Sqrt[b^2
- 4*a*c])/(2*c), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Tan[d +
e*x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*Tan[d + e*x]^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*Tan[
d + e*x]^2)/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])] - (4*
(b - 2*c)*Tan[d + e*x]*(a + b*Tan[d + e*x]^2 + c*Tan[d + e*x]^4))/(1 + Tan[
d + e*x]^2))/(4*(a - b + c)*(-b^2 + 4*a*c)*e*Sqrt[a + b*Tan[d + e*x]^2 + c*
Tan[d + e*x]^4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3597 vs. 2(995) = 1990.

Time = 0.60 (sec) , antiderivative size = 3598, normalized size of antiderivative = 3.67

method	result	size
derivativedivides	Expression too large to display	3598
default	Expression too large to display	3598

```
[In] int(tan(e*x+d)^2/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/e*(-2*c*(1/2/a*b/(4*a*c-b^2)*tan(e*x+d)^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c
*tan(e*x+d))/((tan(e*x+d)^4+b/c*tan(e*x+d)^2+a/c)*c)^(1/2)+1/4*(1/a-(2*a*c-
b^2)/a/(4*a*c-b^2))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*
a*c+b^2)^(1/2))/a*tan(e*x+d)^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*tan(e*x
+d)^2)^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*EllipticF(1/2*tan(e*x+
d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1
/2))/a/c)^(1/2))-1/2*b/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*tan(e*x+d)^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)
^(1/2))/a*tan(e*x+d)^2)^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)/(b+(-
4*a*c+b^2)^(1/2))*(EllipticF(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*tan
(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^
2)^(1/2))/a/c)^(1/2)))+2*c*(1/2*(2*a*c-b^2+b*c)/a/(4*a*c-b^2)/(a-b+c)*tan(e
*x+d)^3+1/2*(3*a*b*c-2*a*c^2-b^3+b^2*c)/a/(4*a*c-b^2)/(a-b+c)/c*tan(e*x+d))
/((tan(e*x+d)^4+b/c*tan(e*x+d)^2+a/c)*c)^(1/2)+1/4*2^(1/2)/(-1/a*b+1/a*(-4*
a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2-2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(
1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2+2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2
)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*EllipticF(1/2*tan(e*x+d)*2^(1/2)*
```

((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(
1/2))/a/(a-b+c)*b-1/4*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*
b*tan(e*x+d)^2-2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+
d)^2+2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x
+d)^4)^(1/2)*EllipticF(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))/a/(a-b+c)*c-3/4*2^(1/2)
/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2-2/a*tan(e*x+d)
^2*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2+2/a*tan(e*x+d)^2*(-4*a*c
+b^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*EllipticF(1/2*ta
n(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b
^2)^(1/2))/a/c)^(1/2))/(4*a*c-b^2)/(a-b+c)*b*c+1/2*2^(1/2)/(-1/a*b+1/a*(-4*
a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2-2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(
1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2+2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2
)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*EllipticF(1/2*tan(e*x+d)*2^(1/2)*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(
1/2))/(4*a*c-b^2)/(a-b+c)*c^2+1/4*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(
1/2)*(4+2/a*b*tan(e*x+d)^2-2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/
a*b*tan(e*x+d)^2+2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)
^2+c*tan(e*x+d)^4)^(1/2)*EllipticF(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))/a/(4*a*c-b^
2)/(a-b+c)*b^3-1/4*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*t
an(e*x+d)^2-2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^
2+2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)
^4)^(1/2)*EllipticF(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))/a*b^2/(4*a*c-b^2)*c/(a-b+c
)+c^2/(a-b+c)/(4*a*c-b^2)*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+
2/a*b*tan(e*x+d)^2-2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(
e*x+d)^2+2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan
(e*x+d)^4)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*tan(e*x+d)*2^(1/2)*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/
2))*a-1/2*c/(a-b+c)/(4*a*c-b^2)*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(1/
2)*(4+2/a*b*tan(e*x+d)^2-2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*
b*tan(e*x+d)^2+2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+b*tan(e*x+d)^2
+c*tan(e*x+d)^4)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*tan(e*x+d)*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/
c)^(1/2))*b^2+1/2*c^2/(a-b+c)/(4*a*c-b^2)*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^
(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2-2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/
2)*(4+2/a*b*tan(e*x+d)^2+2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+b*ta
n(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*tan(e
*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)
^(1/2))/a/c)^(1/2))*b-c^2/(a-b+c)/(4*a*c-b^2)*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b
^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2-2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))
^(1/2)*(4+2/a*b*tan(e*x+d)^2+2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/2)/(a+
b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/2*t
an(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+

```
b^2)^(1/2))/a/c)^(1/2))*a+1/2*c/(a-b+c)/(4*a*c-b^2)*2^(1/2)/(-1/a*b+1/a*(-4
*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2-2/a*tan(e*x+d)^2*(-4*a*c+b^2)^
(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2+2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1/2))^(1/
2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE
(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-
4*a*c+b^2)^(1/2))/a/c)^(1/2))*b^2-1/2*c^2/(a-b+c)/(4*a*c-b^2)*2^(1/2)/(-1/a
*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2-2/a*tan(e*x+d)^2*(-4
*a*c+b^2)^(1/2))^(1/2)*(4+2/a*b*tan(e*x+d)^2+2/a*tan(e*x+d)^2*(-4*a*c+b^2)^
(1/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
*EllipticE(1/2*tan(e*x+d)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4
+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*b-1/(a-b+c)*2^(1/2)/(-1/a*b+1/a*(-4
*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*tan(e*x+d)^2-1/2/a*tan(e*x+d)^2*(-4*a*c+b
^2)^(1/2))^(1/2)*(1+1/2/a*b*tan(e*x+d)^2+1/2/a*tan(e*x+d)^2*(-4*a*c+b^2)^(1
/2))^(1/2)/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(1/2)*EllipticPi(1/2*tan(e*x+d
)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a,(-
1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\tan^2(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^2/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\tan^2(d+ex)}{\left(a+b\tan^2(d+ex)+c\tan^4(d+ex)\right)^{3/2}}\,dx = \int \frac{\tan^2\left(d+ex\right)}{\left(a+b\tan^2\left(d+ex\right)+c\tan^4\left(d+ex\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(e*x+d)**2/(a+b*tan(e*x+d)**2+c*tan(e*x+d)**4)**(3/2),x)
```

```
[Out] Integral(tan(d + e*x)**2/(a + b*tan(d + e*x)**2 + c*tan(d + e*x)**4)**(3/2)
, x)
```

Maxima [F]

$$\int \frac{\tan^2(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \int \frac{\tan(ex+d)^2}{\left(c\tan(ex+d)^4+b\tan(ex+d)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(e*x+d)^2/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(tan(e*x + d)^2/(c*tan(e*x + d)^4 + b*tan(e*x + d)^2 + a)^(3/2), x
)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^2(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(e*x+d)^2/(a+b*tan(e*x+d)^2+c*tan(e*x+d)^4)^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(d+ex)}{(a+b\tan^2(d+ex)+c\tan^4(d+ex))^{3/2}}\,dx = \int \frac{\tan(d+e\,x)^2}{\left(c\tan(d+e\,x)^4+b\tan(d+e\,x)^2+a\right)^{3/2}}\,dx$$

```
[In] int(tan(d + e*x)^2/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(3/2),x)
```

```
[Out] int(tan(d + e*x)^2/(a + b*tan(d + e*x)^2 + c*tan(d + e*x)^4)^(3/2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```